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An Improved ARQ Scheme with Application for Multi-Level Modulation Techniques in MIMO-Systems

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Abstract

In this paper paper we investigate the performance of employing an advanced ARQ scheme for multi-level modulation (MLM-ARQ) in a multi-input multi-output (MIMO) environment. This is realized though rearranging the symbols between different transmissions and using the modulation level M as an extra dimension to improve the quality of the signal and reduce the number of retransmissions per packet. The obtained simulation results show that the proposed scheme outperforms a scheme employing Chase combining.

1. INTRODUCTION

It has been demonstrated in literature that the use of MIMO can offer significant capacity gain over a traditional single-input single-output (SISO) channel. The MIMO systems are today regarded as one of the most promising research areas of wireless communications. To achieve high-quality or error-free transmission for multimedia services in wireless networks it is necessarily to adopt some powerful error control techniques.

In packet based systems, packet retransmission is usually employed when a received packet is erroneously decoded. Pure ARQ systems are easy to implement but lead to variable delays which are not acceptable for some real-time applications. It is well known that introducing packet combining into an ARQ scheme can improve the throughput remarkably. In [1], Chase introduced a packet combining scheme where the individual transmissions are encoded at some code rate R. If the receiver has P packets that have been requested for retransmission, the packet are concatenated to form a single packet of lower rate code of rate R/P. In [2], Harvey et al. proposed a version of packet combining where L copies of the data packet are combined into a single packet of the same length as the original transmitted data packet by averaging the soft decision values from the consecutive copies.

Employing packet retransmissions in MIMO systems is a relative new research area. In [3], Nguyen and Ingram investigated hybrid ARQ protocols for systems that uses recursive space-time codes. In [5], Onggosanusi et al. investigated the possibilities to enhance the efficiency of HARQ in MIMO systems by employing either a zero-forcing or MMSE receiver before (precombining) and after (post-combining) the interference-resistant detection. Their result showed that a precombining scheme outperform a post-combining scheme. Zheng et al. proposed in [4] a new ARQ scheme for MIMO systems where substreams emitted from various transmit antennas encounter different error statistics. By using per-antenna encoders, separating the ARQ process among the substreams, they obtained some throughput improvements.

In this paper we present an advanced ARQ scheme for multi-level modulation techniques such as MPAM, MPSK and MQAM in MIMO systems employing sphere decoding. Usually multi-level modulation is bandwidth efficient but lacks of robustness in noisy channels. Our objective is to improve the robustness by implementing a multi-level modulation ARQ scheme for MIMO systems. In multi-level modulation we have a total of M signal points in the signal space. These M points provide a new dimension to a system that uses packet (or symbol) retransmission. The idea is to design an ARQ scheme that uses the modulation level M as an extra dimension in improving the quality of the signal, and thus reduce the number of retransmissions per packet. By considering the modulation level and number of retransmissions as an augmented signal space and using packet combining as a way of utilizing this new signal space we can increase the robustness of the system in case of retransmission. In this paper we show that by selecting the signal constellation of retransmissions in

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such a way that the resultant signals are well spread in the signal space the gain in terms of throughput is significant.

The paper is organized as follows. In Section 2 we present the system model under consideration. In Section 3 we present the proposed ARQ scheme and also derive a general upper BER bound for the scheme. In Section 4 we present some numerical results and finally in Section 5 we conclude the work.

2. SYSTEM MODEL

We consider an M-transmit, N-receiver antenna system with L transmissions of the packet and for the l^{th} transmission, the receiver obtains

$$\mathbf{y}_l = \mathbf{H}_l \mathbf{s} + \mathbf{n}_l, \tag{1}$$

where $\mathbf{y}, \mathbf{n} \in \mathbb{C}^m$, $\mathbf{s} \in \mathbb{Z}^N[j]$, $\mathbf{H} \in \mathbb{C}^{M \times N}$ has full rank, $M \geq N$, $\mathbb{C}$ and $\mathbb{Z}$ denote the sets of integers and complex numbers; and $\mathbb{Z}[j] := \{a + jb | a, b \in \mathbb{Z}\}$.

In a wireless communication context, $\mathbf{s}, \mathbf{y}$ and $\mathbf{n}$ are the transmitted, received and the additive white Gaussian noise (AWGN) vectors, whereas $\mathbf{H}$ contains the channel coefficients. The distribution of $\mathbf{n}$ is $\mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$, where $\mathcal{CN}(\cdot, \cdot)$ represents the complex Gaussian distribution; and $\mathbf{H}$ is a random matrix often with known statistical properties. Furthermore, instead of the whole integer lattice $\mathbb{Z}^N[j]$, $\mathbf{s}$ is usually drawn from the finite alphabet $\mathcal{S}^N \subset \mathbb{Z}^N[j]$. Under these conditions and assuming $\mathbf{H}$ perfectly known at the receiver, the optimal detector $g : \mathbf{y} \mapsto \hat{\mathbf{s}} \in \mathcal{S}$ that minimizes the average error probability

$$P(e) \triangleq P(\hat{\mathbf{s}} \neq \mathbf{s})$$

is the maximum-likelihood (ML) detector given by

$$\zeta(\hat{\mathbf{s}}) = \arg \min_{\mathbf{s} \in \mathcal{S}} \sum_{l=1}^{L} ||\mathbf{y}_l - \mathbf{H}_l \mathbf{s}||^2. \tag{2}$$

For simplicity, we assume that $\mathcal{S} = \chi^m$, where χ is a pulse amplitude modulation (PAM) signal set of size Q, i.e.,

$$\chi = \{u = 2q - Q + 1 : q \in \mathbb{Z}_Q\} \tag{3}$$

with $\mathbb{Z}_Q \triangleq \{0, 1, ..., Q-1\}$. Under the assumption (3), by applying suitable translation and scaling of the received signal vector, (2) takes on the *normalized* form

$$\zeta(\hat{\mathbf{s}}) = \arg \min_{\mathbf{s} \in \mathbb{Z}_Q^m} \sum_{l=1}^{L} ||\mathbf{y}_l - \mathbf{H}_l \mathbf{s}||^2 \tag{4}$$

where the components of the noise $\mathbf{n}$ have a common variance equal to 1.

Developing efficient sphere decoders to solve the approximate (4) has recently gained renewed attention, mainly because of their applications in MIMO systems. The sphere decoder solves

$$\min_{s \in \Lambda} (\mathbf{s} - \hat{\mathbf{s}})^T H^T H (\mathbf{s} - \hat{\mathbf{s}}), \tag{5}$$

where $\hat{\mathbf{s}}$ and $\mathbf{H}$ are complex, $(\cdot)^H$ denotes the conjugate transpose and Λ is a complex lattice in the sense that each coordinate of $\mathbf{s}$ is chosen from a complex constellation. The complex search is

$$(\mathbf{s} - \hat{\mathbf{s}})^H H^H H (\mathbf{s} - \hat{\mathbf{s}}) \leq r^2. \tag{6}$$

Let $\mathbf{U}_l$ be an upper-triangular matrix that satisfies $\mathbf{U}_l^H \mathbf{U}_l = \mathbf{H}_l^H \mathbf{H}_l$. This is typically accomplished with a Cholesky decomposition. If the elements of $\mathbf{U}_l$ are indicated using $u_{l,ij}$, the metric becomes

$$\sum_{i=1}^{N} \sum_{j=1}^{M} u_{ii,j}^2 \left| s_i - \hat{s}_i + \sum_{j=i+1}^{N} \frac{u_{ij}}{u_{ii}} (s_i - \hat{s}_i) \right|^2 \leq r^2. \tag{7}$$

To apply sphere decoding to our multiple transmission scenario, we define a hypersphere of radius r_ψ, with a modification of the proposed rule by Hochwald et al. in [7]

$$r_\psi^2 = \sum_{l=1}^{L} 2\sigma^2 K \psi - \mathbf{y}^T (\mathbf{I} - \mathbf{H}(\mathbf{H})^T \mathbf{H}^{-1} \mathbf{H}^T) \mathbf{y}. \tag{8}$$

The parameter ψ is chosen to ensure that the hypersphere contains some candidates. From the metric (7), we focus on the label s_ψ by looking only at the $i = \psi$ term of summation. A necessary condition for s to lie inside the sphere is therefore that

$$\sum_{l=1}^{L} u_{l,\psi\psi}^2 |s_{l,\psi} - \hat{s}_{l,\psi}|^2.$$

This condition is equivalent to s_m belonging to the interval

$$\left\lceil \hat{s}_{l,\psi} - \frac{r}{u_{l,\psi\psi}} \right\rceil \leq s_{l,\psi\psi} \left\lfloor \hat{s}_{l,\psi} + \frac{r}{u_{l,\psi\psi}} \right\rfloor. \tag{9}$$

3. PROPOSED ARQ SCHEME

A simplistic way to utilize HARQ and MIMO is to apply a single-antenna HARQ scheme for each data stream. This setup, however, does not fully exploit the characteristic of MIMO channel. We assume that the same symbol vector $\mathbf{s}$ is transmitted L times due to $(L-1)$ repeat requests. This scheme requires that the receiver extracts the substreams independently. Now suppose that the decoder produces a retransmission request for that packet. At the next transmission time, there are k=rank($\mathbf{H}$) subchannels available for the retransmission. In this paper we consider that the retransmission is on the same subchannel as the previous transmission.

Our proposed ARQ scheme employs the technique of rearranging the bit mapping in retransmissions which enables the scheme to even out the error probabilities and improve the performance. The rationale with this proposed scheme is that through proper interaction between retransmission and signal mapping we can obtain a better spreading of the modulation signal points within the space. This better spread of signal points, if done in terms of maximizing the Euclidean distance between the different signal points, can improve the error probability and reduce the number of retransmissions [8]. There are several techniques to arrange this remapping procedure but most of them are suboptimal, i.e., they do not distribute the signal points in the signal space in an optimum way. Observe that the proposed scheme do not need any information about previous retransmissions except an retransmission request.

3.1. Cyclic shifting of Symbol bits

One simple procedure in generating different signal mapping sets for packet retransmission is cyclic shifting the bits of the original signal mapping set. By doing a cyclically shift we will obtain a resulting signal constellation Ω which is better spread over the two-dimensional signal space than the signal constellation Chase combining give. Cyclic shifting achieves a small performance gain compared to Chase combining but does not take the used modulation scheme into account, which indicates that a better signal constellation can be achieved by careful design.

3.2. Proposed Mapping Scheme

Our aim is to find a signal constellation Ω after L retransmissions such that the signal symbols are far better spread in the signal space than with Chase combining and cyclic shifting. By spreading the signal points in a nearly optimum fashion, the signal points s_i in the first retransmission are spread so that they maximize the Euclidean distance δ. To map the signal points of the second transmission, we will consider both set partition and permutation of symbols. The fundamental idea of set partitioning is to group the points of the signal constellation Ω into sub-sets to achieve the maximum Euclidean distance between the points. First we define the squared Euclidean distance within the set as the smallest squared distance between any pair of distinct points:

$$\delta \stackrel{\text{def}}{=} min\left\{\|s_i - s_j\|^2 : \ s_i, s_j \in \Omega, \ i \neq j\right\}. \quad (10)$$

Let $k = 1, 2, ..., k$ represent the partition level, and let $\Omega_j^{(k)} = \omega_L$, be the j^{th} subset in the k^{th} partition level where $\Omega^{(0)} = \Omega$. Also let $M_j^{(k)}$ be the size of Ω_j^k and $M^0 = \sum_{i=1}^{L} M^{(j)}$. Then δ_j^k is the minimum distance within the subset $\Omega_j^{(k)}$. We assume a set size of $M = s^m$ for simplicity, where m is an integer. The principle of set partitioning is to partition the constellation into a series of subsets of diminishing size

$$\begin{aligned}\Omega_j^{(k-1)} &= \bigcup_{i=1,\dots,s} \Omega_{(j-1)s+i}^{(k)}, \quad j = 1, 1, , ..., Ks \\ M^{(k)} &= \frac{1}{s} M^{(k-1)} \end{aligned} \quad (11)$$

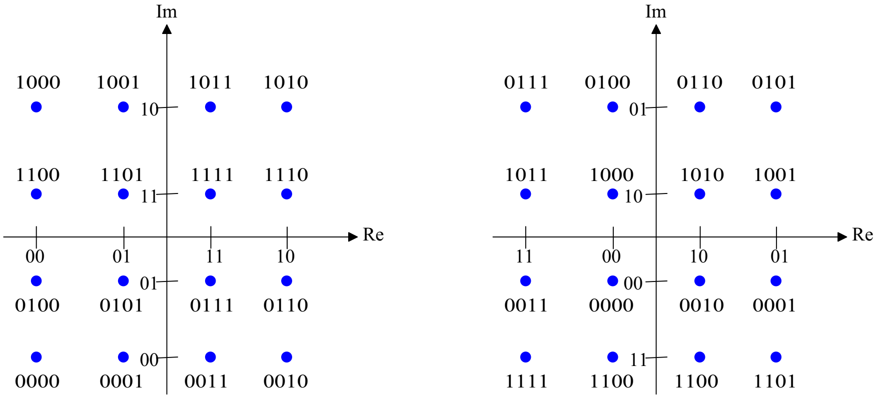
in such way that the minimum Euclidean distance within the subsets increases while increasing the partition level, that is

$$\delta^{(0)} < \delta^{(1)} < \cdots < \delta^{(K)}. \quad (12)$$

The iteration steps can be continued until each subset only contains one signal point. We define π to be a permutation matrix, which is a square matrix of zeros and ones with the property that every row, and column has a single one. The permuting process take place to ensure that the signal points are mapped in such manner that minimum δ is maximal. We can summarize the remapping algorithm as follows:

1. Initialization: Set $j = 0$ and
$\Omega^{(0,0)} = \left\{\omega_l^{(0,0)} = s_l\right\}_{l=0,1,\dots,s^{m-1}}$

2. Partitioning:
for $k = 1, 2, ..., m$
for $j = 0, 1, ..., 2^{k-1} - 1$
$\Omega^{(k,2j)} = \left\{\omega_l^{(k,2j)} = \omega_{2l}^{(k-1,j)}\right\}_{l=0,1,\cdots,2^{m-k}-1}$
$\Omega^{(k,2j+1)} = \left\{\omega_l^{(k,2j+1)} = \omega_{2l+1}^{(k-1,j)}\right\}_{l=0,1,\cdots,2^{m-k}-1}$
(This process continues until we have M subsets and each subset has only one point:
$\Omega^{(M,0)}, \Omega^{(M,1)}, ..., \Omega^{(m,M-1)}$ which we simply denote as $\Omega_0, \Omega_1, ..., \Omega_{M-1}$)



(a) Signal constellation for first transmission.

(b) Signal constellation for second transmission (1st retransmission).

Figure 1: Signal constellations for different transmissions.

3. Permuting π: $\Omega_0, \Omega_1 \cdots \Omega_{M-1} \rightarrow \Omega_{\frac{M}{2}}, \Omega_{\frac{M}{2}+1}, \cdots, \Omega_{M-1}, \Omega_0, \Omega_1 \cdots \Omega_{\frac{M}{2}-1}$

In Figure 1 we can see the difference in the signal constellation between the first and the second transmission. The Euclidean distance is clearly larger than before and we achieve a diversity gain. In the first transmission we use Gray mapping of the packet and natural mapping in the first retransmission. When the same modulation scheme is used, the only difference appears in the mapping of the data bits into the signal points.

3.3. General BER Upper Bound for Independent Fading Channel

In this section we will derive an upper bound in order to minimize the BER. We denote ϖ as the set of real or complex numbers that represent the points s_i in the signal constellation Ω, we also recall that ζ is the minimization metric defined in (4). First, we derive a BER expression for multiple transmissions. The general error expression is

$$Pr\{Error\} = \sum_{s=0}^{|\varpi|-1} Pr\{s\} Pr\{\hat{s} \neq s | s\}. \qquad (13)$$

The function $Pr\{s\}$ denotes the a priori probability that label s is transmitted. To obtain the error detection probability $Pr\{\hat{s} \neq s|s\}$, we apply the union bound which states that

$$Pr\{\hat{s} \neq s | s\} \leq \sum_{\hat{s}=0}^{|\varpi|-1} P(\mathbf{s} \rightarrow \hat{\mathbf{s}}). \qquad (14)$$

This allows us to deal with the pairwise error probability (PEP) $P(\mathbf{s} \rightarrow \hat{\mathbf{s}})$, which is the probability that label $\hat{s}$ is detected when s is transmitted. Then (13) becomes upper bounded by

$$Pr\{Error\} \leq \sum_{s=0}^{|\varpi|-1} \sum_{\substack{\hat{s}=0 \\ s \neq s}}^{|\varpi|-1} Pr\{s\} P(\mathbf{s} \rightarrow \hat{\mathbf{s}}). \qquad (15)$$

To obtain the BER bound, we need to account for the bit errors that result from a label misdetection. We define a function $\xi[s, \hat{s}]$ as the number of differing bits between s and $\hat{s}$, divided by $\log_2 |\zeta|$.

Including $\xi[s, \hat{s}]$ and (14) in (13) we obtain a BER upper bound of

$$\sum_{s=0}^{|\varpi|-1} \sum_{\substack{\hat{s}=0 \\ \hat{s} \neq s}}^{|\varpi|-1} Pr\{s\} \xi[s, \hat{s}] P(\mathbf{s} \rightarrow \hat{\mathbf{s}}). \qquad (16)$$

For independent fading channels, the PEP for L transmissions is obtained as

$$\mathrm{P}(\mathrm{s} \rightarrow \hat{\mathbf{s}}) =$$

$$= P\left(\sum_{l=1}^{L} \{||\mathbf{y}_l - \mathbf{H}_l \hat{\mathbf{s}}_l||^2 - ||\mathbf{y}_l - \mathbf{H}_l \hat{\mathbf{s}}_l||^2 < 0\}\right) =$$

$$= P\left(\sum_{l=1}^{L} \{||\mathbf{H}_l(\mathbf{s}_l - \hat{\mathbf{s}}_l) + \mathbf{n}_l||^2 - ||\mathbf{n}_l||^2 < 0\}\right) =$$

$$= Q\left(\sqrt{\frac{1}{2N_0}\sum_{l=1}^{L}||\mathbf{H}_l(\mathbf{s}_l - \hat{\mathbf{s}}_l) + \mathbf{n}_l||^2}\right).$$

where $Q(.)$ is the Gaussian tail function which is defined as

$$Q(x) = (2\pi)^{-1/2}\int_x^{\infty} exp(-x^2/2)dx. \quad (17)$$

Substituting (17) into (16) gives us the BER upper bound.

4. NUMERICAL RESULTS

To asses the performance of the proposed scheme in MIMO environments, we simulated a 2×2 MIMO system using 16 QAM modulation. We consider a packet size of 1000 bits and the channels $\mathbf{H}_1$ and $\mathbf{H}_2$ are independent, and remain constant through the transmission of the packet. The limit of number of retransmission are set to 10.

Figure 2 depicts the average bit-error rate (BER) vs bit SNR (E_b/N_0) for a 2×2 MIMO system using 16 QAM modulation. We can clearly see the performance gain offered when employing the proposed ARQ scheme as compared to a scheme employing Chase combining. After one retransmission (M=2) and at a BER value of 10^{-2} we gain approximately 2.5 dB when employing the proposed scheme. This gain is obtained by the fact that the signal points are far better spread in the signal space than with the Chase combining scheme. The performance gain then becomes less for every retransmission.

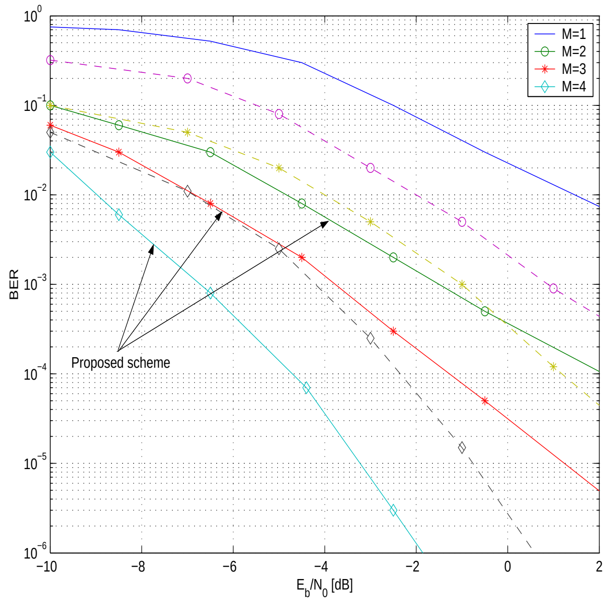


Figure 2: Simulation results for a 2×2 MIMO system using 16 QAM modulation. The solid lines show the proposed scheme while the dotted line represents the Chase combined scheme.

5. CONCLUSIONS

In this paper we proposed an improved ARQ scheme which is suitable for multi-level modulation in a MIMO system. The rationale behind this new scheme is that the signal constellations is rearranged in every retransmission in order to improve the quality of the signal and reduce the number of retransmissions. We demonstrate that the proposed scheme outperforms a scheme employing Chase combining. The complexity is the same for both scheme since there is no need for bigger buffers with the proposed scheme.

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