Performance of Coded Packet Retransmission Diversity Scheme

Mikael Gidlund
Department of Information Technology and Media, Electronics Design Division
Mid Sweden University, 851 70 Sundsvall, Sweden
Email: mikael.gidlund@miun.se

Abstract

This paper considers coded packet retransmission diversity scheme that reduce the probability of error. The proposed retransmission diversity scheme employs bit-interleaved coded modulation (BICM) and varies the bit-to-symbol mapping among multiple transmissions of the same packet (e.g., ARQ). The obtained mappings for forthcoming transmissions is chosen such that the sum of the magnitudes of the LLR of the bits forming the symbols in different (re)transmissions is maximized. Our performance results show that the proposed scheme performs better (in terms of BER) than a retransmission scheme transmitting same mappings all the time (e.g., Chase combining).

I. Introduction

Due to the growing demand for high data-rate mobile and personal communication systems, spectral efficiency is of primary concern in designing wireless communication systems. Consequently, multi-level modulation (M-QAM and M-PSK) has become attractive modulation technique for wireless communication. Bit-interleaved coded modulation (BICM) is bandwidth-efficient coding technique based on serial concatenation of binary error-correcting coding, bit-by-bit interleaving, and multi-level modulation [1]. The two first systems that have implemented the concept of BICM is HIPERLAN/2 and DVB-T [2], [3]. The general idea with BICM is to map the encoded bits, after interleaving, to a certain constellation. The decoding is performed by first computing the log-likelihood ratios (LLRs) of the coded bits from the soft output of the channel. These LLR values are then sent to a soft-input binary decoder.

In packet communication systems, packet retransmission is often requested when a received packet is detected to be in error. This scheme, termed automatic repeat request (ARQ), is intended to ensure extremely low packet error rate. During the ARQ process, the same data is sent until recovered without errors. The efficiency of ARQ can be improved by reusing the data from previous (re)transmissions instead of discarding them, this technique creates a diversity gain. This technique is termed hybrid ARQ and has been well investigated in literature [4]-[7]

An effective method to achieve diversity is to vary the bit-to-symbol mapping in M-ary modulation for each packet (re)transmission. This results in improved packet combining performance in terms of reduced packet error rate (PER) compared to a system without symbol mapping. Ottnes and Maseng proposed the use of nonuniform constellations for retransmitted packets in systems that require unequal error protection [8]. In [9], Gidlund proposed a mapping scheme that view the signal constellations of the modulation scheme in an augmented signal space formed by the modulation signal dimension and the number of retransmissions. That augmented signal space provided a good spread of the modulation signal points and the error probability was increased.

In this letter, we will use the LLR of the bits forming a M-QAM symbol in the optimum selection of mappings. We propose to choose the mappings for multiple (re)transmissions such that the sum of the magnitudes of the LLR of the bits forming the M-QAM symbols in different (re)transmissions is maximized. The rest of the letter is organized like this: In Section II the system model is presented and in Section III we discuss the proposed packet retransmission diversity scheme. In Section IV we discuss numerical results and finally in Section V we conclude the work.

II. System Model

Let us consider a bandwidth efficient M-ary modulation scheme where a data block B consisting of $b = \log_2 M$ bits, which are mapped to a point in the signal constellation via a bit-to-symbol mapping function ψ, and this signal point $\psi(B)$ is transmitted over the channel. In order to achieve packet combining diversity, the same bits may be transmitted more than once. Let L be the number of retransmissions. The data block B can either be retransmitted by using the same bit-to-symbol mapping in all transmissions, or vary the bit-to-symbol mapping in each transmission $\psi_1, \psi_2, \cdots, \psi_l$. Assuming that the transmitted symbol s undergoes fading, the received signal y_l (after multiple transmissions) can then be written as

$$y_i = h\psi_i(s) + n_i, \quad i = 1, 2, \cdots, L, \tag{1}$$

where h is the complex fading coefficient with $E\{||h||^2\} = \Omega$, and the r.v's $||h||$'s for different symbols are assumed to be i.i.d. Rayleigh distributed. A received and combined symbol is first converted into $\log_2 M$ bit metrics by a soft demodulator. Assuming perfect knowledge of the CSI at the receiver, the combined signal output for symbol s_k is given by $\hat{s}_k = hs_k + \zeta_k$, we define ζ as a complex gaussian random variable with zero mean and variance $h\sigma^2$.

III. LLR-based Selection of Signal Constellations

Let us define the log-likelihood ratio of bit $b_i,\ i = 1, 2, \cdots, b_i$ as following:

$$\begin{aligned}\Lambda_{s_k}(b_i) &= \log\left(\frac{Pr(b_i=1|y,h)}{Pr(b_i=0|y,h)}\right)\\ &= \log\left(\frac{Pr(b_i=1|\hat{s}_k,h)}{Pr(b_i=0|\hat{s}_k,h)}\right)\end{aligned} \tag{2}$$

The optimum decision rule is to decide $\hat{b_i} = 1$ if $\Lambda(b_i) \geq 0$, and 0 otherwise. Furthermore, we also assume that all symbols are equally probable and that fading is independent of the transmitted symbols. According to Bayes' rule, we can rewrite (2) as:

$$\Lambda_{s_k}(b_i) = \log\left(\frac{\sum_{\alpha\in S_i^{(1)}} f_{\hat{s}_k|s,h}(\hat{s}_k|s,h=\alpha)}{\sum_{\beta\in S_i^{(0)}} f_{\hat{s}_k|s,h}(\hat{s}_k|s,h=\beta)}\right) \tag{3}$$

where S_i^1 and S_i^0 is defined as the set partitions that compromises symbols with $b_i = 1$ and $b_i = 0$, respectively. Using the well known fact that $f_{\hat{s}_k|s,h}(\hat{s}_k|s,h=\alpha) = \frac{1}{\sigma\sqrt{\pi}}\exp(1/\sigma^2||\hat{s}_k - h\alpha||^2)$ [10], then we can rewrite (3) as

$$\Lambda_{s_k}(b_i) = \log\left(\frac{\sum_{\alpha\in S_i^{(1)}} \exp(-1/\sigma^2||\hat{s}_k - h\alpha||^2)}{\sum_{\alpha\in S_i^{(0)}} \exp(-1/\sigma^2||\hat{s}_k - h\alpha||^2)}\right) \tag{4}$$

To find the optimum mappings for bit-to-symbol mapping diversity we take advantage of the soft information given by the LLRs of the bits forming the QAM symbol. To complicate our model, we are including FEC coding. In this case, we consider bit-interleaved coded modulation and for more practical reason we consider Log-Max demodulation method which can be modeled by replacing the log function in the metrics function with a max operation. Then the metric function can be rewritten as:

$$\Lambda_{s_k}(b_i) = -\min_{\hat{s}\in M:B_l(\hat{s})=c_l} ||\hat{s}_k - h\alpha||^2 \tag{5}$$

where $B_l(s) \in \{0,1\}$ denotes the l-th bit of the symbol s. We will iteratively compute the Lth mapping from the $L-1$ previous mappings. We define the sum of LLRs of a given bit in the previous $L-1$ mappings as

$$\epsilon(i,j) = \sum_{l=1}^{L-1} \overline{\Lambda}_{ij}^{(l)} \tag{6}$$

where $\overline{\Lambda}_{ij}^{(l)}$ is defined as the averaged Λ computed for the ith bit of the jth symbol in the mapping of the lth transmission and the averaging over the noise samples. Furthermore, We define Ψ as the set of mappings ($|\psi| = M!$). To choose the Lth mapping we need to solve the following optimization problem

$$\max_{\psi_L\in\Psi} \sum_{j=1}^{M} \sum_{i=1}^{\log_2 M} |\epsilon(i,j) + \overline{\Lambda}_{ij}^{(L)}|, \tag{7}$$

By using the above optimizing procedure we can construct new optimized constellations for next retransmission with respect to the system requirement. We obtained the optimum mappings for 8QAM by carrying out the optimization in (7). The resulting mappings is shown in Figures 1 and 2. We observe that the newly obtained signal constellation is still Gray mapped.

IV. Performance Analysis

A. Error Probability

We denote the coded symbol sequence of length N by $\mathbf{s} = (s_1, s_2, ..., s_N)$ where the kth element of $\mathbf{s}$, namely, s_k, represents the transmitted symbol at time k. For the case of known channel-state information (CSI) and given a fixed matrix $\mathbf{H}$, consisting of a set of fading coefficients h with fading amplitudes α_l, the conditional pairwise error probability (PEP), namely, the probability of deciding $\hat{\mathbf{s}}$ when indeed $\mathbf{s}$ was transmitted, is given by

$$P(\mathbf{s} \to \hat{\mathbf{s}}|\mathbf{H}) = \frac{1}{2} erfc\left(\sqrt{\frac{1}{4N_0}\sum_{l=0}^{L-1}\alpha_l^2|s_{k,l} - \hat{s}_{k,l}|^2}\right). \tag{8}$$

Following the method described in [12] and use the polar representation of the Gaussian probability integral

$$\frac{1}{2}erfc\sqrt{\gamma} = \frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{\gamma}{\sin^2\theta}\right)d\theta \tag{9}$$

then we get

$$P(\mathbf{s} \to \hat{\mathbf{s}}) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{l=0}^{L-1} R_k\left(\frac{1}{4N_0}\cdot\frac{|s_l - \hat{s}_l|^2}{\sin^2\theta}\right)d\theta, \tag{10}$$

where $R_K(\gamma)$ is the averaged exponential of the Rician fading amplitude α with Rice factor K. The usual union bound for linear binary codes leads to

$$P_b \leq \frac{1}{k_c}\sum_{d=1}^{\infty} W_I(d) P(\mathbf{s} \to \hat{\mathbf{s}}) \tag{11}$$

where $W_I(d)$ denotes the total input weight of error events at Hamming distance d.

B. Capacity

In this paper, we assumes that the symbols in the 2^M-ary modulation M are used with equal probability. The BICM capacity has a general form [11]:

$$C_{BICM}(SNR) = \sum_{l=1}^{\log_2 M} C_l(SNR), \tag{12}$$

where C_l is defined as the effective rate of the l-th bit channel at SNR. When we are using LOG-MAP decoding at the receiver the effective rate of the bit channel is given by

$$C_{l,LogMAP}(SNR) = 1 - \int_{v\in\mathbb{C}} \frac{e^{-|v|^2}}{\pi M} \sum_{\{} s \in M \log_2 \left(1 + \frac{\sum_{\{\hat{s}\in M: B_l(\hat{s})=1-B_l(s)\}} e^{-|v+\sqrt{SNR}(s-\hat{s})|^2}}{\sum_{\{\hat{s}\in M: B_l(\hat{s})=B_l(s)\}} e^{-|v+\sqrt{SNR}(s-\hat{s})|^2}} \right) dv, \tag{13}$$

which is equivalent to the result Caire et al. obtained in [1]. When employing the proposed scheme with changing the bit-to-symbol mapping in each transmission and using a more practical LOG-MAX demodulation, we can rewrite (13) to the following

$$C_{l,LogMAP}(SNR) = 1 - \int_{v\in\mathbb{C}} \frac{e^{-|v|^2}}{\pi M} \sum_{s\in M} \log_2 \left(1 + \frac{\max_{\{\hat{s}\in M: B_l(\hat{s})=1-B_l(s)\}} e^{-|v+\sqrt{SNR}(s-\hat{s})|^2}}{\max_{\{\hat{s}\in M: B_l(\hat{s})=B_l(s)\}} e^{-|v+\sqrt{SNR}(s-\hat{s})|^2}} \right) dv, \tag{14}$$

V. Numerical Results

To assess the performance of the proposed packet retransmission diversity scheme, we simulate a retransmission scenario where packet is originally transmitted using 16QAM. Furthermore, we consider the system using a rate-$1/2$ 16-state convolutional code. LLR information from previous transmission(s) is retained for use with current retransmission. The BER at the receiver when $L = 2, 3, \ldots, 10$ is measured using Monte Carlo simulation of 10000 packets with a packet size of $N = 1024$ bits. The number of transmissions are maximized to $L = 10$. The multipath fading channel is assumed slowly varying Rayleigh distributed and uncorrelated between the transmissions.

Figure 3 illustrates the average bit error probability of the proposed retransmission diversity scheme over Rayleigh fading channels as a function of E_b/N_0 and for different number of transmissions. Also included in the figure is the BER of Chase combining. The results show that the proposed scheme performs better due to a nice spread of signal constellations. For instance, at a BER of 10^{-3} the proposed scheme result in about 2 dB of E_b/N_0 advantage compared to the BER of Chase combining.

VI. Conclusions

In this letter, we have investigate a novel coded packet retransmission diversity scheme which efficiently uses the different bit-to-symbol mapping among different (re)transmissions and improve the overall performance. By changing the mappings in different transmissions we show that we can enhance the system performance significantly compared to system which transmit the same mapping in all transmissions. Furthermore, this proposed scheme does not increase the receiver complexity.

References

[1] G. Caire, G. Taricco, and E. Biglieri, "Bit-Interleaved Coded Modulation," *IEEE Trans. Info. Theory*, vol 44, no. 3, pp. 927-945, May 1998.

[2] ETSI TS 101 475 Broadband Radio Access Networks (BRAN); HIPERLAN Type 2 Physical Layer. *European Telecommunication Standard,* 2000.

[3] ETS 300 744. Digital broadcasting systems for television, sound, and data service; Framing structure, channel coding and modulation for digital terrestrial television. *European Telecommunication Standard,* 1997.

[4] M. J. Miller and S. Lin, "The analysis of some selective-repeat ARQ schemes with finite receiver buffer," *IEEE Trans. Commun.*, vol. 29, no. 9, pp. 1307-1315, Sept. 1981.

[5] E. J. Weldon Jr., "An improved selective-repeat ARQ strategy," *IEEE Trans. Commun.*, vol. 30, no. 3, pp. 480-486, Mar. 1982.

[6] D. Chase, "Code combining - a maximum-likelihood decoding approach for combining an arbitrary number of noisy packets," *IEEE Trans. Commun.*, vol. COM-33, no. 5, pp. 385-393, May 1985.

[7] B. A. Harvey and S. B. Wicker, "Packet combining systems based on the Viterbi decoder," *IEEE Trans. Commun.*, vol. 42, pp. 1544-1557, Feb./Mar./Apr. 1994.

[8] R. Otnes and T. Maseng, "Adaptive data rates using ARQ and nonuniform constellations," In *Proc. IEEE VTC´01-spring*, Rhodes, Greece, May 2001.

[9] M. Gidlund, "*On Packet Retransmission Diversity Schemes for Wireless Networks,*" Licentiate Thesis, TRITA-S3-RST0408, Royal Institute of Technology, Sweden, Dec. 2004.

[10] J. Proakis, "*Digital Communications,*" Prentics-Hall, 3rd Edition, 2003.

[11] J. Huber and U. Wachsmann, "Capacities of equivalent channels in mulitlevel coding schemes," *IEE Elec. Letters,* vol. 30, no. 7, pp. 557-558, Mar. 1994.

[12] M. K. Simon and D. Divsalar, "Some new twists to problems involving the Gaussian probability integral," *IEEE Trans. on Commun.,* Vol. COM-46(2), pp. 200-210, Feb. 1998.

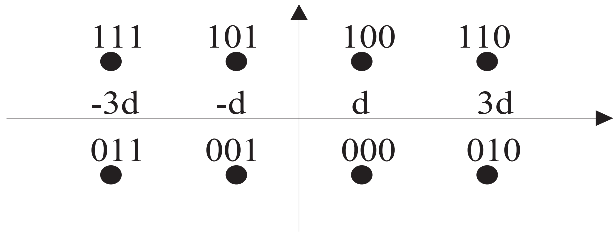

Fig. 1. Bit-to-symbol mapping obtained for first transmission.

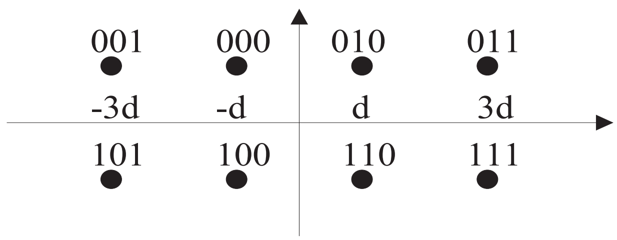

Fig. 2. Bit-to-symbol mapping obtained for second transmission.

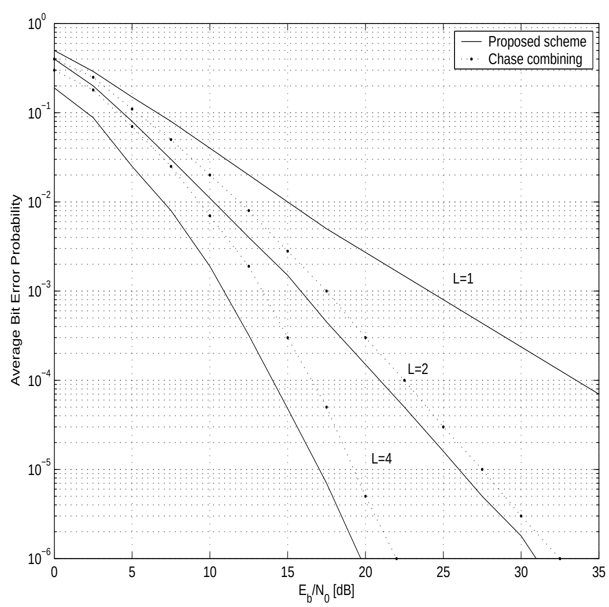

Fig. 3. Average bit error probability for the proposed scheme with 16QAM over Rayleigh fading channels and different number of transmissions.