Packet Combined ARQ Scheme Utilizing Unitary Transformation in Multiple Antenna Transmission

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***Abstract*— In this paper, the objective is to increase the throughput of space-time codes (STC) utilizing automatic repeat request (ARQ) scheme over multiple-input multiple-output (MIMO transmission). The proposal is to use unitary transformation prior to STC to enhance the performance. The transformation is taken from a set of finite predetermined matrices and changes upon retransmission request. Simulation results show a significant performance gain can be obtained by employing unitary transformation prior to a system without transformation. For instance, at a FER=10^{-2} we gain approximately 2 dB by employing unitary transformation.**

I. Introduction

Demands for capacity in wireless communications, driven by cellular mobile, Internet and multimedia services have been rapidly increasing worldwide. It has been demonstrated in literature that the use of *Multiple-Input Multiple-Output* (MIMO) channels can offer significant capacity gain over a traditional single-input single-output (SISO) channel. Space-time coding (STC) achieves bandwidth efficiency through the use of an efficient way of combining forward error correction and diversity transmission to overcome the impairments of wireless channels. There are various approaches in coding structures, including *space-time block codes* (STBC), *space-time trellis codes* (STTC) and *layered space-time* (LST) codes.

In packet based systems, packet retransmission is usually employed when a received packet is erroneously decoded. It is well known that introducing packet combining into an ARQ scheme can improve the throughput remarkably. In [3], Chase introduced a packet combining scheme where the same symbol is transmitted upon a repeat request and soft decision statistics from all retransmissions are combined. In [4], Harvey *et al.* proposed a version of packet combining where L copies of the data packet are combined into a single packet of the same length as the original transmitted data packet by averaging the soft decision values from the consecutive copies.

Employing packet retransmissions in MIMO systems is a relative new research area. In [5], Nguyen and Ingram investigated hybrid ARQ protocols for systems that uses recursive space-time codes. In [6], Onggosanusi *et al.* investigated the possibilities to enhance the efficiency of HARQ in MIMO systems by employing either a zero-forcing (ZF) or minimum mean-square error (MMSE) receiver before (pre-combining) and after (post-combining) the interference-resistant detection. Their result showed that a pre-combining scheme outperform a post-combining scheme. Zheng *et al.* proposed in [7] a new ARQ scheme for MIMO systems where substreams emitted from various transmit antennas encounter different error statistics. By using per-antenna encoders, separating the ARQ process among the substreams, they obtained some throughput improvements. In [8], Gidlund proposed an ARQ scheme for multi-level modulation in MIMO-systems. The rationale with that scheme was that they changed the bit mapping in every retransmission and achieved significant diversity gain.

In this paper, we evaluate the performance of Packet Combined ARQ scheme over a MIMO system. The proposed scheme introduces an unitary transformation prior to the encoder, in order to create an artificially diversity. The transformation is taken from a set of predetermined matrices and changes upon request. Computer simulations confirm that the ARQ scheme can overcome the throughput degradation which is produced by the fading channel. The rest of the paper is organized as follows: The system model is described in Section II. The ARQ scheme and the unitary transformation is described in Section III, the numerical results is presented in Section IV and finally we conclude the work in Section V.

Notation: Column vectors (matrices) are denoted by boldface lower (upper) case letters. Superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ stand for transpose, conjugate, Hermitian transpose, respectively. We will use $\mathbf{I}_N$ to denote the $N \times N$ identity matrix, $Tr(\mathbf{\Theta})$ the trace of $\mathbf{\Theta}$ and superscript ^{c} denotes complex quantities.

II. Preliminaries

In this paper, we consider a frequency-flat fading M-transmit N-receive multiple-input multiple-output (MIMO) channel according to Fig. (1). Let us focus on the transmission of the current information message. The message is encoded by a space-time encoder, and mapped into a sequence of L matrices, $\{\mathbf{X}_l^c \in \mathbb{C}^{M\times T} \ : \ l = 1, \cdots, L\}$. The complex baseband model of our channel is defined as

$$\mathbf{y}_{l,t}^c = \sqrt{\frac{\rho}{M}}\mathbf{H}_l^c \mathbf{x}_{l,t}^c + \mathbf{w}_{l,t}^c, \tag{1}$$

where the index $l = 1, 2, \cdots,$ counts the protocol rounds and $t = 1, \cdots, T$ counts the channel uses in each block, $\{\mathbf{x}_{l,t}^c \in$

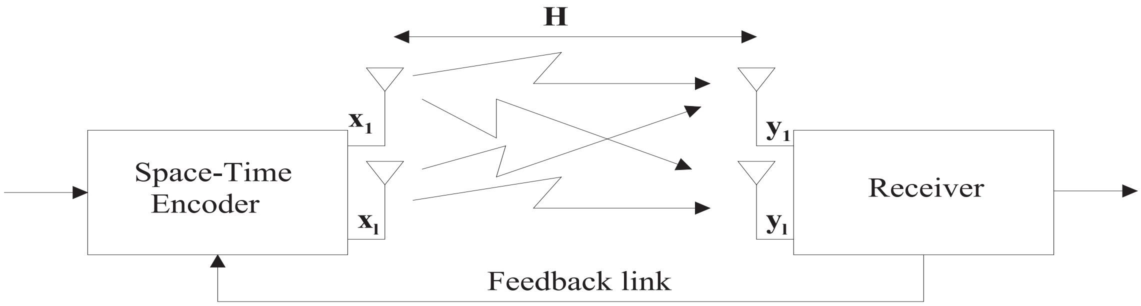

Fig. 1. System model

$\mathbb{C}^M : t = 1, \cdots, T\}$ are the columns of the l-th block $\mathbf{X}_l^c$, $\{\mathbf{w}_{l,t}^c \in \mathbb{C}^M : t = 1, \cdots, T\}$ and $\{\mathbf{y}_{l,t}^c \in \mathbb{C}^M : t = 1, \cdots, T\}$ denote the channel noise and the corresponding received signal block, respectively. The channel noise is assumed to be temporally and spatially white with i.i.d. entries $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$. The channel in l-th round is characterized by the matrix $\mathbf{H}_l^c \in \mathbb{C}^{N \times M}$ with the (i, j)-th element $h_{ij,l}^c$ representing the fading coefficient between the j-th transmit and the i-th receive antenna. The fading coefficients are assumed to be i.i.d. $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$ and remain fixed over each block. For simplicity, we rewrite (1) to the equivalent channel model after l transmissions rounds and the total received signal is given by

$$\mathbf{y}_l = \sqrt{\frac{\rho}{M}} \mathbf{H}_l \mathbf{x} + \mathbf{w}_l, \tag{2}$$

where ρ is the total signal-to-noise ratio of the number of transmit antennas. Furthermore, the vector $\mathbf{y}_l = [y_0 y_1 \cdots, y_{L-1}]^T \in \mathbb{R}^{2NTl}$ represents the signal received overall transmitted blocks from 1 to l. We define $\mathbf{x} = (\mathbf{x}_{1,1}^T, \cdots, \mathbf{x}_{1,1}^T, \cdots, \mathbf{x}_{L,1}^T, \cdots, \mathbf{x}_{L,T}^T)^T$ with $\mathbf{x}_{l,t}^T = [\text{Re}\{\mathbf{x}_{l,t}^c\}^T, \text{Im}\{\mathbf{x}_{l,t}^c\}^T]^T$, and $\mathbf{w} = (\mathbf{w}_{1,1}^T, \cdots, \mathbf{w}_{1,1}^T, \cdots, \mathbf{w}_{L,1}^T, \cdots, \mathbf{w}_{L,T}^T)^T$ with $\mathbf{w}_{l,t}^T = [\text{Re}\{\mathbf{w}_{l,t}^c\}^T, \text{Im}\{\mathbf{w}_{l,t}^c\}^T]^T$.

The channel matrix $\mathbf{H}_l$ has dimensions $2NTl \times 2MTL$, and is formed by talking the first $2NTl$ rows of the matrix

$$\begin{aligned} \mathbf{H}_l \;=\; & \sqrt{\frac{\rho}{M}} \,\text{diag}\left(\mathbf{I}_T \otimes \begin{bmatrix} \text{Re}\{\mathbf{H}_1^c\} & \text{-Im}\{\mathbf{H}_1^c\} \\ \text{Im}\{\mathbf{H}_1^c\} & \text{Re}\{\mathbf{H}_1^c\} \end{bmatrix}, \cdots, \right. \\ & \left. \mathbf{I}_T \otimes \begin{bmatrix} \text{Re}\{\mathbf{H}_L^c\} & \text{-Im}\{\mathbf{H}_L^c\} \\ \text{Im}\{\mathbf{H}_L^c\} & \text{Re}\{\mathbf{H}_L^c\} \end{bmatrix}\right) \end{aligned} \tag{3}$$

which is composed by L diagonal blocks. Each block has also a block-diagonal form, with T diagonal blocks equal to the $2N \times 2M$ real expansion of the complex channel matrix $\mathbf{H}_l$. Notice that for $l < L$ the matrix $\mathbf{H}_l$ can be partitioned into two blocks. The leftmost $2NTl \times 2MTl$ block is block-diagonal while the rightmost $2NTl \times 2MT(l - L)$ block is zero. This is due to the fact that at round l the blocks $\mathbf{X}_{l+1}, \cdots, \mathbf{X}_L$ have not been transmitted yet and in the model they appear as multiplied by a zero channel matrix.

Under these conditions and assuming $\mathbf{H}$ perfectly known at the receiver, the optimal detector $g : \mathbf{r} \mapsto \hat{\mathbf{x}} \in \mathcal{S}$ that minimizes the average error probability

$$P(e) \triangleq P(\hat{\mathbf{x}} \neq \mathbf{x})$$

is the maximum-likelihood (ML) detector (for L transmissions) given by

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \mathcal{S}^N} \sum_{l=1}^{L} ||\mathbf{y}(l) - \sqrt{\frac{\rho}{N}} \mathbf{H}(l) \mathbf{x}||^2. \tag{4}$$

The complexity of maximum likelihood detector in (4) is high (proportional to $\mathcal{D}^N$, where $\mathcal{D}$ is the modulation order), so other signal separation techniques such as MMSE detector with ordered interference cancellation where a symbol with the highest SNR is detected by using a linear MMSE filter, and then subtracted from the received signals. This procedure is repeated until all transmitted symbols are detected as follows [9],[10]:

1) $\bar{\mathbf{H}} = \mathbf{H}$
2) for $i = 1 : M$ do
3) $\mathbf{\Xi} = \left(\frac{\rho}{M} \bar{\mathbf{H}}^H \bar{\mathbf{H}} + \mathbf{I}\right)^{-1}$ (MMSE Criterion)
4) $k_i = \arg\min\{\Xi_{j,j}\}$
5) $\mathbf{w} = (\bar{\mathbf{H}} \mathbf{\Xi})(:, k_i)$
6) $z_{k_i} = \mathbf{w}^H \mathbf{y}$ (Nulling operation)
7) $\hat{x}_k = \mathcal{Q}(z_{k_i})$
8) $\mathbf{y} = \mathbf{y} - \sqrt{\frac{\rho}{N}} \mathbf{H}(:, k_i) \hat{x}_k$ (Cancellation operation)
9) $\bar{\mathbf{H}} = \mathbf{H}_{k_i}$

III. Hybrid ARQ and Packet combining

We consider the following ARQ protocol. The transmitter has an infinite buffer of information messages to send. The information message to be transmitted is encoded by a space-time encoder and then mapped into a sequence. If successful decoding is achieved, a positive acknowledgement signal (ACK) is sent back to the transmitter whereas a negative acknowledgement (NACK) signal is sent in case of detection of decoding failure. Upon reception of the ACK, the transmitter sends the first block of the next message in the buffer whereas the reception of the NACK triggers the transmission of the next block of the current message. The only exception to the above rule is when the maximum number of protocols rounds, L, is reached. In this case, a NACK bit will be interpreted as an error, the current message is removed from the transmitter buffer and the transmission of next message is started anyway.

Error in the system occur either when the decoder makes a decoding error at retransmission $l < L$ and it fails to detect it (undetected error event) or when the decoder makes a decoding error at retransmission L. For simplicity we consider that the retransmissions are made on the same antennas as the previous one.

When the MMSE detection algorithm described earlier is employed, $z(l)$ is defined as the decision static corresponding to the ith symbol s_i and the lth transmission. The combined decision can now be expressed as

$$z_i = \sum_{l=1}^{L} w_i(l) z_i(l). \quad (5)$$

In this paper we consider maximum ratio combining (MRC), and the combining weight $w_i(l)$ is proportional to the signal-to-noise ratio, i.e. $w_i(l) = \frac{1}{\Xi_{k_i,k_i}}$. The combining can easily be modified, for instance, if equal gain combining is considered the combining weight is set to $w_i(l) = 1$ for all $l = 1, 2, ..., L$.

For slow fading channels, the signal and interference components remain approximately constant ($\mathbf{H}_1 = \mathbf{H}_2 = \cdots = \mathbf{H}_l$) upon retransmission which limits the ARQ gain. To increase the ARQ gain we consider to use a precoder to create an artificially diversity. One method is to effectively employ a Vandermode matrix as precoder in both SISO and MIMO systems. Other precoding options is to consider a permutation matrix, which shuffles the label-transmit antenna assignments for each transmissions. A second option is an FFT (or IFFT) matrix, which is both unitary and Vandermonde. It spreads the symbol energy evenly among the L transmit antennas so that the effect of any deep fades (i.e. small values in $\mathbf{H}_l$) are alleviated.

In this paper, we introduce the use of unitary transformation prior $\mathbf{\Theta}$ to each transmission. A unitary $\mathbf{\Theta}$ corresponds to a rotation and preserves the distance among the M-dimensional constellation points. On the contrary, a nonunitary $\mathbf{\Theta}$ draws some pairs of constellation points closer (and some farther). This distance preserving property of unitary transformation also guarantees that if such rotated constellations are to be used over an AWGN channel, performance will remain invariant. In practice, the channel condition can also vary between the two extremes of AWGN and Rayleigh fading, in which case a unitary precoder may be preferred [14]. The symbol s is multiplied by a $P \times P$ unitary matrix $\mathbf{\Theta}$ [15]. In this paper, the unitary transformation $\mathbf{\Theta}$ is for simplicity taken from a set of finite predetermined matrices $\mathcal{S} = \{\mathbf{\Theta}_0, \mathbf{\Theta}_1, ..., \mathbf{\Theta}_{L-1}\}$ instead of using an exhausted search. Observe that $\mathbf{\Theta}$ must be *unitary* to avoid any increase in the transmitter power. Upon retransmission request l, different matrix is chosen from the set $\mathcal{S}$. Given a certain ordering of the set elements, a certain matrix "hopping" pattern can be chosen. This method artificially introduces diversity in quasi-static channels. A simple hopping pattern can be given as

$$\Psi(n) = mod(\left\lfloor \frac{n}{2} \right\rfloor, N), \quad n = 0, 1, ... \quad (6)$$

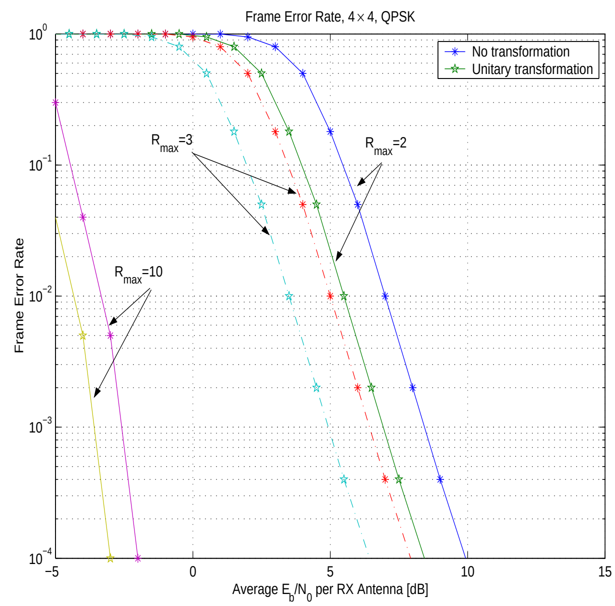

Fig. 2. Residual FER versus average E_b/N_0 in Rayleigh fading channel with different numbers of retransmissions. QPSK and $4Tx \times 4Rx$ antennas are considered. Number of retransmissions is limited to $R_{max} = 10$.

where the selected matrix is $\mathbf{\Theta}_{\Psi(n)}$ and $N = 6$. For a given set of cardinality N, the elements of matrix set $\mathcal{S}$ can be chosen to maximize the performance at each soft packet combining stage. The following matrix sets are used:

$$\mathbf{\Theta}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{\Theta}_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$
$$\mathbf{\Theta}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{\Theta}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$
$$\mathbf{\Theta}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{\Theta}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (7)$$

The chosen transformation for the n-th retransmission can now be defined as $\Theta(n) = \Theta_{\Psi(n)}$. Observe that the transformation changes upon every NACK with the hopping pattern in (6). The received signal can be determined as

$$\mathbf{r} = \sqrt{\frac{\rho}{N}} \begin{bmatrix} \mathbf{H}\mathbf{\Theta}_{\Psi(1)} \\ \vdots \\ \mathbf{H}\mathbf{\Theta}_{\Psi(N)} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{n}_{(1)} \\ \vdots \\ \mathbf{n}_{(N)} \end{bmatrix} \quad (8)$$

When unitary transformation is employed, the MMSE criterion can be expressed as $\mathbf{\Xi} = \left(\frac{\rho}{M}\mathbf{\Theta}^H\bar{\mathbf{H}}^H\bar{\mathbf{H}}\mathbf{\Theta} + \mathbf{I}\right)^{-1}$.

IV. Numerical Results

To verify the performance, we have simulated a MIMO system with $M = 4$ transmit and $N = 4$ receive antennas signaling over a quasi-static flat fading channel with quadrature phase shift keying (QPSK) modulation. For simplicity, ACK and NACK control signals are assumed to returned error free to the sender where the feedback channel is assumed to be zero-delay. The retransmission interval is set to two frame lengths to ensure that high temporal correlations between initial transmission and retransmissions. The data is encoded with a $R_c = 1/2$ convolutional code (133,171) with a constraint length 7, and minimum code distance 10. Furthermore, we consider the number of repetitions including the initial transmission to be limited to $L_{max} = 10$.

Figure 2 shows the residual frame error rate (FER) performance as a function of average E_b/N_0 per RX antenna in quasi-static channel. The figure plots the FER performance with limitation of maximum transmission times L_{max}, which is the residual FER after error detection, retransmission, and combining of the transmitted frames. When employing unitary transformation, a performance gain of 2.2 dB can be achieved between 2nd and 3rd transmission.

V. Conclusion

In this paper, we have investigated the performance of employing packet combined ARQ schemes over a MIMO architecture. Prior to the space-time coding, an unitary transformation take place to artificially create diversity in an quasi-static channel. The transformation is taken from a finite set of predetermined set of matrices and is changed upon retransmission requests. The obtained simulation results show that employing ARQ with unitary transformation can improve the performance significantly. It should also be mentioned that the proposed scheme does not introduce to much complexity into the system.

References

[1] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp. 744-765, March 1998.

[2] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Area Commun.,* vol. 16, no. 8, pp. 1451-1458, October 1998.

[3] D. Chase, "Code combining - a maximum-likelihood decoding approach for combining an arbitrary number of noisy packets," *IEEE Trans. Commun.*, vol. COM-33, no. 5, pp. 385-393, May 1985.

[4] B. A. Harvey and S. B. Wicker, "Packet combining systems based on the Viterbi decoder," *IEEE Trans. Commun.*, vol. 42, pp. 1544-1557, Feb./Mar./Apr. 1994.

[5] A. V. Nguyen and M. A. Ingram, "Hybrid ARQ protocols using Space-Time Codes,"In *Proc. IEEE VTC'01-fall*, Atlantic City, NJ, USA, Oct. 2001.

[6] E. Onggosanusi, A. Dabak, Y. Hui, and G. Geong, "Improving Hybrid ARQ transmission and combining for Multiple-Input Multiple-Output systems," In *Proc. IEEE ICC'03*, Vol. 4, Anchorage, Alaska, USA, May 2003.

[7] H. Zheng, A. Lozano and M. Haleem, "Multiple ARQ processes for MIMO systems," In *Proc. IEEE PIMRC'02*, Bejing, China, 2002.

[8] M. Gidlund, "An improved ARQ scheme with application for Multi-Level Modulation in MIMO-systems," In *Proc. ISITA'04,* Parma, Italy, Oct. 2004.

[9] G. D. Golden, G. J. Foschini, R. A. Valenzuela, and P. W. Wolniansky, "Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture," *IEE Electronics Letters,* 35(1):14-15, 1999.

[10] I. Berenguer and X. Wang, "Space-time coding and signal processing for MIMO communications," *Journal of Computer Science and Technologym* 18(6):689-702, 2003.

[11] J. G. Proakis, *Digital Communications,* 2nd edition, New York: McGraw-Hill, 1989.

[12] D. Rainish, "Diversity transform for fading channels," *IEEE Trans. Commun.*, vol. 44, pp. 1653-1661, Dec. 1996.

[13] V. M. DaSilva and E. S. Sousa, "Fading-resistant modulation using several transmitter antennas," *IEEE Trans. Commun.*, vol. 45, pp. 1236-1244, Oct. 1997.

[14] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: information-theoretic and communications aspects," *IEEE Transaction of Information Theory,* vol. 44, pp. 2619-2692, Oct. 1998.

[15] X. Li, Z. Wang, and G. B. Giannakis, "Space-time constellation-rotating codes maximizing diversity and coding gains," in *Proc. IEEE GLOBECOM'01*, Dec 2001.

[16] G. J. Foschini and M. J Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, Vol. 6, No. 3, March 1998.