On Packet Retransmission Diversity Scheme with M-QAM in Fading Channels

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***Abstract*— Achieving high data rates and low error rates are vital in wireless communications. High data rates can be achieved using higher-order modulation such as M-ary QAM. Low error rates can be achieved by multiple transmissions of the same packet. In this paper we propose to use a packet combining method based on bit-to-symbol mapping, where the mapping is varied for different transmissions of the same packet to achieve diversity. Analytical results are shown to agree very well with simulation results. Our simulation results show that the proposed method outperform a packet combining scheme employing Chase combining where the same mapping is used in all transmissions. For example, at a BER of 10^{-3} the LLR based mappings result in about 2 dB of E_b/N_0 advantage compared to the Chase combined scheme**

I. Introduction

The recent rapid growth in wireless communications has led to the demand of high data rates and reliable communications. Unfortunately, the wireless channel medium contains multipath fading which can limit the systems performance. Higher order modulation (e.g., M-QAM, M-PSK) is attractive to employ for wireless communications due to the high spectral efficiency it provides [1]. In packet communication systems, packet retransmission is often requested when a received packet is detected to be in error. This scheme, termed automatic repeat request (ARQ), is intended to ensure extremely low packet error rate. During the ARQ process, the same data is sent until recovered without errors. The efficiency of ARQ can be improved by reusing the data from previous (re)transmissions instead of discarding them. This technique is termed hybrid ARQ. In [2], Chase introduced a packet combining scheme where the individual transmissions are encoded at same code rate R. If the receiver has L packets that have been caused by retransmission requests, the packet are concatenated to forms a single packet of lower rate code if rate R/L. Other works on packet combining methods include [3]- [7]. In [3], Harvey *et al* proposed a version of packet combining where L packets are combined into a single packet of the same length as the original transmitted data packet by averaging the soft decision values from the constituent packets. In [4], Kumagi *et al* presented a maximal ratio combining (MRC) frequency diversity ARQ scheme for OFDM systems which works such that in every retransmission, the different symbols of the OFDM blocks are transmitted on different subcarriers, and then employs MRC on the previous versions of the packet. In [5], Zhang and Kassam outlined a hybrid ARQ protocol for rate-compatible codes in fading channels that selectively combines a subset of L received transmissions. In [6], Narayanan and Stuber developed an ARQ receiver using error correcting codes where the extrinsic information from the decoding of previous packet is reused. In [7], Gidlund showed that packet combining can effectively enhance the performance of IEEE 802.11a WLAN system. When the signal constellation is the same in each transmission carried over an time-invariant channels, the so-called maximum ratio combining (MRC) is optimal and boils down to averaging of the received signals. The average data corresponds to a transmission over the channel with higher signal-to-noise ratio (SNR) which increases T-times after L transmissions.

A method to achieve packet combining diversity is to employ *bit-to-symbol mapping diversity*, where the bit-to-symbol mapping in M-ary modulation is varied for each packet (re)transmission. This results in improved packet combining performance in terms of reduced packet error rate (PER) compared to a system without symbol mapping diversity [8]. In that paper, the authors proposed to view the signal constellations of the modulation scheme in an augmented signal space formed by the modulation signal dimension and the number of retransmissions. That augmented signal space provides a good spread for the modulation signal points and the error probability is increased. One open question in the bit-to-symbol mapping diversity scheme is how to choose the optimum mappings for different (re)transmissions. For an M-ary constellation, there are $M!$ possible mappings to choose from.

In this paper, we will use the *log-likelihood ratios* (LLR) of the bits forming a M-QAM symbol in the optimum selection of mappings. We propose to choose the mappings for multiple (re)transmissions such that the sum of the magnitudes of the LLR of the bits forming the M-QAM symbols in different (re)transmissions is maximized. The rest of the paper is organized as follows: The system model and the mappings based on LLRs is described in Section II. In Section III an BER analysis of the proposed scheme is presented, the numerical results is presented in Section IV and finally we conclude the work in Section V.

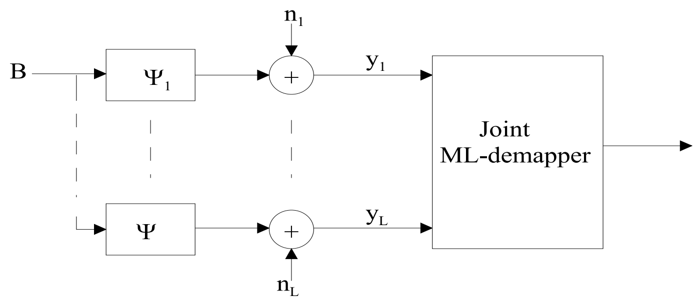

Fig. 1. Bit-to-symbol mapping diversity scheme.

II. System Model and Mapping Based on bit LLR

Let us consider a bandwidth efficient M-ary modulation scheme as in Fig. 1, where a data block B consisting of $b = \log_2 M$ bits, which are mapped to a point in the signal constellation via a bit-to-symbol mapping function ψ, and this signal point $\psi(B)$ is transmitted over the channel. In order to achieve packet combining diversity, the same bits may be transmitted more than once. Let L be the number of retransmissions. The data block B can either be retransmitted by using the same bit-to-symbol mapping in all transmissions, or vary the bit-to-symbol mapping in each transmission $\psi_1, \psi_2, \cdots, \psi_l$. Assuming that the transmitted symbol s undergoes fading, the received signal y_l (after multiple transmissions) can then be written as

$$y_i = h\psi_i(s) + n_i, \quad i = 1, 2, \cdots, L, \tag{1}$$

where h is the complex fading coefficient with $E\{||h||^2\} = \Omega$, and the r.v's $||h||$'s for different symbols are assumed to be i.i.d. Rayleigh distributed. Assuming perfect knowledge of the CSI at the receiver, the combined signal output for symbol s_k is given by $\hat{s}_k = hs_k + \zeta_k$, we define ζ as a complex gaussian random variable with zero mean and variance $h\sigma^2$. Let us define the log-likelihood ratio of bit $b_i,\ i = 1, 2, \cdots, b_i$ as following:

$$\begin{aligned} \Lambda_{s_k}(b_i) &= \log\left(\frac{Pr(b_i = 1|y, h)}{Pr(b_i = 0|y, h)}\right) && (2) \\ &= \log\left(\frac{Pr(b_i = 1|\hat{s}_k, h)}{Pr(b_i = 0|\hat{s}_k, h)}\right) && (3) \end{aligned}$$

The optimum decision rule is to decide $\hat{b_i} = 1$ if $\Lambda(b_i) \geq 0$, and 0 otherwise. Furthermore, we also assume that all symbols are equally probable and that fading is independent of the transmitted symbols. According to Bayes' rule, we can rewrite (2) as:

$$\Lambda_{s_k}(b_i) = \log\left(\frac{\sum_{\alpha \in S_i^{(1)}} f_{\hat{s}_k|s,h}(\hat{s}_k|s, h = \alpha)}{\sum_{\beta \in S_i^{(0)}} f_{\hat{s}_k|s,h}(\hat{s}_k|s, h = \beta)}\right) \tag{4}$$

where S_i^1 and S_i^0 is defined as the set partitions that compromises symbols with $b_i = 1$ and $b_i = 0$, respectively. We know from [1], that $f_{\hat{s}_k|s,h}(\hat{s}_k|s, h = \alpha) = \frac{1}{\sigma\sqrt{\pi}} \exp(1/\sigma^2||\hat{s}_k - h\alpha||^2)$, then we can rewrite (4) as

$$\Lambda_{s_k}(b_i) = \log\left(\frac{\sum_{\alpha \in S_i^{(1)}} \exp(-1/\sigma^2||\hat{s}_k - h\alpha||^2)}{\sum_{\alpha \in S_i^{(0)}} \exp(-1/\sigma^2||\hat{s}_k - h\alpha||^2)}\right) \tag{5}$$

The expression in (5), can be further simplified by using the approximation $\log(\sum_j \exp(-x_j)) \approx -\min_j(x_j)$. By defining $z = \frac{\hat{s}_k}{h} = s + \hat{n}$, where $\hat{n}$ is a complex Gaussian r.v with variance $\sigma^2/||h||^2$ and using the above approximation we can simplify (5) to the following:

$$\begin{aligned} \Lambda_{s_k}(b_i) \quad = \quad & \frac{||h||^2}{4}\Bigg[\min_{\beta \in S_i^{(0)}} (||\beta||^2 - 2z_I\beta_I - 2z_Q\beta_Q) - \\ & \min_{\alpha \in S_i^{(1)}} (||\alpha||^2 - 2z_I\alpha_I - 2z_Q\alpha_Q)\Bigg] \qquad (6) \end{aligned}$$

where $z = z_I + z_{jQ}, \alpha = \alpha_I + j\alpha_Q$ and $\beta = \beta_I + j\beta_Q$. If considering a square or rectangular QAM constellations, the set partitions $S_i^{(1)}$ and $S_i^{(0)}$ is delimited by horizontal or vertical boundaries. To find the optimum mappings for bit-to-symbol mapping diversity we take advantage of the soft information given by the LLRs of the bits forming the QAM symbol. We will iteratively compute the Lth mapping from the $L-1$ previous mappings. We define the sum of LLRs of a given bit in the previous $L-1$ mappings as

$$\epsilon(i, j) = \sum_{l=1}^{L-1} \overline{\Lambda}_{ij}^{(l)} \tag{7}$$

where $\overline{\Lambda}_{ij}^{(l)}$ is defined as the averaged Λ computed for the ith bit of the jth symbol in the mapping of the lth transmission and the averaging over the noise samples. Furthermore, We define Ψ as the set of mappings ($|\psi| = M!$). To choose the Lth mapping we need to solve the following optimization problem

$$\psi_L \in \Psi \sum_{j=1}^{M} \sum_{i=1}^{\log_2 M} |\epsilon(i, j) + \overline{\Lambda}_{ij}^{(L)}|, \tag{8}$$

By using the above optimizing procedure we can construct new constellations for next retransmission. To show the results of this procedure we consider 16QAM and the first transmission is given in Figure 2. If an error is caused we can construct the next retransmission as outlined above and the result is showed in Figure 3. With a close inspection we can see that the squared Euclidean distance has increased between the signal points in this second mapping compared to if we should have been using the same mapping as in first transmission (Figure 2).

III. Derivation of Probability of Bit Error

We derive the probability of error for a bit $b_i, i = 1, 2, 3, 4$, forming a 16-QAM symbol. Let us consider the signal mapping in Figure (2) and we focus on the LLR bit decision. Consider $2d$ as the spacing between adjacent symbols and based on the definition of bit LLRs in previous section, the LLRs for the bits b_1, b_2, b_3 and b_4 for the first mapping ψ_1 can be obtained as

$$\Lambda_{s_k}(b_1) = \begin{cases} -||h||^2 z_I d & |z_I| \leq 2d \\ 2||h||^2 d(d - z_I) & z_I > 2d \\ -2||h||^2 d(d + z_I) & z_I < -2d \end{cases} \tag{9}$$

$$\Lambda_{s_k}(b_2) = \begin{cases} -||h||^2 z_Q d & |z_Q| \leq 2d \\ 2||h||^2 d(d - z_Q) & z_Q > 2d \\ -2||h||^2 d(d + z_Q) & z_Q < -2d \end{cases} \tag{10}$$

$$\Lambda_{s_k}(b_3) = ||h||^2 d(|z_I| - 2d) \tag{11}$$

$$\Lambda_{s_k}(b_4) = ||h||^2 d(|z_Q| - 2d) \tag{12}$$

The probability of error for bit b_1 in symbol $s_k, k = 1, 2, \cdots, K$. Then we can write $P^k_{b_I}$ as

$$\begin{aligned} P^k_{b_1} = \; & P^k_{b_1|s_{k_I=-d}} \cdot Pr\{s_{k_I} = -d\} + \\ & P^k_{b_1|s_{k_I=-3d}} \cdot Pr\{s_{k_I=-3d}\} + \\ & P^k_{b_1|s_{k_I=d}} \cdot Pr\{s_{k_I} = d\} + \\ & P^k_{b_1|s_{k_I=3d}} \cdot Pr\{s_{k_I=3d}\} \end{aligned} \tag{13}$$

Let s_{k_I} represents the real-part of s_k. Consider that $P^k_{b_1|s_{k_I=-d}}$ is given by

$$P^k_{b_1|s_{k_I=-d}} = \overline{P^k_{b_1|s_{k_I=-d}}} \tag{14}$$

where the overline indicates averaging over the complex R.V $\{h_{i,j} \cdot P^k_{b_1|s_{k_I=-d}}\}$.

$$\begin{aligned} P^k_{b_1|s_{k_I=-d,H}} &= Pr\{\Lambda_{s_k}(b_1) < 0|s_{k_I}\} \\ &= Pr\left(\frac{\zeta_{k_I}}{\sqrt{||h||^2}} \leq d\right) \\ &= Q\left(\frac{d\sqrt{||h||^2}}{\sigma_I}\right) \\ &= \overline{Q\left(\sqrt{\frac{4E_b||h||^2}{5N_0}}\right)} \\ &= \frac{1}{2}\left(1 - \sqrt{\frac{2E_b/N_0}{5 + 2E_b/N_0}}\right) \end{aligned} \tag{15}$$

where $\sigma_I = \sigma^2/2$. Similarly, the error probability for $P^k_{b_1|s_{k_I=-3d,H}}$ is given by

$$\begin{aligned} P^k_{b_1|s_{k_I=-3d,H}} &= Pr\{\Lambda_{s_k}(b_1) < 0|s_{k_I}\} \\ &= Pr\left(\frac{\zeta_{k_I}}{\sqrt{||h||^2}} \leq 3d\right) \\ &= Q\left(\frac{3d\sqrt{||h||^2}}{\sigma_I}\right) \\ &= \overline{Q\left(\sqrt{\frac{36E_b||h||^2}{5N_0}}\right)} \\ &= \frac{1}{2}\left(1 - \sqrt{\frac{18E_b/N_0}{5 + 18E_b/N_0}}\right) \end{aligned} \tag{16}$$

The BER expression for the bits b_1, b_2, b_3, b_4 of the symbol s_k can hence be written as:

$$P^k_{b_1} = P^k_{b_2} = \frac{1}{2}(P^k_1 + P^k_2) \tag{17}$$

$$P^k_{b_3} = P^k_{b_4} = \frac{1}{2}(2P^k_1 + P^k_2 + P^k_3) \tag{18}$$

Hence, P_{b_1} can be given as

$$P_{b_1} = \frac{1}{2}\left[1 - \frac{1}{2}\sqrt{\frac{2E_b/N_0}{5 + 2E_b/N_0}} - \frac{1}{2}\sqrt{\frac{18E_b/N_0}{5 + 18E_b/N_0}}\right] \tag{19}$$

The error probabilities for P_{b_3} and P_{b_4} ($P_{b_3} = P_{b_4}$) can be obtained as

$$\begin{aligned} P_{b_3} = \; & \frac{1}{2}\left[1 - \frac{1}{2}\sqrt{\frac{2E_b/N_0}{5 + 2E_b/N_0}} - \frac{1}{2}\sqrt{\frac{18E_b/N_0}{5 + 18E_b/N_0}} + \right. \\ & \left. \frac{1}{2}\sqrt{\frac{50E_b/N_0}{5 + 50E_b/N_0}}\right]. \end{aligned} \tag{20}$$

Then, for the second mapping ψ_2 and taking into account the different decision boundaries for a symbol in the two different mappings, $P_{b_1}|s_1 = 0$ can be written as

$$P_{b^k_i|s_1=0} = Pr\{\Lambda 1(b_1) + \Lambda 2(b_2) \geq 0|s_{k_{I_1}}, s_{k_{I_2}}\} \tag{21}$$

and $P_{b_1}|s_1 = 1$ is given by

$$P_{b_i^k|s_1=0} = Pr\{\Lambda 1(b_1) + \Lambda 2(b_2) \leq 0 | s_{k_{I_1}}, s_{k_{I_2}}\} \qquad (22)$$

where $s_{k_{I_1}}, s_{k_{I_{12}}}$ are the real parts of the symbol sent in the two mappings ψ_1, ψ_2, respectively. Then we follow the same procedure as previous and finally the average BER of the system, P_b, is given by

$$P_b = \frac{1}{K}\sum_{k=1}^{K} P_b^k \qquad (23)$$

We see that the complexity of the analysis increases with L. This is because in the this case the decision boundaries for a given data block is changing from one transmission to another.

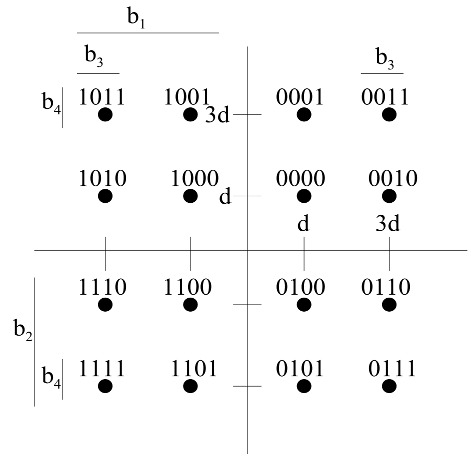

Fig. 2. First mapping

IV. Numerical Results

To evaluate the performance of the proposed packet retransmission scheme, we will compare the scheme with an ARQ scheme employing Chase combining. The scheme is evaluated over a Rayleigh fading channel which is described by Jake's model [9]. The carrier frequency f_c is set to 5.8 GHz and the sampling rate f_s as 12.5 KHz. In Fig. 4, we illustrate the BER performance of the scheme using LLR based bit decision for the case of $L = 2$ for 16-QAM. The optimum mapping based through maximizing the LLR metric are used. It is observed that both analytical and simulations results agree.

In Fig. 5, we show the BER performance comparison between our LLR based mappings versus the Chase combined scheme. We can observe that the LLR based mappings result in a better BER performance than the Chase combining approach. For example, at a BER of 10^{-3} the LLR based mappings result in about 2 dB of E_b/N_0 advantage compared to the Chase combined scheme. Since it difficult to perform an analysis for $L = 3, 4, ..., L-1$, we obtained the performance plots for $L = 3$ through simulation.

The derived LLRs can be used as soft inputs to an Viterbi decoder for decoding convolutional codes when QAM modulation is used. An example of such application is IEEE 802.11a. Here, we consider decoding of a convolutional code of constraint length 7 using Viterbi algorithm. Figure 6 shows the BER performance comparison using LLRs as soft inputs to the Viterbi decoder versus the performance hard decision inputs. It is observed that when the LLRs are used as soft decision inputs the performance clearly improves when compared to using hard decision inputs. For instance, at a BER of 10^{-3} the soft decision inputs result in approximately 1 dB advantage compared to Chase combining.

V. Concluding Remarks

In this paper we have addressed the problem of finding the bit-to-symbols mappings for multiple transmissions by using the log-likelihood ratios of the bits forming a M-QAM symbol. The mappings are chosen such that the sum of magnitudes of the LLR of the bits forming the M-QAM symbols in different transmissions is maximized. The obtained simulation results shows that the proposed method with selecting the mappings by the LLR methods results in better BER performance than utilizing Chase combining.

Furthermore, the proposed method also works for M-PSK modulation. We observe, that even if the constellations are optimized using the LLR method, the optimized mappings give reasonable gain of 1 dB for coded transmission which may justify additional complexity required by the detection algorithm.

Acknowledgment

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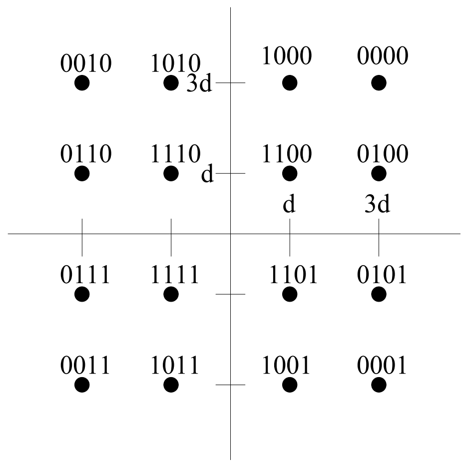

Fig. 3. Second mapping

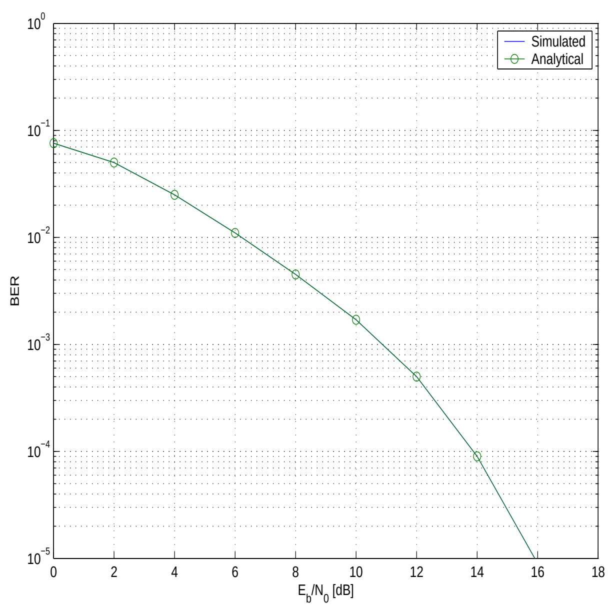

Fig. 4. BER performance of the LLR based scheme for 16QAM, $L = 2$. Analysis and simulation over AWGN Channel.

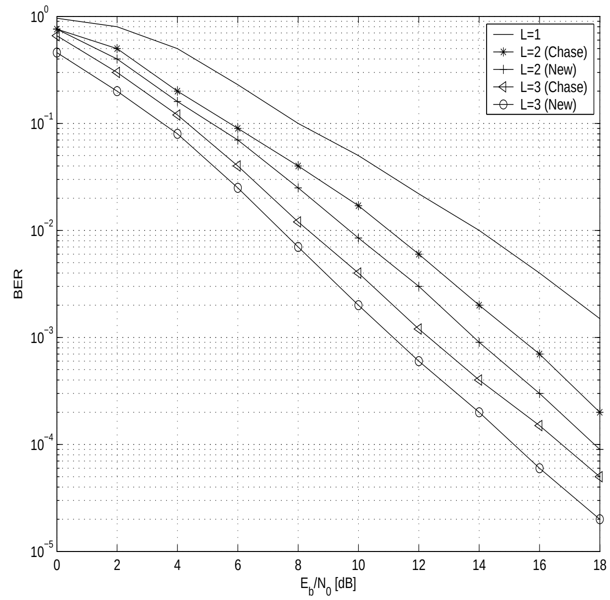

Fig. 5. BER performance comparison between LLR based mappings and Chase combining in Rayleigh fading. $L = 1, 2, 3$

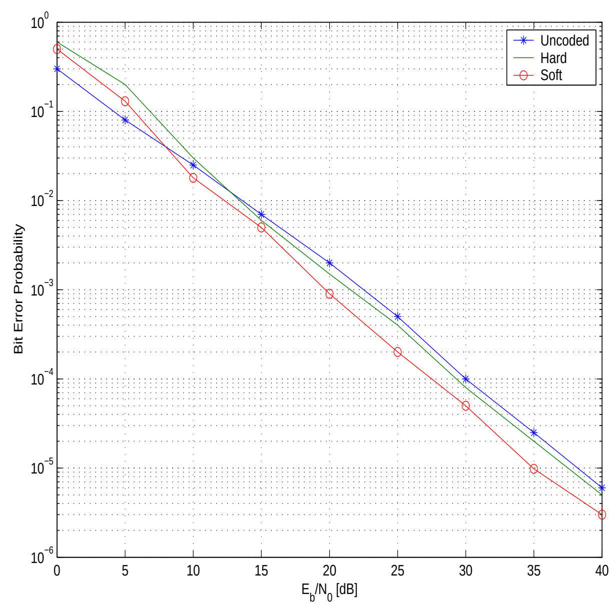

Fig. 6. Comparison of the BER performance of 16-QAM with a) LLRs as soft inputs, b) hard decision inputs to the Viterbi decoder in i.i.d Rayleigh fading channel. Uncoded 16-QAM system performance is also shown.