Retransmission Diversity Schemes for Multicarrier Modulations

Mikael Gidlund

Radio Communication Systems LAB, Department of Signals, Sensors and Systems

Royal Institute of Technology (KTH), 164 40 Kista, Stockholm

Email: mikael@ii.uib.no

Abstract

In this paper, we present some simple techniques for enhancing the diversity provided by retransmissions in multi-carrier modulation. The primary technique of interest are symbol interleaving and symbol mapping diversity for retransmissions. Symbol interleaving involves retransmitting packet symbols through different subcarriers of the same channel, while symbol mapping diversity involves adapting the bit-to-symbol mapping for each retransmission. An analysis of the ability of interleaving to produce lower bit error rates is provided. Also, a discussion of optimally adapting the mappings and their application to OFDM is presented. Simulation results validate the efficiency of these methods in reducing BER and increasing throughput.

I. Introduction

High data rate wireless access systems are currently under discussion since the demand for wireless multimedia communication is rapidly increasing due to strong advances in Internet services. In such systems severe degradation is caused by by the inter-symbol interference (ISI) generated by multipath propagation in the wireless channel. Orthogonal frequency division multiplexing (OFDM) is a promising technique to combat ISI even when the delay spread is large compared to the symbol duration [1]. OFDM is a type of multi-carrier transmission which splits the nominal frequency band into a suitable number of subcarriers, each modulated with a low modulation rate. Furthermore, the OFDM signal allows us to insert adequate guard intervals between successive OFDM symbols which mitigates the effect of ISI.

In packet based systems, packet retransmission is usually employed when a received packet is erroneously decoded. It is well known that introducing packet combining into an ARQ scheme can improve the throughput remarkably. In [2], Chase introduced a packet combining scheme where the same symbol is transmitted upon a repeat request and soft decision statistics from all retransmissions are combined. In [3], Harvey *et al.* proposed a version of packet combining where L copies of the data packet are combined into a single packet of the same length as the original transmitted data packet by averaging the soft decision values from the consecutive copies.

Packet combining for multi-carrier modulation has also been considered previously. In [4], Kumagi et al. presented a maximal ratio combining frequency diversity ARQ scheme for OFDM systems, where at every new retransmission, the different symbols of the OFDM block are transmitted on different subcarriers (frequency interleaving), and then maximum ratio combining with the previous versions of each packet is performed followed by a detection attempt until the packet is accepted or ignored after a certain number if retransmissions. The advantage of the proposed method is that one can take advantage of the frequency variations of the radio channel and we add frequency diversity to the time diversity. The drawback is that it requires the identity of the subcarrier symbol since at every retransmission the symbol is transmitted on a different subcarrier. In [5], Gidlund et al. proposed a hybrid ARQ scheme which take advantage of changing the bit interleaving mode is such manner that the data sequence is changed. Since the coded bit is assigned to different subcarriers and positions of a modulation symbols in different retransmissions we can exploit both frequency- and time diversity effect. Wengerter et al. presented HARQ methods that employ code combining and adapting among a set of Gray mappings for retransmissions [6].

The purpose of this paper is to extend previous ARQ combining methods to multi-carrier methods. To enhance the diversity among the L retransmissions, we interleave the symbols within a packet and using different bit-to-symbol mapping. The interleaving process allows for symbols that were initially transmitted over poor subchannels to be later transmitted over better subchannels. By adapting the mapping for retransmissions increases the average Euclidean distance between any two labels, thereby significantly reducing the bit error rate.

The paper is organized as follows: Section II describes the system model. In Section III we derive an upper BER bound which is used to select the mapping for each (re)transmission. In Section IV, symbol interleaving for OFDM is discussed. In Section V numerical results is provided and finally in Section VI we conclude the work.

II. System Model

Consider the system shown in Fig. 1, where a packet consisting of N symbols $\psi(s_1), \dots, \psi(s_N)$ is transmitted using OFDM through a frequency-selective channel. The mapping ψ maps a group of $\log_2 |\mathcal{C}|$ bits, denoted by s_n, to a symbol in the constellation $\mathcal{C}$. we refer to a group of $\log_2 |\mathcal{C}|$ bits as an label. Herein, it is assumed that the constellation $\mathcal{C}$ has unit energy. The channel is modeled as an FIR filter v with K independent coefficients $v_1, \dots, v_K$. Succintly stated, OFDM uses an N-point IFFT and a cyclig prefix at the transmitter with an N-point FFT at the receiver to transform the channel into a set of parallel, flat subchannels. The gains, $h_1, \dots, h_N$ of the particular subchannels correspond to the FFT of the channel response. Across the subchannels, the gains follow a Rayleigh distribution.

Let L denote the number of total transmissions of a given packet, all trough the same frequency-selective channel. To enhance the diversity provided by the retransmissions, we propose interleaving the symbols within a packet and using a different bit-to-symbol mapping. The interleaving process allows for symbols that were initially transmitted over poor subchannels to be later transmitted over better subchannels. Moreover, symbols that initially went through good subchannels can afford to experience poor subchannels in subsequent transmissoions. Effectively, over all transmissions, interleaving provides a label with a Rayleigh fading channel rather than a fixed AWGN-only channel. Adapting the mapping for retransmissions increases the average Euclidean distance between any two labels, thereby significantly reducing the bit error rate.

The entire packet of symbols is uniquely interleaved and m,apped for the l-th transmission using the interleaver $\pi_l : 1, \ldots, N \rightarrow 1, \ldots, N$ and mapping $\psi_l : 0, \ldots, \mathcal{C} \rightarrow \mathcal{C}$. Consequently, symbols $\psi_1(s_{\pi_1[n]}), \ldots, \psi_L(s_{\pi_L[n]})$ are sent over the n-th subchannel over the L transmissions. By default, $\pi_1[n] = n$. After deinterleaving, the receiver obtains $y_n^l = h_{\pi_l^{-1}} \psi_l(s_n) + v_{\pi_l^{-1}[n]^{(l)}}$ for detection of the n-th symbol of the packet. the deinterleaver is specified by π_l^{-1}. The noise $v_n^{(l)}$ is assumed white Gaussian with zero mean and variance σ_v^2. Demapping is performed using the maximum likelihood rule

$$\zeta(\hat{s}) = \min_{0,1,\cdots,M-1} \sum_{i=1}^{L} |y_n^l - h_{\pi_l^{-1}} \psi_l(s_n)|^2. \tag{1}$$

III. Applying Mapping Diversity

One way to obtain good mappings is to choose the mappings that minimize the BER of the system. To be able to use that kind of method, an expression for the BER of the diversity scheme with L transmissions is required. Although, it is difficult to obtain a exact BER expression for this scheme, an upper bound on the BER can be used to obtain the mappings. The union bound, using the metric $\zeta_L(s)$ defined in (1) states that [7]

$$Pr\{\hat{s} \neq s|s\} \leq \sum_{k=0}^{M-1} Pr\{\zeta_L(k) < \zeta_L(s)|s\}.$$

Assuming independence of the Gaussian noise variable n_i, the pairwise error probability (PEP) of the transmitted symbol s is being decoded as symbol k can be described as follows:

$$P(\zeta(s) \rightarrow \zeta(k)) = Q\left(\sqrt{\frac{1}{4\sigma^2} \sum_{i=1}^{L=1} \mathcal{D}^2[\psi_i(s), \psi_i(k)]}\right) \tag{2}$$

where $\zeta_L(s)$ is the minimization metric given in (1), $\mathcal{D}[a, b]$ is the Euclidean distance between points a and b. The Q-function is defined as

$$Q(x) = \frac{1}{2\pi} \int_x^{\infty} e^{-t^2/2} dt.$$

To be able to determine the BER upper bound we denote the variable $\chi(s, k)$ as function that accounts for the number of bit errors caused by the block error. Including (2) and $\chi(s, k)$ we can express the upper BER bound as following [5]

$$P_b(L) \leq \frac{1}{M} \sum_{a=0}^{M-1} \sum_{\substack{b=0 \\ b \neq a}}^{M-1} \chi(s,k) Q \left(\sqrt{\frac{1}{4\sigma^2} \sum_{i=1}^{L=1} \mathcal{D}^2[\psi_i(a), \psi_i(b)]} \right). \tag{3}$$

Our problem is to determine the L optimal symbol mappings $\psi_1(s) = \psi_2(s) = \cdots = \psi_L(s)$, which minimize the BER upper bound in (3). This optimization is stated as

$$\min_{\psi_L \in \Psi} \frac{1}{M} \sum_{s=0}^{M-1} \sum_{\substack{k=0 \\ \neq s}}^{M-1} \chi(s,k) Q \left(\sqrt{\frac{1}{4\sigma^2} \sum_{i=1}^{L=1} \mathcal{D}^2[\psi_i(s), \psi_i(k)]} \right), \tag{4}$$

with Ψ denoting the set of all possible mappings. This minimization will become a massive combinatorial optimization problem whose solution space contains $(M!)^L$ solutions. To overcome this problem, a simpler sub-optimal iterative solution can be used by computing the Lth mapping from the previous $L-1$ mappings, where the optimization problems simplifies to

$$\min_{\psi_L \in \Psi} \frac{1}{M} \sum_{s=0}^{M-1} \sum_{\substack{k=0 \\ k \neq s}}^{M-1} g[s, \psi_L(s), \psi_L(k)], \tag{5}$$

with Ψ denoting the set of all possible mappings and $g[s,a,k,b]$ is the pairwise BER that results from mapping label s to symbol $a \in \mathcal{C}$ and label k to symbol $b \in \mathcal{C}$ in the Lth mapping,

$$g[s,a,k,b] = Pr\{s\} B[s,k] Pr\{\delta < 0\}.$$

A. OFDM

For OFDM transmissions, the PEP for any label is defined as

$$E_{\mathbf{f}} \left\{ Q \left(\sqrt{\frac{1}{2\sigma_v^2} \sum_{m=1}^{M} |f_m|^2 |d_m[s,k]|^2} \right) \right\}. \tag{6}$$

where $f_m = h_{\pi_m^{-1}}, d_m[s,k] = \gamma_m[s] - \gamma_m[k]$ and $\mathbf{f} = \{f_1, \ldots, f_2\}$. The variable f_m represents the Rayleigh fading gain of the m^{th} transmission of a label. The expectation over $\mathbf{f}$ is necessary since these fading gains are not known to the transmitter.

For optimization posed in (4),

$$\delta = (\alpha_{M-1}[k] - \alpha_{M-1}[s]) + |y_n^{(M)} - f_M b|^2 - |y_n^{(M)} - f_M a|^2,$$

leading to a PEP $Pr\{\delta < 0\}$ of

$$E_{\mathbf{f}} \left\{ Q \left(\sqrt{\frac{1}{2\sigma_v^2} \left(\sum_{m=1}^{M} |f_m|^2 |d_m[s,k]|^2 + |f_M|^2 |a-b|^2 \right)} \right) \right\}. \tag{7}$$

This PEP can be numerically computed to provide the function $g[s,a,k,b]$ to solve (4), to producing the optimal mappings for fading channels.

IV. OFDM Symbol Interleaving

In this section, we intend to show how symbol interleaving provides improved performance. Intuitively, a label (or symbol) experiences a Rayleigh fading channel instead of a possibly bad AWGN channel, over all transmissions. Alternative, one can view symbol interleaving as the equivalent of the channel varying for each retransmission.

Our goal is to illustrate that symbol interleaving provides a lower BER. We intend to show that upper bound in (3) is smaller than interleaving. Proving that the PEP with interleaving is less than the PEP without interleaving is sufficient to accomplish this. The PEP with interleaving is found in (7), while the PEP without interleaving is

$$E_{f_1} = \left\{ Q\left(\sqrt{\frac{1}{2\sigma_v^2}|f_1|^2 \sum_{m=1}^{M} |d_m[s,k]|^2} \right) \right\}. \tag{8}$$

The objective is to show the following PEP relation

$$E_{\epsilon_I}\{Q\sqrt{\epsilon_I}\} < E_{\epsilon_{NI}}\{Q\sqrt{\epsilon_{NI}}\} \tag{9}$$

where

$$\epsilon_I = \sum_{m=1}^{M} |f_m|^2 a_m, \quad \epsilon_{NI} = |f_1|^2 \sum_{m=1}^{M} a_m,$$

and $a_m = |d_m[s,k]|^2/2\sigma_v^2$ come from (7) and (8). These are stated as using an alternative formulation of the $Q(\cdot)$-function

$$E_\epsilon\{Q(\sqrt{\epsilon})\} = \frac{1}{\pi}\int_0^{\pi/2} \phi_\epsilon\left(\frac{1}{2\sin^2\theta}\right)$$

where

$$\phi_{\epsilon_I}(x) = E\{\exp(-x\epsilon)\}$$

is the moment generating function (MGF) of the random variable ϵ. The MGF with symbol interleaving is

$$\phi_\epsilon(x) = \prod_{m=1}^{M} \exp\left[\frac{-x\mu_h^2 a_m}{1+x\sigma_h^2 a_m}\right] \frac{1}{1+x\sigma_h^2 a_m},$$

while the MGF without symbol interleaving is expressed as

$$\phi_{\epsilon_{NI}}(x) = \exp\left[\frac{-x\mu_h^2\sum_{m=1}^{M} a_m}{1+x\sigma_h^2\sum_{m=1}^{M} a_m}\right] \frac{1}{1+x\sigma_h^2\sum_{m=1}^{M} a_m}.$$

Both of the MGF expressions are for Rician fading, and thus are also valid for Rayleigh fading. To show 9, it is sufficient to prove that

$$\phi_\epsilon\left(\frac{1}{2\sin^2\theta}\right) < \phi_{\epsilon_{NI}}\left(\frac{1}{2\sin^2\theta}\right), \quad 0 \le \theta \le \frac{\pi}{2}.$$

or equivalently

$$\phi_{\epsilon_I}(x) < \phi_{\epsilon_{NI}}(x), \quad \frac{1}{2} \leq x \leq \infty. \tag{10}$$

Herein, it is assumed that σ_h^2 and $a_1, \ldots, a_M$ are positive real numbers, and $M > 1$.

The inequality in 10 can be decomposed into two separate inequalities. First,

$$\prod_{m=1}^{M} \frac{1}{1 + x\sigma_h^2 a_m} < \frac{1}{1 + x\sigma_h^2 \sum_{m=1}^{M} a_m} \tag{11}$$

is simple to demonstrate by expanding the left-hand side to show that

$$\prod_{m=1}^{M} \frac{1}{1 + x\sigma_h^2 a_m} = \frac{1}{1 + 1 + x\sigma_h^2 \sum_{m=1}^{M} a_m + \sum_{m=2}^{M} c_m x^m}$$

where all c_m are positive. The second inequality,

$$\prod_{m=1}^{M} \exp\left[\frac{-x\mu_h^2 a_m}{1 + x\sigma_h^2 a_m}\right] < \exp \frac{-x\mu_h^2 \sum_{m=1}^{M} a_m}{1 + x\sigma_h^2 \sum_{m=1}^{M} a_m}, \tag{12}$$

has a simpler form,

$$\sum_{m=1}^{M} \frac{x a_m}{1 + x a_m \sigma_h^2} \leq \frac{x \sum_{m=1}^{M} a_m}{1 + x\sigma_h^2 \sum_{m=1}^{M} a_m}. \tag{13}$$

Using the Chebyshev sum inequality,

$$\sum_{m=1}^{M} \frac{x a_m}{1 + x a_m \sigma_h^2} > \frac{1}{M} \left(x \sum_{m=1}^{M} a_m \right) \left(\sum_{m=1}^{M} \frac{1}{1 + x a_m \sigma_h^2} \right),$$

and the inequality in (13) reduces to

$$\frac{1}{M} \sum_{m=1}^{M} \frac{1}{1 + x a_m \sigma_h^2} > \frac{1}{1 + x\sigma_h^2 \sum_{m=1}^{M} a_m}.$$

Defining $a_{\max} = \max_m a_m$, note that

$$\frac{1}{M} \sum_{m=1}^{M} \frac{1}{1 + x a_m \sigma_h^2} \geq \frac{1}{1 + x\sigma_h^2 a_{\max}} > \frac{1}{1 + x\sigma_h^2 \sum_{m=1}^{M} a_m}.$$

This proves the inequalities in (13) and (12), which in turn finally proves the inequality originally posed in (9). Therefore, we conclude that symbol interleaving always provides a lower BER.

V. Numerical Results

In this section, results are presented to validate the methods previously discussed. We assume that the channel has length $L = 15$ taps. Each tap is complex-valued Gaussian random variable (zero mean, unit variance). that varies per packet, but remains constant for retransmissions. The channel is normalized to unit energy to maintain the desired E_b/N_0. In all cases, $N = 256$ subcarriers used.

In Figs. 2, 3 and 4 the obtained BER results obtained via Monte Carlo simulations using 10000 packets at each E_b/N_0 value. Each packet contains 1024 bits, represented by 256 16QAM symbols. Comparisons are made against simple retransmissions made with no symbol interleaving and identical Gray mappings for all transmissions. All symbol interleavers are uniform interleavers. Both symbol mapping and symbol interleaving provide significant gains, and appear to do so independently. For example, with two-transmissions ($L = 2$), symbol mapping diversity produces about 2 dB gain and symbol interleaving produces about 13 dB gain at a BER of 10^{-4}. When both methods are employed, this gain increases to approximately 15 dB. The interleaving gains are fairly consistent with those found in previous work.

VI. Conclusion

In conclusion, some packet combining techniques were presented to improve system performance for OFDM modulation. In particular, dramatic improvements were discovered by uniquely interleaving symbols in OFDM retransmissions while adapting the bit-to-symbol mapping could improve a system using DMT. Moreover, an analysis of the effectiveness of symbol interleaving was introduced.

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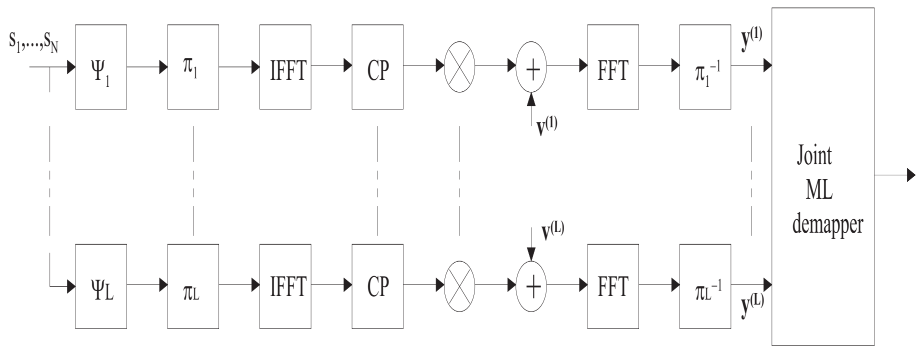

Fig. 1. System model of multiple OFDM transmissions of a packet.

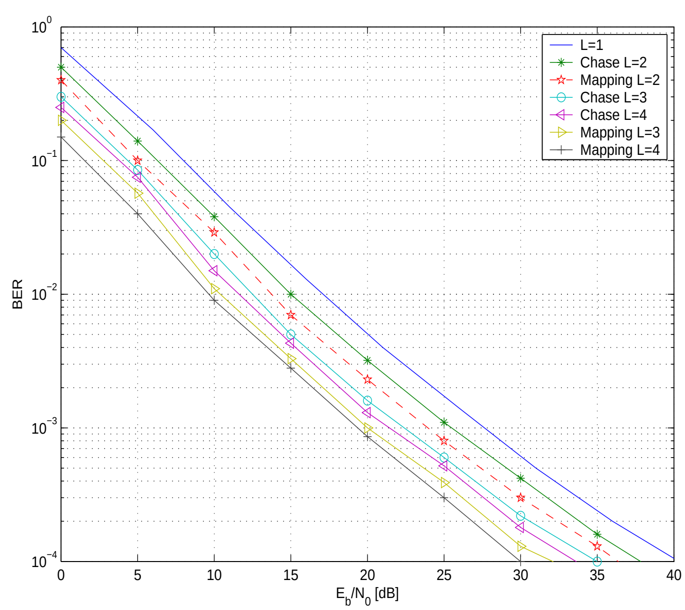

Fig. 2. BER result for OFDM applying symbol mapping diversity compared to retransmission same packet in every transmission (Chase).

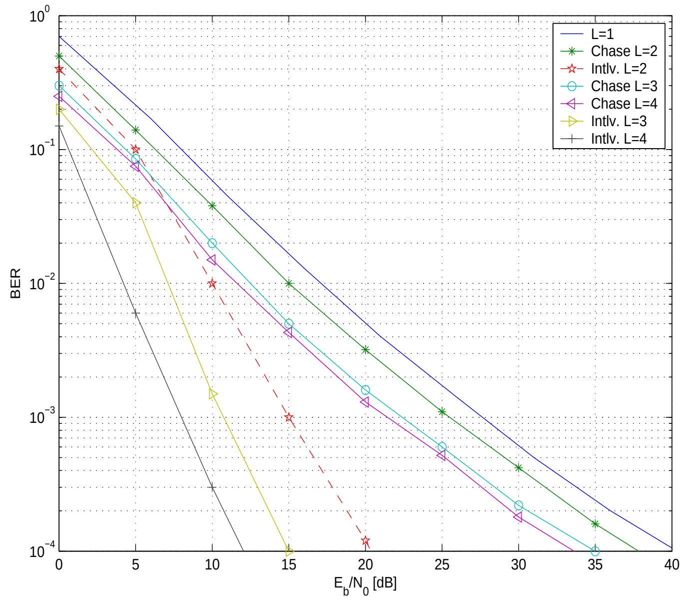

Fig. 3. BER result for OFDM applying symbol interleaving compared to retransmission same packet in every transmission (Chase).

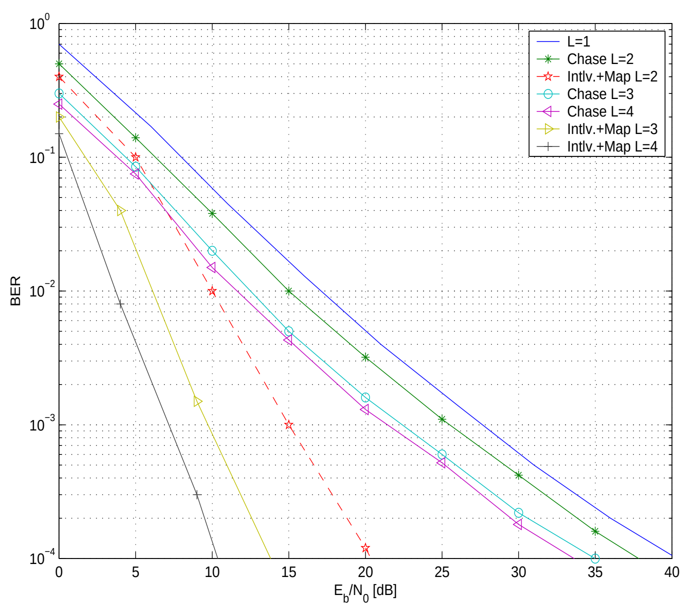

Fig. 4. BER result for OFDM applying symbol mapping diversity and interleaving compared to retransmission same packet in every transmission (Chase).