Tenta 2006-03-25

1a) $\dfrac{6x^3y+4xy}{2xy} = \dfrac{2\cancel{xy}(3x^2+2)}{2\cancel{xy}} = 3x^2+2$

b) $\dfrac{c-1}{1-c} = \dfrac{-\cancel{(1-c)}}{\cancel{1-c}} = -1$

c) $\dfrac{a^4b^2-25}{a^2b+5} = \dfrac{\cancel{(a^2b+5)}(a^2b-5)}{\cancel{a^2b+5}} = a^2b-5$

2 a) $f(x) = x^4+2x^3-7x+1 \Rightarrow f'(x) = 4x^3+6x^2-7$

b) $f(x) = x^{1,2}+e^{3x} \Rightarrow f'(x) = 1,2x^{0,2}+3e^{3x}$

c) $f(x) = 4\sqrt{x} \Rightarrow f'(x) = \dfrac{4}{2\sqrt{x}} = \dfrac{2}{\sqrt{x}}$

3a) $\dfrac{x^7}{5} = 78 \Leftrightarrow x^7 = 78\cdot 5 = 390 \quad x = 390^{1/7} \approx 2,345$

b) $11^x - 20 = 0 \Leftrightarrow 11^x = 20 \Rightarrow \ln 11^x = \ln 20$

$\Leftrightarrow$

$x \ln 11 = \ln 20$

$\Leftrightarrow$

$x = \dfrac{\ln 20}{\ln 11} \approx 1,249$

3c) $\frac{1}{x} = x+1 \iff \frac{1}{x} = \frac{x^2+x}{x} \iff 0 = \frac{x^2+x-1}{x}$

$\iff 0 = x^2+x-1 = \left(x+\frac{1}{2}\right)^2 - \frac{1}{4} - 1 = \left(x+\frac{1}{2}\right)^2 - \left(\frac{\sqrt{5}}{2}\right)^2 =$

$= \left(x+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\left(x+\frac{1}{2}-\frac{\sqrt{5}}{2}\right)$

Alltså $x = -\frac{1}{2} + \frac{\sqrt{5}}{2}$ el. $x = -\frac{1}{2} - \frac{\sqrt{5}}{2}$

$\approx 0{,}618$ $\qquad \approx -1{,}618$

4a) $y = 2+2x-x^2 \Rightarrow y' = 2-2x$

Tangentens lutning i $x=4$:

$y'(4) = 2-2\cdot 4 = 2-8 = -6$

Tangentens ekvation fås vi enpunktsformeln:

$y - y(4) = -6(x-4)$ $\quad (y(4) = 2+2\cdot 4 - 4^2 = -6)$

$\iff$

$y+6 = -6x+24$

$\iff$

$\underline{\underline{y = -6x+18}}$

b) $y = x^2 - 5x + 3 \Rightarrow y' = 2x - 5$

Eftersom tangenten ska vara parallell med $y = x - 2$ så måste vi hitta de x så att $y' = 1$, dvs

$$2x - 5 = 1 \Leftrightarrow x = 3 \Rightarrow y(3) = 3^2 - 5 \cdot 3 + 3 = -3$$

Enpunktsformeln ger att

$$y - (-3) = 1 \cdot (x - 3)$$

$$\Leftrightarrow$$

$$y = x - 3 - 3 = \underline{\underline{x - 6}}$$

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5 a) $f(x) = 4$: $\underline{\underline{x = -2 \wedge x = 1}}$

b) $f'(x) = 0$: $\underline{\underline{x = -2 \wedge x = 0}}$

c) $f'(x) < 0$: ~~x = -2 ∧ x = 0~~

$\underline{\underline{-2 < x < 0}}$

6a) $a_1 = 7 \quad a_3 = 12$ aritmetisk talföljd

$a_{17} = ?$

$$12 = a_3 = a_1 + 2d \iff 12 = 7 + 2d$$

$$\; \iff d = \frac{5}{2}.$$

$$a_1 + 2d = 7 + 2d$$

Detta ger att

$$a_{17} = 7 + \frac{5}{2} \cdot 16 = \underline{\underline{47}}$$

b) $a_1 = 3$, $a_2 = 12$ geometrisk talföljd

Beräkna S_7. Vi har att $a_n = c \cdot r^{n-1}$

så

$$3 = a_1 = c \cdot r^0 = c \iff c = 3$$

och

$$12 = a_2 = 3 \cdot r \iff r = 4$$

Alltså $a_n = 3 \cdot 4^{n-1}$, så

$$S_7 = 3 \cdot \frac{1 - 4^7}{1 - 4} = \underline{\underline{16383}}$$

7 a) $N(t) = 500e^{0,2t}$

$N(0) = 500e^{0,2\cdot 0} = 500$

b) $N(t) = \cancel{}\ 1660$

$\Leftrightarrow$

$500 \cdot e^{0,2t} = 1660$

$\Leftrightarrow$

$e^{0,2t} = \frac{1660}{500}$

$\Rightarrow$

$\ln e^{0,2t} = \ln\left(\frac{1660}{500}\right)$

$\Leftrightarrow$

$0,2t = \ln\left(\frac{1660}{500}\right)$

$\Leftrightarrow$

$t = \frac{\ln\left(\frac{1660}{500}\right)}{0,2} \approx \underline{\underline{6\text{ h}}}$

c) $N'(t) = 500 \cdot 0,2\, e^{0,2t} = 100e^{0,2t}$

$N'(2) = 100e^{0,2\cdot 2} = 100e^{0,4} \approx 149$ bakterier/h.

8) $f(x) = x^4 - 2x^3 \Rightarrow f'(x) = 4x^3 - 6x^2$

Kritiska punkter: $f'(x) = 0 \Leftrightarrow 0 = 4x^3 - 6x^2 = 2x^2(2x-3)$

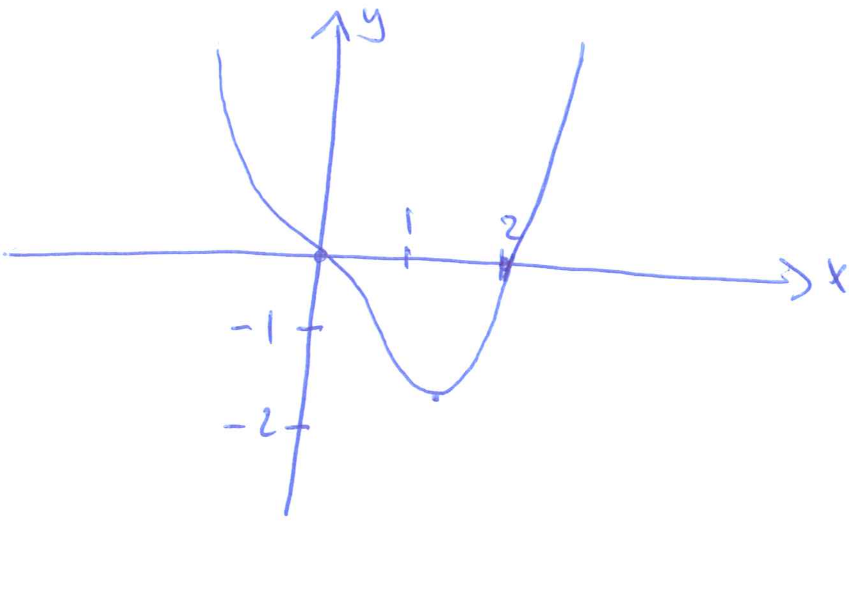
så $x = 0$ el $2x - 3 = 0 \Leftrightarrow x = 3/2$

Teckenschema:

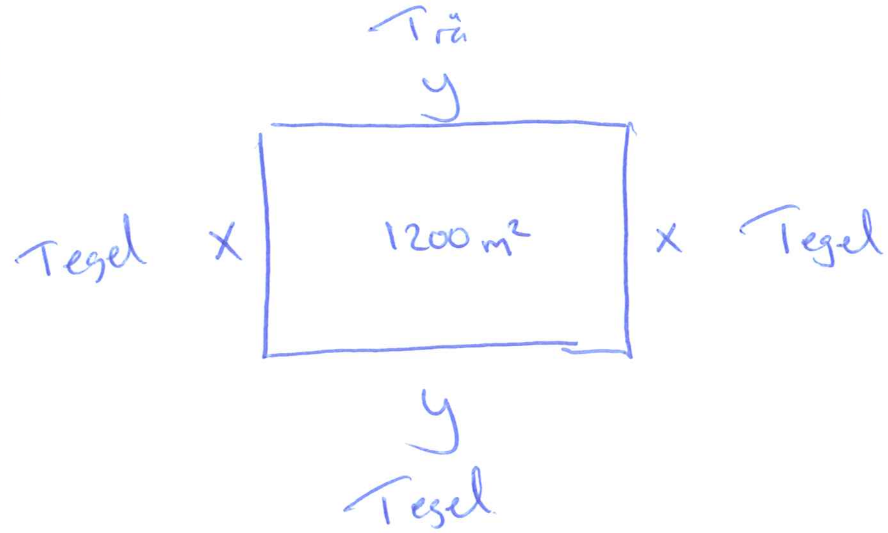
x		0		$3/2$	
$f'(x)$	$-$	0	$-$	0	$+$
$f(x)$	$\searrow$	$\rightarrow$	$\searrow$	$\rightarrow$	$\nearrow$
		Terasspunkt		lokal minpunkt	

$f(x) = 0 \Leftrightarrow x^3(x-2) = 0 \Leftrightarrow x = 0$ el $x = 2$

$f(0) = 0 \qquad f(3/2) \approx -1,69$



a)



$x \cdot y = 1200 \iff y = \frac{1200}{x}$

Kostnad: $K(x) = 400x + 400y + 400x + 200y$

$= 800x + 600y = 800x + \frac{720000}{x}$

$\uparrow$

$y = \frac{1200}{x}$

Derivering ger att

$K'(x) = 800 - \frac{720000}{x^2}$

Kritiska punkter ges av $K'(x) = 0$, dvs

$0 = 800 - \frac{720000}{x^2} = \frac{800x^2 - 720000}{x^2}$

$\iff$

$800x^2 - 720000 = 0 \iff x^2 = 900 \iff x = (\pm)\, 30$

Om $x = 30 \Rightarrow y = 40$.

30x40 m ska ~~[illegible]~~ trädgården vara. En sida som är 40 m ska vara i trä.

10)

$$\frac{1+2+3+\dots+n}{1+2+3+\dots+100} = \frac{1703}{1010}$$

Aritmetiska summor:

$$\frac{\frac{n(1+n)}{2}}{\frac{100(1+100)}{2}} = \frac{1703}{1010}$$

$$\Longleftrightarrow$$

$$\frac{n(1+n)}{100\cdot 101} = \frac{1703}{1010}$$

$$\Longleftrightarrow$$

$$n(1+n) = \frac{17200300}{1010} = 17030$$

$$\Longleftrightarrow$$

$$0 = n^2+n-17030 = \left(n+\frac{1}{2}\right)^2 - \frac{1}{4} - 17030 =$$

$$= \left(n+\frac{1}{2}\right)^2 - \frac{68121}{4} = \left(n+\frac{1}{2}\right)^2 - \left(\frac{261}{2}\right)^2 =$$

$$= \left(n+\frac{1}{2}+\frac{261}{2}\right)\left(n+\frac{1}{2}-\frac{261}{2}\right) = (n+131)(n-130)$$

Alltså $n=130$ och $n=-131$, men $n>0$ så $n=130$ är lösningen.