A Novel Combined Packet Retransmission Diversity and Multi-Level Modulation Scheme

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Abstract

In this paper, we present a simple bandwidth efficient packet retransmission diversity scheme suitable for multi-level modulation schemes such as MPAM, MPSK and MQAM. Our proposed retransmission diversity scheme consider the interaction between the packet retransmission and the signal space of the used modulation scheme to improve the performance. The proposed scheme rearrange the symbols between the different transmissions and also use the modulation level M as an extra dimension to improve the quality of the signal in order to reduce the number of retransmissions per packet. Our idea with seeing the modulation level and number of retransmissions as an augmented signal space and employ packet combining will improve the robustness and performance. Our performance results show that the proposed scheme improves the system performance in both additive white Gaussian noise and fading multipath channels.

I. Introduction

The demand for wireless multimedia communication is rapidly increasing due to strong advances in wireless Internet services. reliable high-speed data communications over insufficient channel bandwidth is one of the major challenges of harsh wireless environments that push the achievable spectral efficiency far below theoretical limits. To achieve high-quality or error free transmission for multimedia services in wireless networks it is necessarily to adopt some powerful tools such as diversity and error control techniques.

Diversity is a very good technique for improving the performance of wireless communication links. Orthogonal transmitter diversity such as frequency diversity and time diversity have some properties that are quite attractive in wireless communications as they can provide a diversity gain without the need for multiple transmit/receive antennas. In general, diversity tries to solve signal fading problem in wireless communication by transmit several redundant replicas of the information signal that undergo different multipath profiles. The idea is that the repetition of the information, together with an appropriate combining of the received signals, will greatly reduce the negative effects of the radio channel fading. In error control techniques, we say that ARQ can been seen as an orthogonal transmitter diversity scheme since it creates time diversity.

There are two fundamental techniques used for error control: forward error correction (FEC) and automatic repeat request (ARQ). In FEC schemes, the receivers have the ability to correct erroneous data while in pure ARQ schemes the receivers can only detect erroneous data in the transmissions. To overcome the individual drawbacks of FEC and ARQ, such as channel utilization, efficiency and data integrity; the schemes are combined into one scheme which is called hybrid ARQ and has received a lot of attention from researchers during the last decade [1]-[6]. It is well known that introducing packet combining into an ARQ scheme creates a diversity gain and can improve the throughput remarkably. In [7], Chase introduced a packet combining scheme where the individual transmissions are encoded at same code rate R. If the receivers has L packets that have been caused by retransmission requests, the packet are concatenated to form a single packet of lower rate code of rate R/L. In [8], Harvey *et al* proposed a version of packet combining where L copies of the data packet are combined into a single packet of the same length as the original transmitted data packet by averaging the soft decision values from the constituent copies.

By varying the bit-to-symbol mapping for each packet (re)transmission, the diversity is enhanced among L transmissions. Benelli conducted the first known work that considered the effects of modulation; an ARQ protocol for continuous-phase modulation [9]. Ottnes and Maseng proposed the use of nonuniform constellations for retransmitted packets in systems that require unequal error protection [10]. Wengerter *et al.* presented HARQ methods that employ code combining and adapting among Gray mappings for re-transmissions [11]. However their method did not take the modulation level and number of retransmissions in consideration. In the past, the problem in finding optimal symbol mappings for a single transmission has been studied. The prime example is the work of Ungerboeck on trellis-coded modulation (TCM), with the development of set-partitioning mapping [12]. Wesel *et al.* developed mappings for linear encoders that minimize a constellations edge profile [13].

In this paper we consider combined retransmission and multi-level bandwidth efficient modulation techniques and changing the mapping in every retransmission. The main idea is to try to take advantage of the extra dimension provided by the retransmission diversity scheme in improving the power efficiency of the used modulation without altering the diversity order of the system. Considering the overall scheme as one entity we can obtain a transmission scheme that can perform very well in both additive Gaussian and fading multipath channels. This combining procedure does not increase the complexity at the receiver and consecutive received symbols can still be treated independently. The full performance potential of this scheme is obtained by employing a detector based on the Maximum Likelihood Sequence Estimation (MSEE) algorithm on a symbol-by-symbol basis.

The rest of the paper is organized as follows. In Section II we present the novel idea of the paper and the necessarily background. Section III gives an detailed description of the proposed combined packet retransmission diversity scheme and multi-level modulation, in Section IV a performance analysis of the scheme is carried out. Then in Section V we discuss numerical results and finally in Section VI we conclude our work.

II. Preliminaries

As mentioned earlier in this paper, the need for high data rates and bandwidth efficient multi-level modulation is becoming more important in today's and future wireless networks to support new multimedia services.

Let us consider a bandwidth efficient M-level modulation scheme where the information stream of bits b are mapped to constellation symbols $\{s_0, s_1, \cdots, s_{M-1}\}$ where M in an integer denoting the modulation level. We let $\mathcal{A}$ denote the constellation alphabet, and $|\mathcal{A}|$ its size. Every $\log_2 |\mathcal{A}|$ consecutive symbol of b is mapped to one constellation symbol when s_n is binary. When the constellation is multilevel, i.e., $|\mathcal{A}| > 2$, the constellation mapping policy will affect the performance, depending on the detection scheme. Assuming transmission takes place over a fading channel and we denote a transmitted packet x of length N with $x = x_0, x_1, \cdots, x_{N-1}$ where $x_i \in \{s_0, s_1, \cdots, s_{M-1}\}, \forall i$. The received packet after demodulation can then be expressed as

$$r_{i,l} = h_i s_i + n_{i,l}, \tag{1}$$

where h as the complex fading coefficient with $E\{||h||^2\} = \Omega$, and the r.v's $||h||'s$ for different symbols are assumed to be i.i.d Rayleigh distributed. Furthermore, $n_{i,l}$ is complex Gaussian with zero mean and variance N_0, l refers to transmission l of the same packet (or equivalently retransmission $l-1$). Assuming perfect knowledge of the CSI at the receiver, the combined signal output for symbol s_i is given by $\hat{s}_i = hs_i + \xi_i$, we define ξ as a complex Gaussian random variable with zero mean and variance $h\sigma^2$

At the receiver we employ maximum likelihood (ML) method to combine the different replicas of the received packets. The ML metric of a particular symbol of the packet can be written as follows:

$$\zeta(s_n, \hat{s}_n) = \sum_{l=0}^{L-1} |r_{l,n} - h_n \hat{s}_n|^2 \tag{2}$$

For every transmitted symbol, the detector selects the set of signals points $\{\hat{s}_n\}$ that minimizes the above metric. A decision error will occur if the detector finds a set $\{\hat{s}_n\} \neq \{s_n\}$ with a metric smaller than that of $\{s_n\}$.

Assuming an AWGN channel and that the components $n_{l,n}$ are uncorrelated complex Gaussian random variables, the error probability can be written as follows:

$$P_2\left(s_n \rightarrow \hat{s}_n\right) = Q\left(\sqrt{\frac{L|s_n - \hat{s}_n|^2}{2N_0}}\right) \tag{3}$$

which is a function of the squared Euclidean distance between the two sequences of signal points $\{s_n, \hat{s}_n\}$, and the number of transmissions L.

Compared to the case of no retransmission ($L = 1$) we notice that the only advantage obtained from packet retransmission, in this case, is the accumulated signal power. This accumulated signal power is translated into a dB gain of $10\log_{10}(L)$ due to the coherent addition of the replicas of the transmitted symbols after combining. Note that this obtained dB gain is independent of the modulation scheme used.

As indicated above, the gain in performance obtained from packet retransmission and packet combining depends only on the number of retransmissions and is completely independent from the modulation scheme used. This is clearly not optimum because the combination does not take full advantage of the available signal space. With packet retransmission we basically add an extra dimension to the modulation scheme used. Hence, with a proper interaction between retransmission and signal mapping we can obtain a better mapping of the modulation signal points within the space. This better spread of signal points, if done in terms of maximizing the Euclidean distances between the different signal points, can improve the error probability and hence reduces the average number of retransmissions within the system. Here, this interaction and design will depend on the modulation level used and its signal dimension. For binary modulation the number of signal points is only two which fully exploits the signal space and this case retransmission will not add any extra dimension to the combined binary signal. Hence, the above combining method is optimum for binary modulation and the dB gain obtained in this case is related to the number of transmissions only. However, for the case of multi-level modulation where the number of signal points exceeds the number of transmissions, a good combination between signal retransmission and modulation can provide a better use of the augmented signal space (obtained from the combination of the modulation signal space and the transmissions of the signal). We believe that using a different signal mapping scheme for every retransmission can bring us toward this interaction and can improve the system performance further.

Now, we consider that we change the signal constellation within the retransmissions, we can rewrite the ML metric in (2) as follows:

$$\zeta(s_n, \hat{s}_n) = \sum_{l=0}^{L-1} |r_{l,n} - h_n \hat{s}_{l,n}|^2 \tag{4}$$

where l refers to transmission l of the same packet (or equivalently retransmission $l-1$). The error probability can now be described as

$$P_2\left(s_n \rightarrow \hat{s}_n\right) = Q\left(\sqrt{\frac{\sum_{l=0}^{L-1} |s_{l,n} - \hat{s}_{l,n}|^2}{2N_0}}\right) \tag{5}$$

which is a function of the squared Euclidean distance between the two sequences of signal points $\{s_{0,n}, s_{1,n}, \cdots, s_{L-1,n}\}$, $\{\hat{s}_{0,n}, \hat{s}_{1,n}, \cdots, \hat{s}_{L-1,n}\}$, and the number of transmissions L.

Compared to the case of no retransmission ($L = 1$) we notice that the above error probability is now dependent on the number of retransmissions and the set of signal points used through the different retransmissions. This combination of signal mapping and retransmission is translated into a dB gain given by

$$10 \log_{10} \left(\frac{\sum_{l=0}^{L-1} |s_{l,n} - \hat{s}_{l,n}|^2}{|s_{0,n} - \hat{s}_{0,n}|^2} \right). \tag{6}$$

The above obtained dB gain indicates that the signal mapping used at the different transmissions of the packet has a big influence on the possible improvement that can be achieved. It is clear from (6) that the modulation level will play an important role on the possible gain that can be achieved as a function of the number of transmissions L and will also decide on the maximum number of signal mapping sets that can be used. Since the number of signal points of a bandwidth-efficient multi-level modulation exceeds the dimension of the signal space, they are at different distances from each other. Hence, changing the signal mapping from transmission to the next will increase the dB gain given in (6) as compared to the case of using the same signal mapping procedure for all transmissions of the packet. We can easily see that, for the case of binary modulation, changing signal mapping over the different transmissions will not provide any extra gain as compared to the regular case of always using the same signal mapping. Hence, for binary modulation the dB gain is related to the number of transmissions only and given by $10 \log_{10}(L)$. This indicates that the number of possible signal mapping sets depends on the modulation level M. Denoting by $\mathcal{Q}$ the number of possible signal mapping sets, we can say that $\mathcal{Q} = 1$ for the case of binary modulation. For higher values of M, the value of $\mathcal{Q}$ will depend on the distribution of the signal points within the augmented signal space and this will change when the number of transmissions increases. Given the number of signal mapping sets $\mathcal{Q}$, they can be used in the given order along the different retransmissions of the packet and if the number of required transmissions exceeds the number of sets $\mathcal{Q}$ then the set is reused in a periodic manner. For instance, for 4PAM we find that $\mathcal{Q} = 8$. This procedure will maximize the system performance and facilitates the combining and detection procedure at the receiver.

III. Combined Packet Retransmission Diversity and Multi-Level Modulation

In previous section we saw that for systems with packet retransmission in conjunction with efficient multi-level modulation schemes it is preferable to use a different signal mapping set at every retransmission. There exist different ways to achieve remapping of signal constellations. In this section we will discuss how to generate new mappings for each retransmission.

Since the selected mapping sets depends on the bandwidth efficient modulation, we will first for simplicity consider a 2^m-PAM constellation, described by the following sequence

$$\left(s_m^{(0)}, s_m^{(1)}, \ldots, s_m^{(2^m-1)}\right) = (-2^m + 1/2, -2^m + 3/2, \ldots, -1/2, 1/2, \ldots, 2^m - 1/2) \tag{7}$$

of rational numbers, for a positive integer m. PAM is a one-dimensional modulation scheme which should facilitate the design of the proposed design of signal mapping sets. As consecutive transmissions of packet in a wireless data system are orthogonal, we can see the retransmitted versions of the modulated symbols as an augmented signal space. For a total of L transmissions ($L-1$ retransmissions) and pulse amplitude modulation, the augmented signal space dimension is equal to L. The idea is, for a given value of L, identify the signal mapping sets that maximizes the Euclidean distance between the different signal points of the modulation scheme after signal combining. Since the integer L is a random variable, the signal mapping set used a transmission i should optimize the Euclidean distance between the different signal points of the combined signal taking into account the previous signal mapping sets only. In other words, the signal mapping set of the first transmission should be selected assuming that there is only one transmission (Gray mapping for instance), the signal mapping set for the first retransmission should be selected assuming that there is a total of two transmissions and designed taking into account the signal mapping set of the first transmission, and so on. With this procedure we can take full advantage of the available signal space and reduce the probability of loosing packets in wireless data communication systems.

For simplicity, let us consider the case of 4-PAM which is Gray mapped as in Figure 1. If this modulation scheme is using signal combining according to (2) then the signal constellations of the combined signal becomes two-dimensional as shown in Figure 2. Note, in an AWGN environment, the signal space still appears as a one-dimensional signal space even though we are using two dimensions. This indicates that this retransmission procedure does not take full advantage of the system dimension (which is two in this case). By taking advantage of the available system may enhance the system performance and this increase in dimension of the signal space will ensure that the distance is increased between the signal points and improving the bit error probability over different channels.

By simple manipulating the signal points in the next transmission, the combined signal can be optimized to use the complete signal space as shown in Figure 3. It can clearly be seen that the obtained signal points are much better spread within the signal space and the minimum squared Euclidean distance is increased. This remapping procedure will provide a gain of 4 dB compared to conventional case and we have taken full advantage of the augmented signal space. If a third transmission is required, our signal space will have a dimension equal to three and in the case of 4PAM will form a cube in the signal space. The new signal mappings can easiest be found by computer search. The above procedure can be repeated for finding the following signal mapping sets. In Table I, we have listed the optimum signal mapping sets for eight retransmissions [1] and then stopped since we found that set 9 is equal to set 1.

Our methodology can be readily applied to other signal constellations. For instance, it also applies for QAM since we know that a square 2^{2m}-QAM constellation of size 2^{2m} is the sequence of complex

[1]Note that there are other signal mapping sets that give the same distance distribution between the different signal points of the modulation scheme.

TABLE I

OPTIMUM SIGNAL MAPPING SETS FOR 4-LEVEL PAM SCHEME.

Level	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8
$-3d$	00	01	10	00	00	10	00	01
$-d$	01	10	00	10	11	00	01	00
$+d$	11	00	11	01	10	01	10	11
$+3d$	10	11	01	11	01	11	11	10

TABLE II

OPTIMUM SIGNAL MAPPING SETS FOR 8-LEVEL PSK SCHEME.

Level	Set 1	Set 2	Set 3
s_1	000	000	000
s_2	001	010	111
s_3	011	101	110
s_4	010	001	100
s_5	110	110	011
s_6	111	100	001
s_7	101	011	101
s_8	100	111	010

numbers where the real and imaginary parts are both taken from (7), for a positive integer m. By employing the obtained signal constellation for 4PAM, we can easily obtain the signal constellation for 16QAM as shown in Figures 4 and 5. From inspection it can be seen that the Euclidean distance has increased and the performance gain is also 4 dB in this case.

For a 2^m-PSK constellation we can describe the sequence in following way

$$\left(s_m^{(0)}, s_m^{(1)}, \ldots, s_m^{(2^m-1)}\right) = \left(A_m, A_m \exp\left(\frac{\sqrt{-1}\pi}{2^{m-1}}\right), \ldots, A_m \exp\left(\frac{\sqrt{-1}(2^m-1)\pi}{2^{m-1}}\right)\right) \quad (8)$$

of complex numbers with $A_m = 1/(2\sin(\pi/2^m))$, for a positive integer m. Applying the same method as previously, we will obtain the optimum mappings for 8PSK as shown in Table II.

IV. Performance Analysis

In this section, we will analyze the performance of the proposed combined retransmission and multi-level modulation scheme for the case with ideal feedback. We will both derive an upper bound on the symbol error probability (SEP) and analyze the throughput for the proposed scheme.

A. Upper Bound on SEP

Consider an AWGN channel and that the detector make an erroneous decision in (4), we can write the pairwise error probability as

$$P_2\left(s_n \to \hat{s}_n\right) = Q\left(\sqrt{\frac{\lambda^2(s_{l,n}, \hat{s}_{l,n})}{2N_0}}\right) \tag{9}$$

where $Q(.)$ is the well-known Gaussian integral which is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp(-x^2/2)\ dx \tag{10}$$

and λ is defined as the squared Euclidean distance between the transmitted sequence $\{s_n\}$ and the chosen sequence $\{\hat{s}_n\}$ and given by

$$\lambda^2 = \frac{1}{L}\sum_{l=0}^{L-1} |s_{l,n} - \hat{s}_{l,n}|^2.$$

We can find an upper bound of the average symbol error probability for the proposed scheme by averaging over all possible candidate symbols and all transmitted symbols. Hence, the bound is then given by

$$P_s \leq \frac{1}{M}\sum_{s_n=1}^{M-1}\sum_{\hat{s}_{l,n}\neq s_{l,n}} Q\left(\sqrt{\frac{\lambda^2(s_{l,n}, \hat{s}_{l,n})}{2N_0}}\right) \tag{11}$$

For the 4PAM scheme discussed in previous section an upper bound on the SEP can be derived an is given by

$$P_s \leq \begin{cases} 2Q\left(\sqrt{\frac{4E_b}{5N_0}}\right) & L\text{=}1 \\ 2Q\left(\sqrt{\frac{2E_b}{5N_0}}\right) + 2Q\left(\sqrt{\frac{4E_b}{N_0}}\right) & L\text{=}2 \\ 2Q\left(\sqrt{\frac{2.4E_b}{N_0}}\right) + 2Q\left(\sqrt{\frac{3.6E_b}{N_0}}\right) + 2Q\left(\sqrt{\frac{4.4E_b}{N_0}}\right) + 4Q\left(\sqrt{\frac{5.6E_b}{N_0}}\right) & L\text{=}3 \end{cases} \tag{12}$$

For the case of 16 QAM scheme, the modulation sets can be determined following the procedure described in the previous section and an upper bound on the SEP can be derived and is given by

$$P_s \leq \begin{cases} 2Q\left(\sqrt{\frac{4E_b}{5N_0}}\right) & L\text{=}1 \\ 4Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + 2Q\left(\sqrt{\frac{4E_b}{N_0}}\right) & L\text{=}2 \\ 2Q\left(\sqrt{\frac{2.4E_b}{N_0}}\right) + 2Q\left(\sqrt{\frac{2.6E_b}{N_0}}\right) + 2Q\left(\sqrt{\frac{3E_b}{N_0}}\right) & L\text{=}4 \\ 6Q\left(\sqrt{\frac{8E_b}{3N_0}}\right) & L\text{=}6 \end{cases} \tag{13}$$

Note that when the number of transmissions increases, the squared Euclidean between the different points of the combined signal become more balanced an in the limit of all signal points become equidistance from each other in the new augmented signal space (similar to orthogonal signaling schemes).

For the case with a Rayleigh fading channel an upper bound on the SEP for the proposed scheme can be derived in the same manner as for the case of AWGN channel. We can then write the SEP as follows

$$P_s(\epsilon|\mathbf{h}) \le \frac{1}{M}\sum_{s_n=1}^{M-1}\sum_{\hat{s}_{l,n}\neq s_{l,n}} Q\left(\sqrt{\frac{\lambda_{\mathbf{h}}^2(s_{l,n},\hat{s}_{l,n})}{2N_0}}\right) \tag{14}$$

where

$$\lambda_{\mathbf{h}}^2 = \frac{1}{L}\sum_{l=0}^{L-1}\alpha_l^2|s_{l,n}-\hat{s}_{l,n}|^2$$

is the squared Euclidean distance between the transmitted sequence $\{s_n\}$ and the chosen sequence $\{\hat{s}_n\}$. Furthermore, α is the fading amplitude and

$$\mathbf{h} = \{h_0, h_1, \cdots, h_{L-1}\}$$

is the set of fading coefficients during the observed symbol interval. Averaging the above expression over the probability density functions of the fading coefficients, the upper bound on the symbol error probability over Rayleigh fading becomes [14]

$$\begin{aligned} P_s \;\le\; & \frac{1}{M}\left[\sum_{j=1}^{L}\begin{pmatrix} 2L-j-1 \\ L-1 \end{pmatrix}\frac{2^{j-2L}}{(1+x_{min})^2}\right] \\ & \times \sum_{s_n=1}^{M-1}\sum_{\hat{s}_{l,n}\neq s_{l,n}}\prod_{i=0}^{L-1}\left(\frac{1}{1+\frac{2\sigma^2|s_{l,n}-\hat{s}_{l,n}|^2}{4LN_0}}\right) \end{aligned} \tag{15}$$

where

$$x_{min} = \min_{\forall s_n\neq k}\sqrt{\frac{2\sigma^2|s_n-s_k|^2/(4N_0)}{L+2\sigma^2|s_n-s_k|^2/(4N_0)}} \tag{16}$$

where the product distance is depending on the selected set. When the modulation signal constellation sets are properly selected, a better product distance is obtained and the overall system performance is improved.

Applying the upper bound in (15) to the 16QAM scheme we get

$$P_s \le \left[\frac{1}{1+x_1}\right]\frac{1}{1+0.4\gamma_0}, \quad N=1,$$

$$P_s \le \left[\frac{1}{1+x_2}+\frac{1}{(1+x_2)^2}\right]\frac{1}{1+0.2\gamma_0}\times\left[\frac{1}{1+0.8\gamma_0}+\frac{0.5}{1+1.8\gamma_0}\right], \quad N=2,$$

where

$$x_N = \sqrt{\frac{0.4\gamma_0}{N+0.4\gamma_0}}, \text{ and } \gamma_0 = \frac{2\sigma^2 E_b}{N_0}$$

B. Throughput

Let us consider a wireless data communication link employing a selective-repeat request ARQ scheme. Furthermore, for simplicity we assume that an ideal feedback channel is available and in the analysis we have neglected the CRC bits in each packet since they introduce negligible redundancy relative to the number of payload bits.

We define N_s as the number of symbols in a transmitted packet and $P_{p,i}$ as the packet error probability of the combined packet at transmission number i. The maximum number of retransmissions is set to $L-1$. Now we can derive the system throughput, η, as follows:

$$\eta = N_s \left[(1-P_{p,0}) + \frac{1}{2}(1-P_{p,1})P_{p,0} + \frac{1}{3}(1-P_{p,2})P_{p,0}P_{p,1} + \cdots + \frac{1}{L}(1-P_{p,L-1}) \prod_{i=0}^{L-2} P_{p,i} \right], \qquad \text{symbols/packet.} \tag{17}$$

When no error control coding is employed on the transmitted symbols and if the packet symbols are assumed uncorrelated, the packet error probability can be written as $PER = 1-(1-P_s)^{N_s}$, where P_s is the symbol error probability of the packet symbol. When FEC is employed, the packet error probability will depend on the kind of FEC used and its error correction capabilities.

V. Numerical Results

The main purpose of the numerical evaluations is to draw insight on how the proposed ARQ scheme performs in terms of increasing the throughput when maximizing the Euclidean distance between re-transmissions. In order to compare the performance of the proposed ARQ scheme we will use a ARQ scheme employing Chase combining as reference. The fading multipath fading channel is assumed slowly varying Rayleigh distributed and uncorrelated between the transmissions. Furthermore, we will also the performance under ideal channel conditions, i.e., an AWGN channel.

In Figures 6 and 7, the throughput for 16QAM with two transmission respective 4 transmissions is plotted as a function of E_b/N_0. The result clearly show the throughput gain the new scheme provides when the signal mapping is proper selected in different transmissions, especially when the channel detoriates. It can also be observed from the same figure that with increasing SNR less errors are caused by the channel and both the proposed ARQ scheme and the Chase combined ARQ scheme will have the same throughput. From the figures it is observed that we gain most between first and second transmission, and then decreases when the number of transmissions increases which can be seen in the case of four transmission. However, when the number of transmissions exceeds the modulation level, no more gain can be achieved.

From the figures, we note a knee in the range between 6 and 8 dB, exactly at $1/2$ of the max throughput. This is due to the packet always needs two transmissions to be accepted and the rough granularity of the retransmission (always a full packet). This knee also appears at $1/4$ of the max throughput in the case of four transmissions.

Figure 8 shows both simulation results and upper bounds for the average symbol error probability of the proposed scheme for 16QAM over AWGN channels as a function of E_b/N_0 and for different number of branches. From the figure, we can observe that a better error performance is obtained when the number of transmissions increases. Compared to a Chase combined scheme, a gain about 4dB is obtained when two transmissions are used. This gain has increased to about 4.7 dB after four transmissions.

Figure 9 illustrates the average bit error probability of the combined scheme over Rayleigh fading channels as a function of E_b/N_0 and for different number of transmissions. Also included in the figure is the BER of employing Chase combining. Although the both schemes have same diversity order, the proposed scheme outperforms the Chase combined scheme due to the nice spread of the signal constellations of the combined scheme that gives larger product distance.

VI. Conclusions

In this paper we have discussed the use of multi-level linear modulation in conjunction with retransmission diversity scheme. It is well known that multi-level linear modulation are bandwidth efficient but lacks in power efficiency due to the limited dimension of its signal space which is less or equal to two. By viewing the modulation level and the number of retransmissions as an augmented signal space we can achieve a good spread for the modulation signal points and can be quite efficient for high-level linear modulation techniques. The obtained simulation showed that the performance gain increases with the number of transmissions and modulation level. This proposed scheme can be used to solve the bandwidth efficiency loss seen retransmission diversity schemes without increase in receiver complexity. It can also provide good power saving even when the diversity branches of the wireless system are very uncorrelated.

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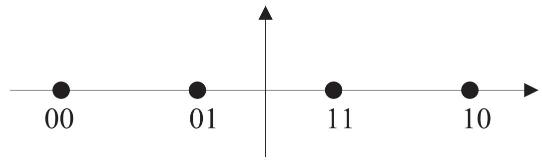

Fig. 1. Signal constellations of 4PAM.

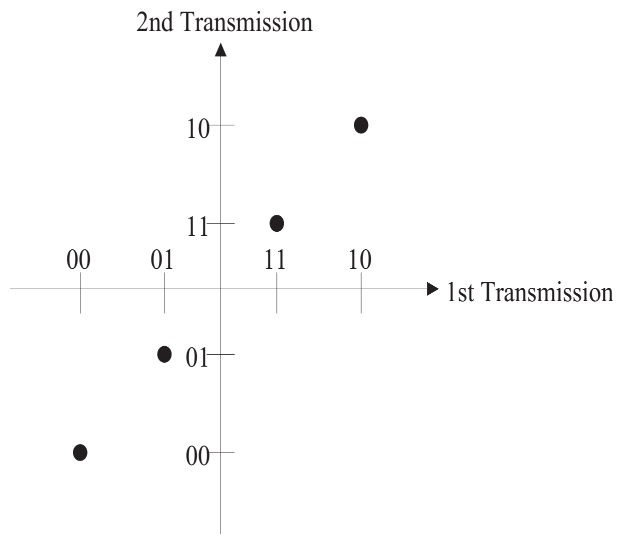

Fig. 2. Signal constellations of the combined signal with 4PAM and Chase combining.

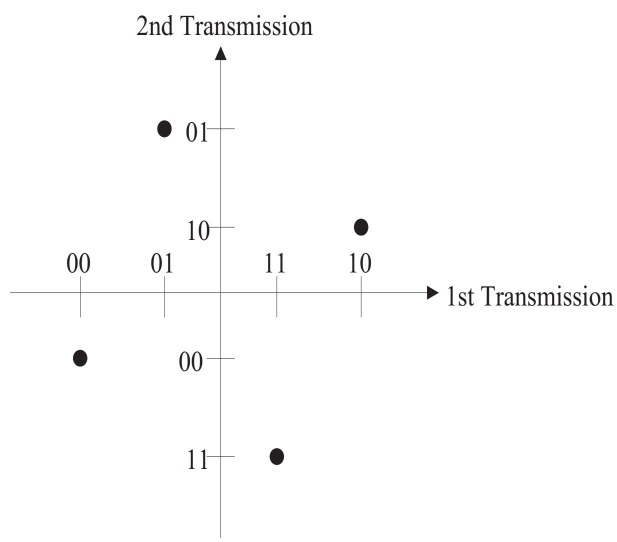

Fig. 3. Signal constellations of the proposed scheme.

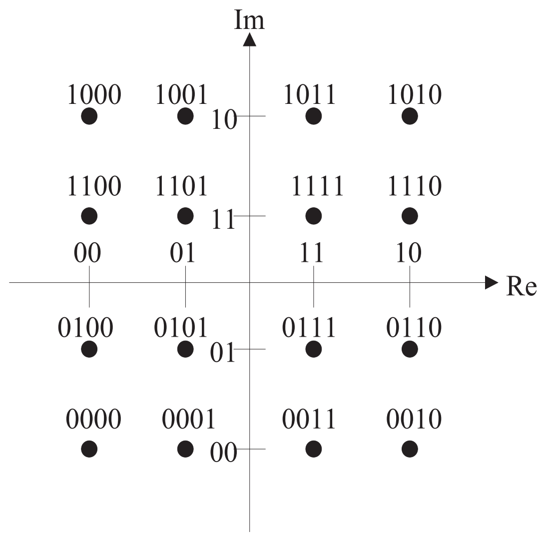

Fig. 4. Initial transmission for 16QAM

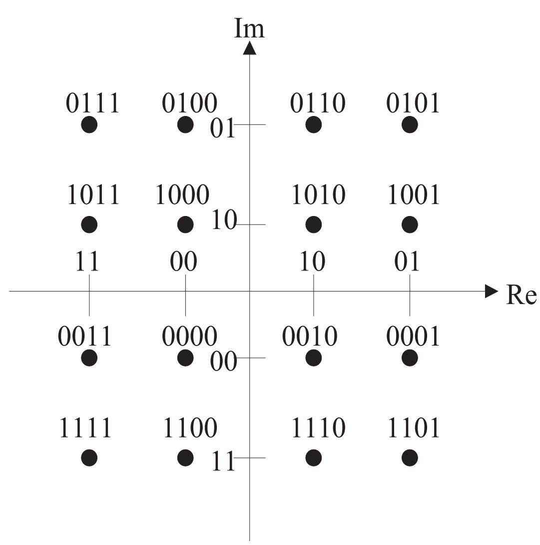

Fig. 5. second transmission for 16QAM

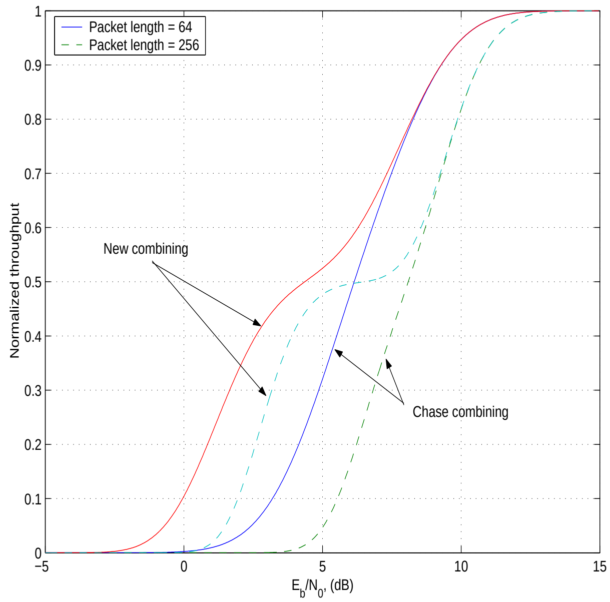

Fig. 6. Normalized throughput of the new ARQ scheme as a function of E_b/N_0, for 16QAM modulation and different packet length in AWGN channels. Maximum number of transmissions are limited to two.

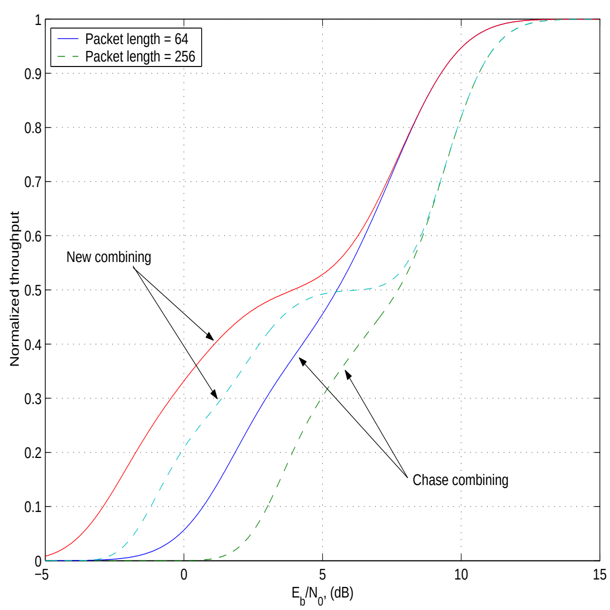

Fig. 7. Normalized throughput of the new ARQ scheme as a function of E_b/N_0, for 16QAM modulation and different packet length in AWGN channels. Maximum number of transmissions are limited to four.

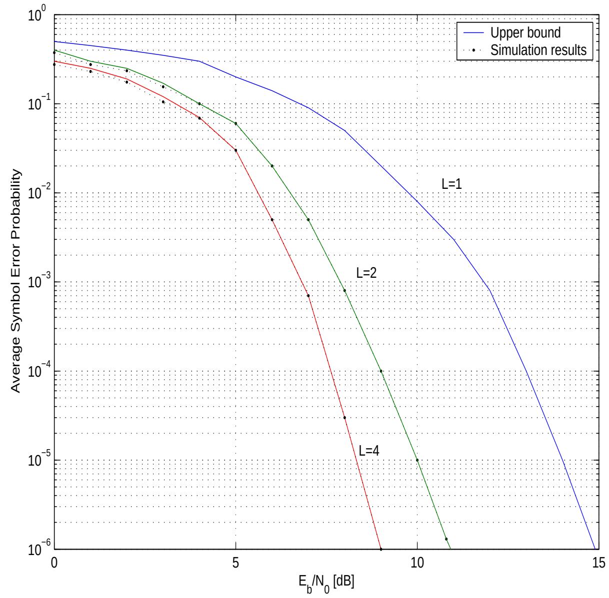

Fig. 8. Symbol error probability of proposed scheme with 16QAM over an AWGN channels and for different number of transmissions.

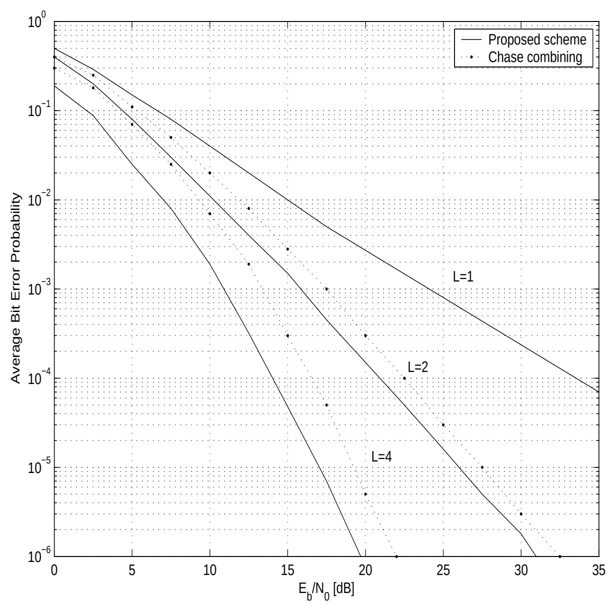

Fig. 9. Average bit error probability for the proposed scheme with 16QAM over Rayleigh fading channels and for different number of transmissions.