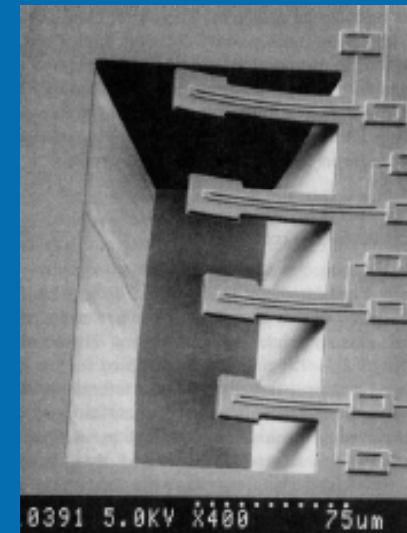
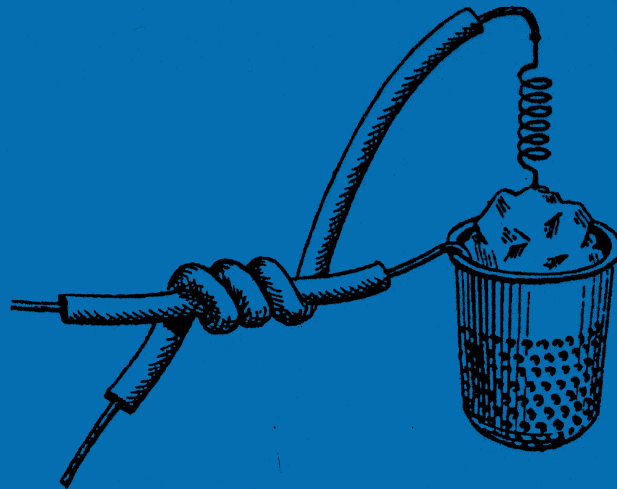


SENSOR DEVICES

MECHANICAL SENSORS



OUTLINE

- **4 Mechanical Sensors**
 - **Introduction**
 - **General mechanical properties**
 - **Piezoresistivity**
 - **Piezoresistive sensors**
 - **Capacitive sensors**
 - **Applications**

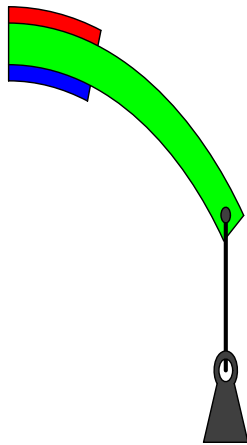
INTRODUCTION

- MECHANICAL SEMICONDUCTOR SENSORS
 - Combine electronic properties of semiconductors with its excellent mechanical properties
- TWO MAJOR CLASSES OF MECHANICAL SENSORS
 - **Piezo**resistive sensors (material property in silicon)
 - Capacitive sensors (relative motion of electrodes)
- APPLICATIONS
 - Pressure sensors
 - Accelerometer
 - Flow sensors

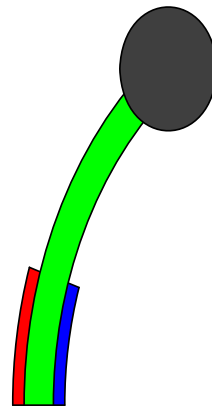
"Piezo" = "squeeze" or "press"

INTRODUCTION

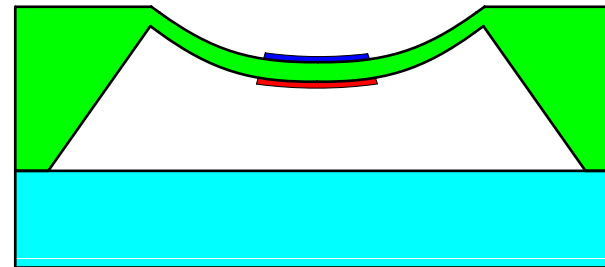
Piezoresistive sensing applications



Load cell

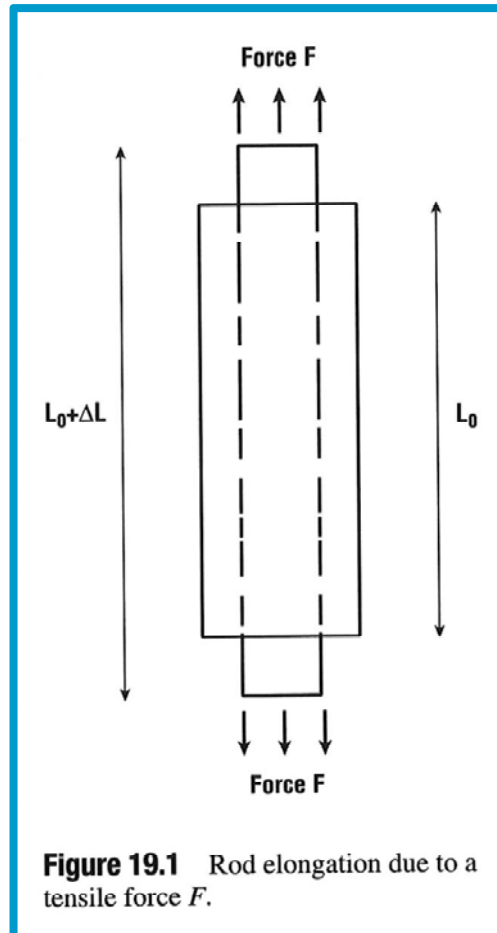


Accelerometer



Pressure sensor

General mechanical properties



Hooke's law: $\sigma = E\varepsilon$

Stress: $\sigma = F / \text{Area}, \quad [\text{N/m}^2]$

Strain: $\varepsilon = \Delta L / L$

→ Young's module: $E, \quad [\text{N/m}^2 = \text{Pa}],$

Volumetric change:

$$\frac{\Delta V}{V} \approx \frac{\Delta L}{L} - \frac{2\Delta L_l}{L} = (1 - 2\nu) \frac{\Delta L}{L}$$

→ Poisson's ratio: $\nu = \frac{\Delta L_l}{\Delta L} = -\frac{\varepsilon_l}{\varepsilon}, \quad l = \text{lateral}$

Silicon: $E_s \approx 190 \text{ GPa}, \quad \nu_s \approx 0.28$

General mechanical properties

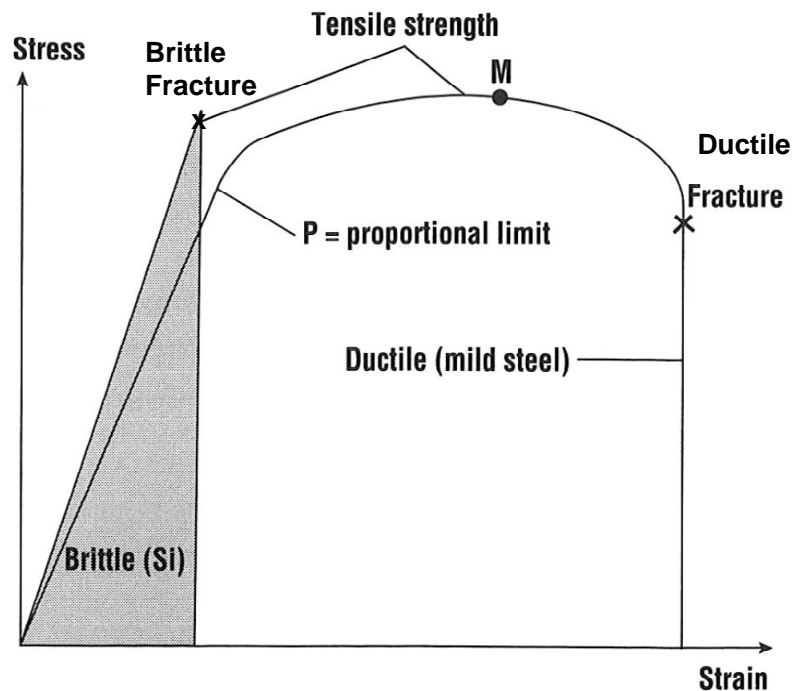


Figure 19.2 Stress–strain curve for a typical metal as well as for a brittle material like silicon (high Young’s modulus and no plastic deformation region) (*reprinted with premission from Madou [7]. Copyright CRC Press*).

- **Silicon**

- Linear elasticity until fracture
- No plastic deformation
- Excellent for sensor applications

- **Metals**

- Linear elastic behaviour for small strain only

- **Strength**

- Yield strength = tensile stress where plastic deformation starts (non-reversible)
- Tensile strength = maximum tensile stress before facture

General mechanical properties

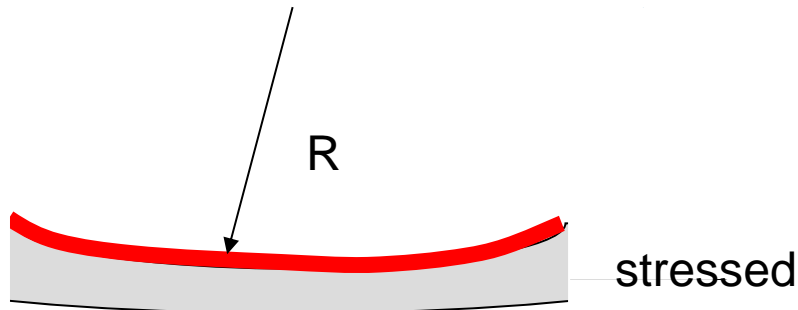
Table 19.1 Properties of materials

	Yield Strength (10⁹ Pa)	Young's Modulus (10⁹ Pa)	Density (g/cm³)	Thermal Conductivity (W/cm °C)	Thermal Expansion (10⁻⁶/°C)
Diamond (single crystal)	53.0	1035.0	3.5	20.0	1.0
SiC (single crystal)	21.0	700.0	3.2	3.5	3.3
Si (single crystal)	7.0	190.0	2.3	1.6	2.3
Al ₂ O ₃	15.4	530.0	4.0	0.5	5.4
Si ₃ N ₄ (single crystal)	14.0	385.0	3.1	0.2	0.8
Gold	—	80.0	19.4	3.2	14.3
Nickel	—	210.0	9.0	0.9	12.8
Steel	4.2	210.0	7.9	1.0	12.0
Aluminum	0.2	70.0	2.7	2.4	25.0

General mechanical properties

Stress in thin film

Stress in thin film cause a curvature of the sample, which can be measured using a laser system



$$\frac{1}{R} = 6(1-\nu) \frac{\Delta s}{ET^2}$$

$$\Delta s = \sigma t$$

= surface stress [N/m]

R= radius

E= Young's module in substrate

T= substrate thickness

t = thin film thickness

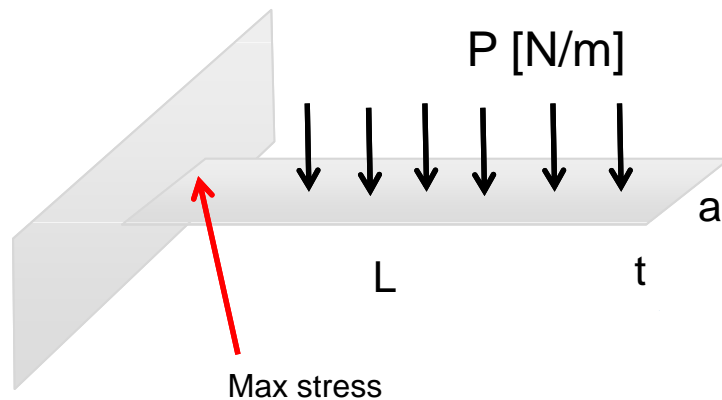
ν = Poisson's ratio in substrate

General mechanical properties

- Cantilever beams
 - Max deflection
 - Max longitudinal stress
 - Resonant frequency
- Square membranes
 - Max deflection
 - Max longitudinal and transverse stress
 - Resonant frequency

General mechanical properties

Cantilever beam with uniform distributed load ($P=F/\Delta x$)



SENSOR:

$$P = at\rho \cdot \text{acceleration}$$

$$\Delta P = at\rho_{\text{measurand}} \cdot g$$

Beam equation: $EI \frac{d^4 w(x)}{dx^4} = P$

Beam stiffness: $EI = Eat^3/12$

$$I = at^3/12 \quad (\text{2nd moment of inertia})$$

Deflection: $w(x) = \frac{P}{24EI} x^2 (6L^2 - 4Lx + x^2)$

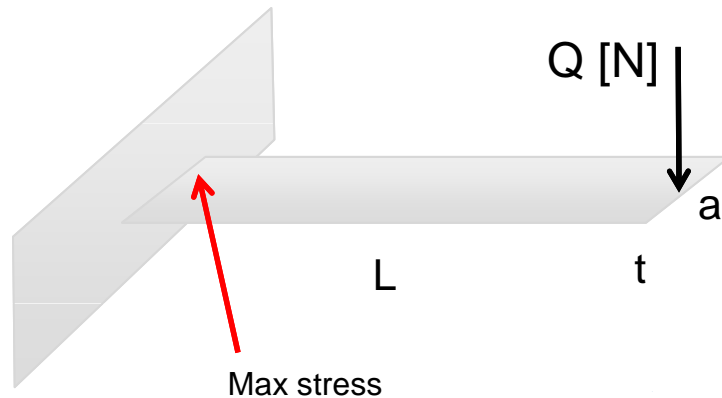
$$w(L) = \frac{PL^4}{8EI}$$

Surface stress: $\sigma(x) = -\frac{tE}{2} \frac{d^2 w(x)}{dx^2}$

Max stress: $\sigma(0) = \frac{PL^2 t}{4I} = \frac{3PL^2}{at^2}$

General mechanical properties

Cantilever beam point load at the end



$$\text{Deflection: } w(x) = \frac{Qx^2}{6EI}(3L - x)$$

$$w(L) = \frac{QL^3}{2EI}$$

$$\text{Max stress: } \sigma(0) = \frac{QLt}{2I} = \frac{6QL}{at^2}$$

Resonant frequency:

$$F_0 = 0.161 \frac{t}{L^2} \sqrt{\frac{E}{\rho}} = 0.161 \frac{t}{L} \sqrt{\frac{Eta}{ML}}$$

Quasi-static sensing

$$f_{\text{measure}} < F_0$$

SENSOR:

$$Q = \text{mass} \cdot \text{acceleration}$$

General mechanical properties

SQUARE MEMBRANES (UNIFORM LOAD)

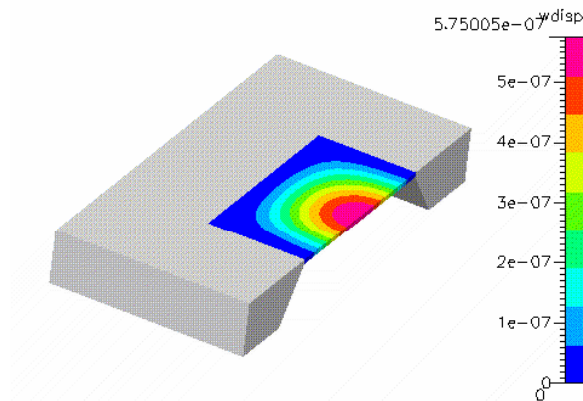
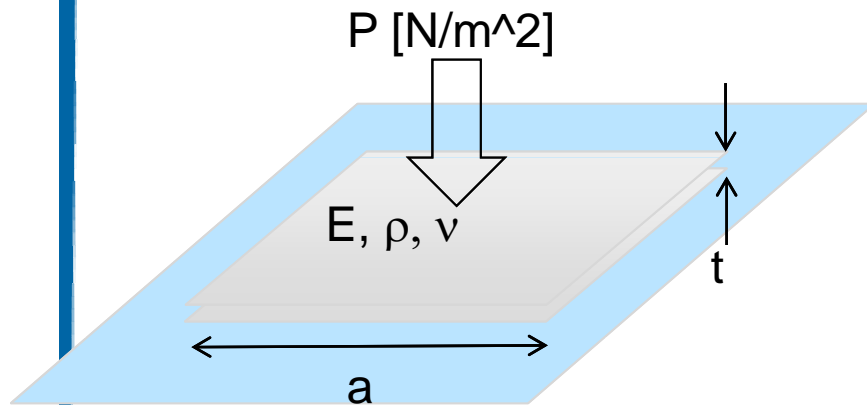


Plate equation: $D \nabla^4 w(x, y) = P$

Membrane stiffness: $D = \frac{Et^3}{12(1-\nu^2)}$

Max deflection: $w_{\max} = 0.001265 \frac{Pa^4}{D}$

Max longitud. stress: $\sigma_l = 0.3081 \frac{Pa^2}{t^2}$

Max transverse stress: $\sigma_t \approx \nu \sigma_l$

Resonant frequency: $F_0 = 1.654 \sqrt{\frac{12D}{\rho t a^4}}$

Piezoresistivity

$$R = \rho \frac{L}{a^2}$$

Resistivity change in semiconductor:

- Resistivity change due to mechanical stress
 - **piezoresistive effect**
- Large resistivity change
- Dependence on
 - Doping (n- or p-type, doping concentration)
 - Temperature
 - Direction of force and direction of current flow (anisotropic effect)

$$\rho = \rho(\sigma)$$

$$R(\sigma) = R + \rho' \sigma \frac{L}{a^2}$$

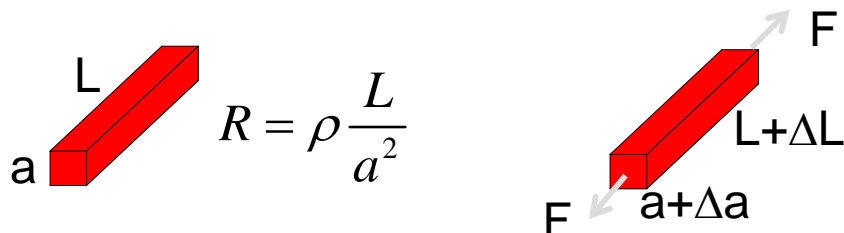
$$\Delta R/R = (\rho'/\rho) \sigma$$

$$\equiv \pi \sigma$$

$$= \pi E \varepsilon$$

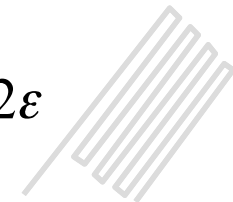
$$\Delta R/R \approx 100 \varepsilon$$

Resistivity change in **metal strain gauge** – mainly due to geometric effect



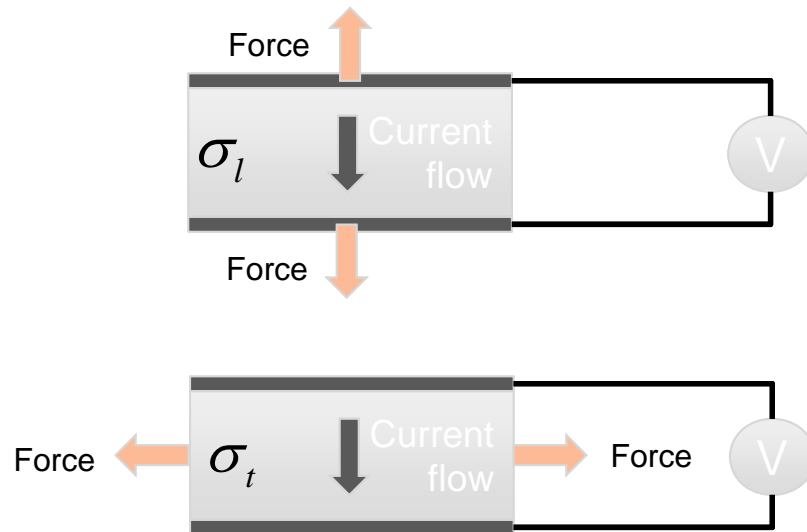
$$R = \rho \frac{L}{a^2}$$

$$\frac{\Delta R}{R} = (1 + 2\nu) \frac{\Delta L}{L} \sim 2\varepsilon$$



Piezoresistivity

Longitudinal and transverse piezoresistance coefficients



- Longitudinal stress: $\Delta R/R = \pi_l \sigma_l$
 π_l = longitudinal piezoresistance coefficient

- Transverse stress: $\Delta R/R = \pi_t \sigma_t$
 π_t = transverse piezoresistance coefficient

Stress: $\sigma = \text{Force}/\text{Area}$

$$E_i = \sum_j \rho_{ij} j_j$$

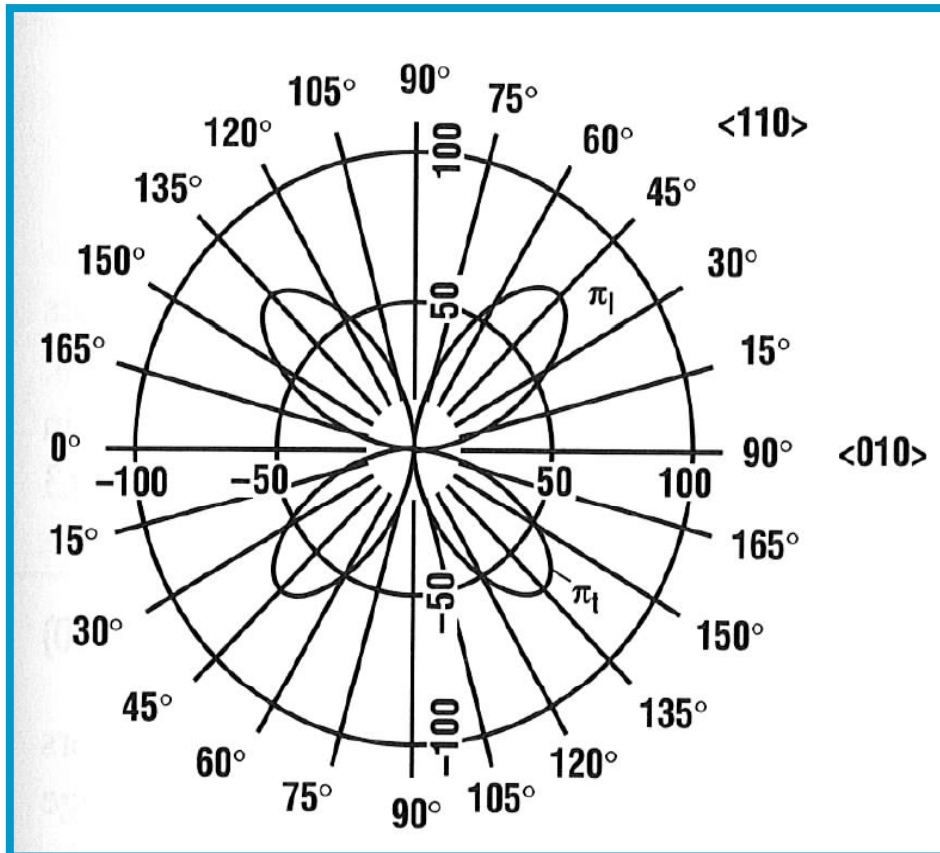
$$\rho_{ij} = \sum_k \sum_l \pi_{ijkl} \sigma_{kl}$$

- In general we have both longitudinal and transverse stresses:

$$\Delta R/R = \pi_t \sigma_t + \pi_l \sigma_l$$

PIEZORESISTIVITY

Piezoresistance coefficients in
p-type silicon [10^{-11} /Pa]



- 100 plane
- Upper half
 - Longitudinal coefficient π_l
- Lower half
 - Transverse coefficient π_t

In sensor application the π should be as large as possible, i.e. resistors along <110> direction

$$\pi_l = \frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44})$$

$$\pi_t = \frac{1}{2}(\pi_{11} + \pi_{12} - \pi_{44})$$

PIEZORESISTIVITY

Resistors along $\langle 110 \rangle$ direction in (100) wafers (common for bulk micromachining)

$$\pi_l = \frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44})$$

$$\pi_t = \frac{1}{2}(\pi_{11} + \pi_{12} - \pi_{44})$$

$$\frac{\Delta R}{R} = \pi_l \sigma_l + \pi_t \sigma_t$$

$$\approx \frac{\pi_{44}}{2}(\sigma_l - \sigma_t)$$



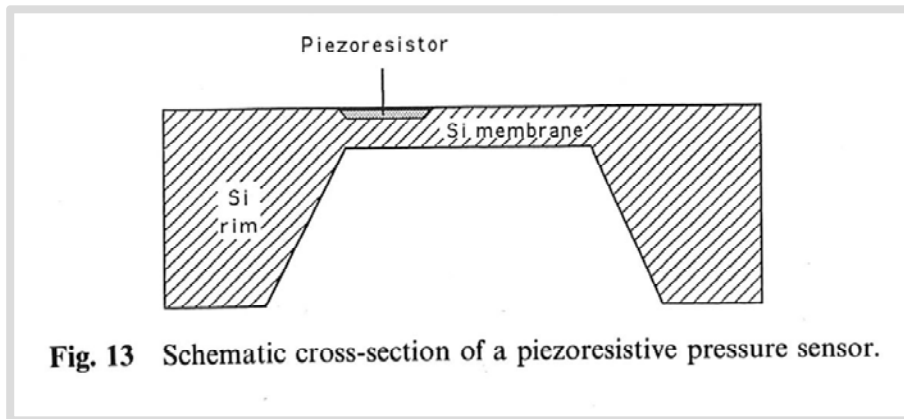
Table 18.1. Typical room-temperature piezoresistance coefficients for n- and p-type silicon [98].

Type	Resistivity	π_{11}	π_{12}	π_{44}
Units	$\Omega\text{-cm}$	10^{-11} Pa^{-1}	10^{-11} Pa^{-1}	10^{-11} Pa^{-1}
n-type	11.7	-102.2	53.4	-13.6
p-type	7.8	6.6	-1.1	138.1

$$\frac{\Delta R}{R} \approx \frac{\pi_{44}}{2} \sigma \approx \frac{\pi_{44}}{2} E \varepsilon \approx 100 \varepsilon$$

Piezoresistive Sensors

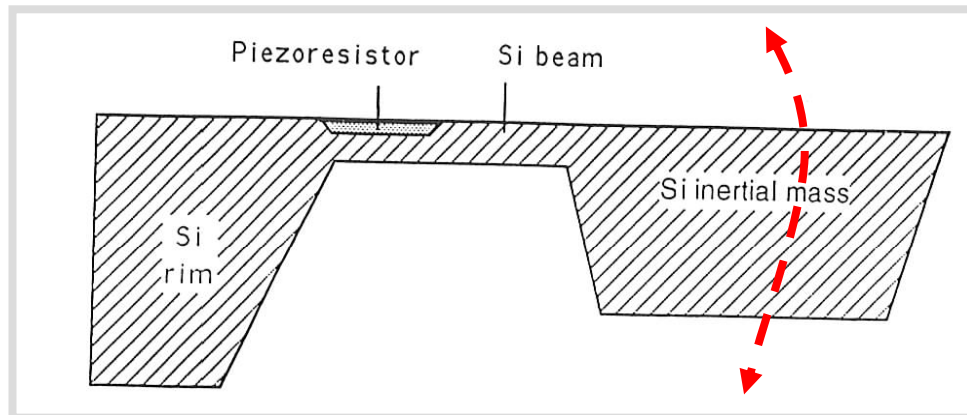
Piezoresistive pressure sensor



- Membrane fabrication
 - Anisotropic etch
- Piezoresistor fabrication
 - doped area
 - or deposited polysilicon resistor on an insulator (SiO_2 or Si_3N_4)
- Piezoresistor position
 - at the edges of the membrane where the stress is maximal

Piezoresistive Sensors

Piezoresistive accelerometer



- The piezoresistor must be placed where the stress is maximal
- To increase the sensitivity an inertial mass is included

Piezoresistivity

$$R = \rho \frac{L}{w \cdot t}$$

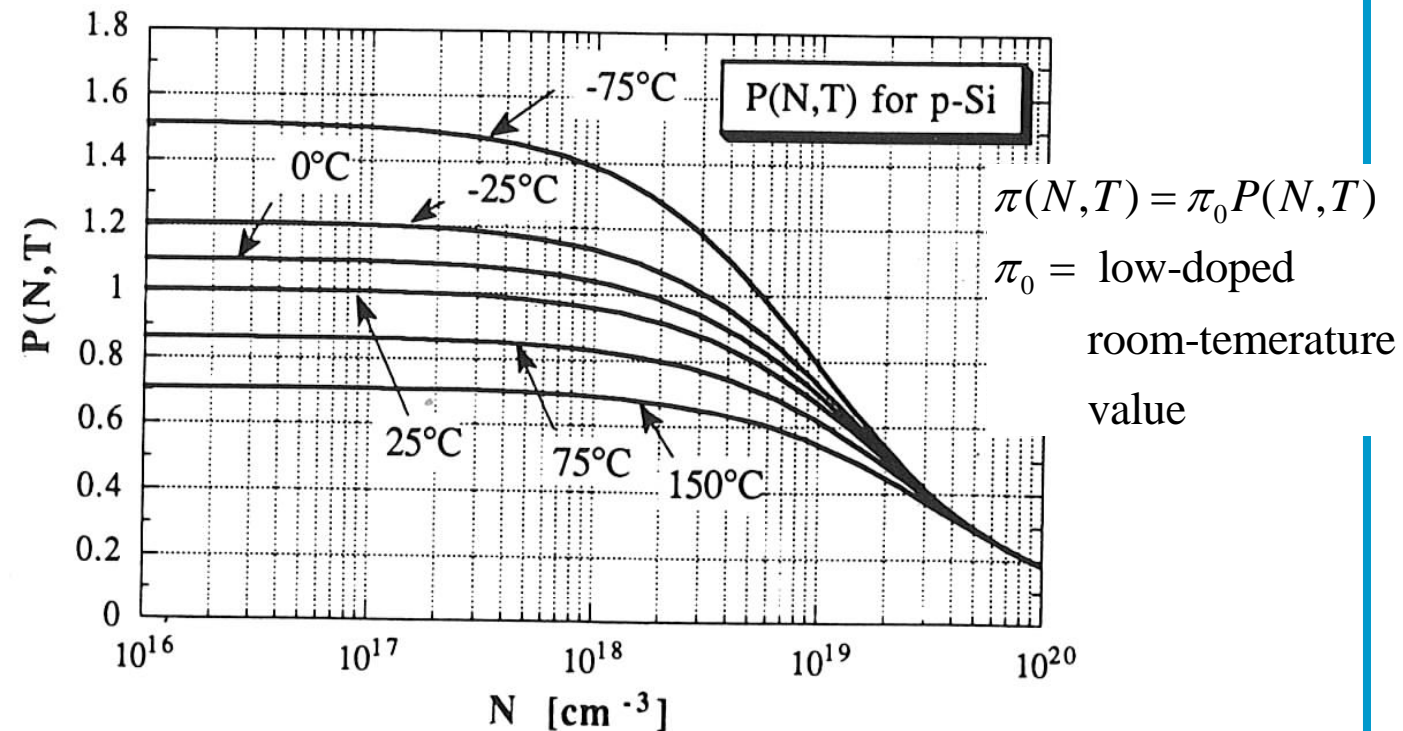


Fig. 10 Piezoresistance factor $P(N, T)$ as a function of impurity concentration and temperature for p -type silicon. (After Ref. 49)

Piezoresistiv Sensor

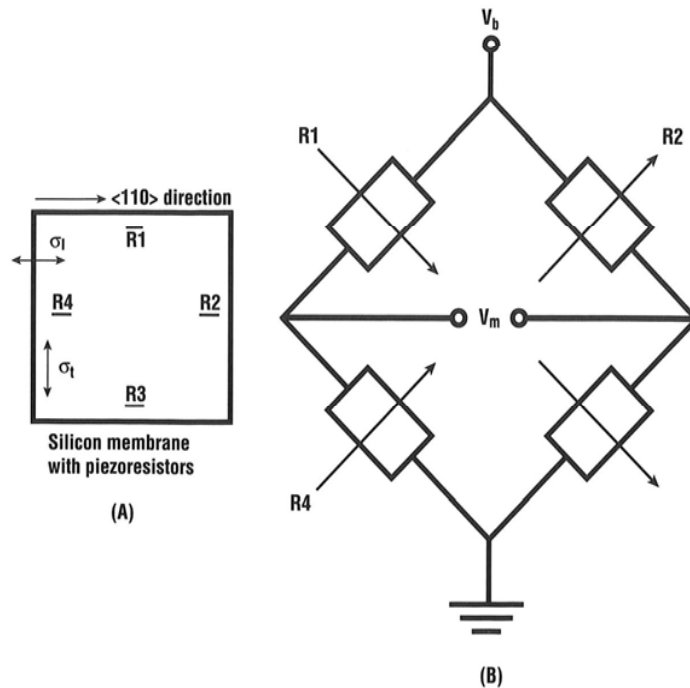


Figure 19.7 (A) Schematic drawing of the position and orientation of four piezoresistance elements on a silicon membrane with sides defined by anisotropic silicon etching of a (100) silicon wafer; (B) Wheatstone bridge configuration of the four piezoresistive elements, with arrow directions indicating resistance increases or decreases for membrane deflection downward (after Sze [15], © 1994, reprinted with permission of John Wiley & Sons, Inc.).

Wheatstone Bridge Configuration

- R1 and R3 under lateral stress and decrease
- R2 and R4 under longitudinal stress and increase

$$\frac{\Delta R}{R} \approx -\frac{\pi_{44}}{2} \sigma$$

$$\frac{\Delta R}{R} \approx \frac{\pi_{44}}{2} \sigma$$

$$R_1 = R_3 = R - \Delta R$$

$$R_2 = R_4 = R + \Delta R$$

$$\frac{\Delta R}{R} = \frac{\pi_{44}}{2} (\sigma_l - \sigma_t)$$

$$V_m = V_b \left(\frac{\Delta R}{R} \right) = V_b \frac{\pi_{44}}{2} (\sigma_l - \sigma_t)$$

Piezoresistive Sensors

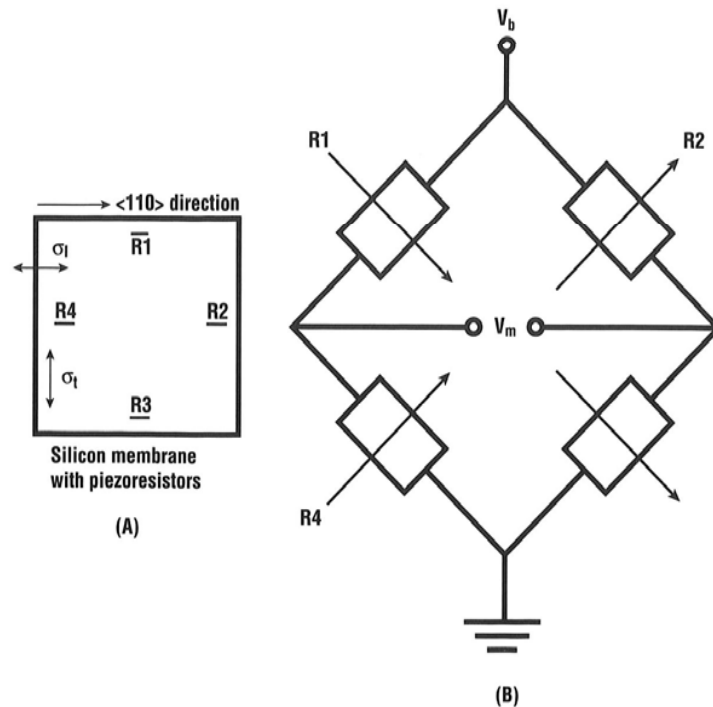


Figure 19.7 (A) Schematic drawing of the position and orientation of four piezoresistance elements on a silicon membrane with sides defined by anisotropic silicon etching of a (100) silicon wafer; (B) Wheatstone bridge configuration of the four piezoresistive elements, with arrow directions indicating resistance increases or decreases for membrane deflection downward (after Sze [15], © 1994, reprinted with permission of John Wiley & Sons, Inc.).

Pressure sensitivity for constant V_b :

$$S_v = \frac{\Delta V / V_b}{\Delta P} \text{ [mV/V-bar]}$$

$$= \frac{\Delta R / R}{\Delta P} = \frac{1}{2\Delta P} \pi_{44} (\sigma_l - \sigma_t)$$

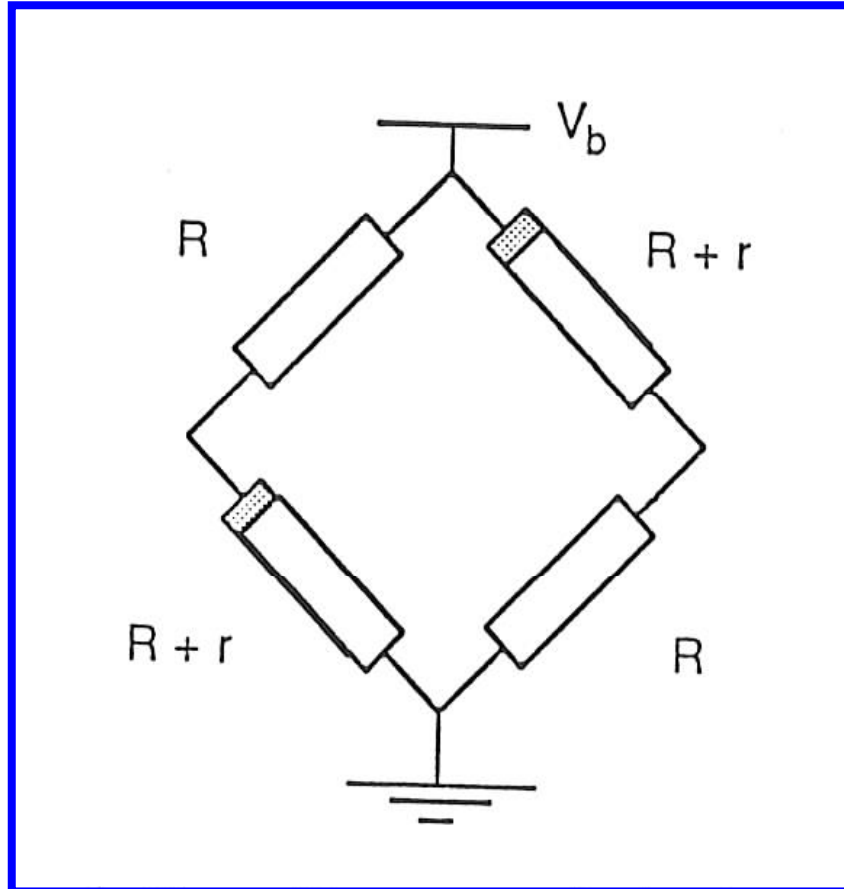
Pressure sensitivity for constant I_b :

$$S_i = \frac{\Delta V / I_b}{\Delta P} \text{ [mV/mA-bar]}$$

$$= \frac{\Delta R}{\Delta P} = \frac{1}{2\Delta P} R \pi_{44} (\sigma_l - \sigma_t)$$

Piezoresistive Sensors

Offset voltage



Symmetrical mismatch of the resistors, caused by difference in layout (parallel and perpendicular to the edges of the membrane)

$$R_1 = R_3 = R$$

$$R_2 = R_4 = R + r$$

$$V_o = V_b \left(\frac{r}{2R + r} \right)$$

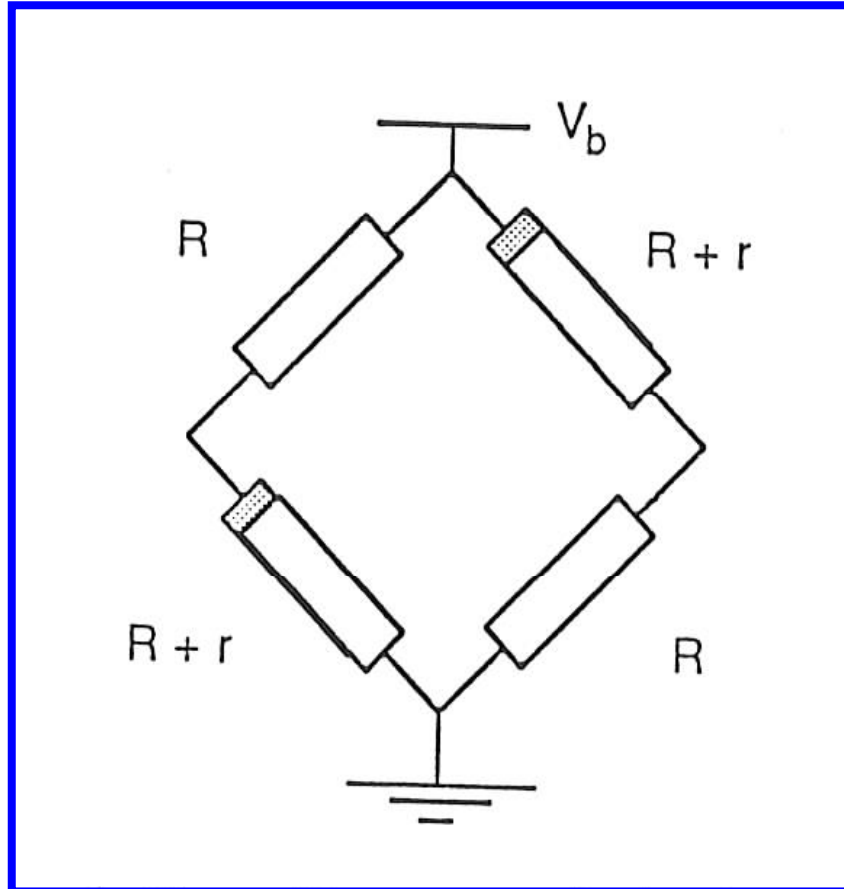
$$O = \frac{V_o}{V_b} = \frac{r}{2R + r} = \frac{1}{1 + 2R/r}$$

Offset also caused by residual stress on resistors

- pre-stress due to passivation layers and packaging

Piezoresistive Sensors

Temperature coefficient of offset



$$\frac{\partial O}{\partial T} = \frac{2Rr}{(2Rr + r)^2} \left(\frac{\dot{r}}{r} - \frac{\dot{R}}{R} \right)$$

If resistors have equal temperature coefficients

$$\frac{\dot{r}}{r} = \frac{\dot{R}}{R}, \quad \frac{\partial O}{\partial T} = 0$$

However, temperature dependence of pre-stress might be significant.

Piezoresistive Sensors

Temperature coefficient of sensitivity

Constant bridge
voltage

$$S \propto \pi_{44}(\sigma_l - \sigma_t)$$

$$\text{TCS}_v = \frac{1}{S} \frac{\partial S}{\partial T} = \frac{1}{\pi_{44}} \frac{\partial \pi_{44}}{\partial T} + \frac{1}{\sigma_l - \sigma_t} \frac{\partial (\sigma_l - \sigma_t)}{\partial T}.$$

Temperature coefficient for π_{44} can be high for p-type silicon

Constant bridge
current

$$S \propto \pi_{44} R (\sigma_l - \sigma_t)$$

$$\text{TCS}_i = \frac{1}{S} \frac{\partial S}{\partial T} = \frac{1}{\pi_{44}} \frac{\partial \pi_{44}}{\partial T} + \frac{1}{R} \frac{\partial R}{\partial T} + \frac{1}{\sigma_l - \sigma_t} \frac{\partial (\sigma_l - \sigma_t)}{\partial T}.$$

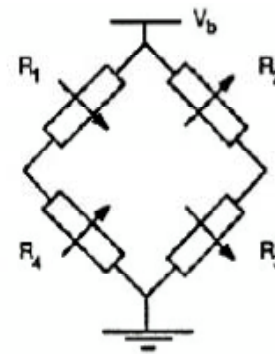


Piezoresistance Sensors

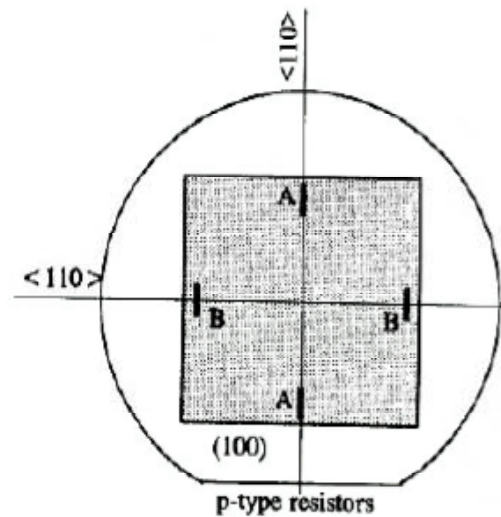
Place your resistors this way:

$\sigma_l = 10 \text{ MPa}$, $\sigma_t = 50 \text{ MPa}$:
 $\Delta R/R = 3\%$

Up to about 10 mV/kPa sensitivity
 of a final pressure sensor



A



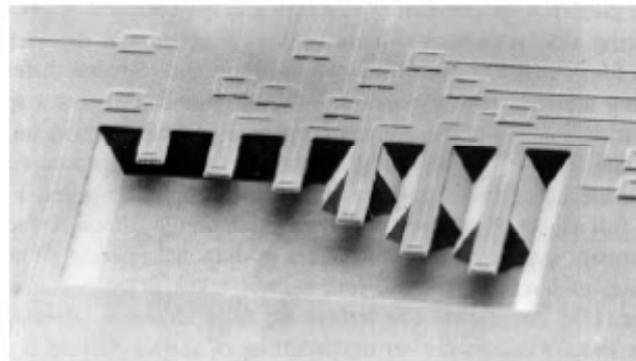
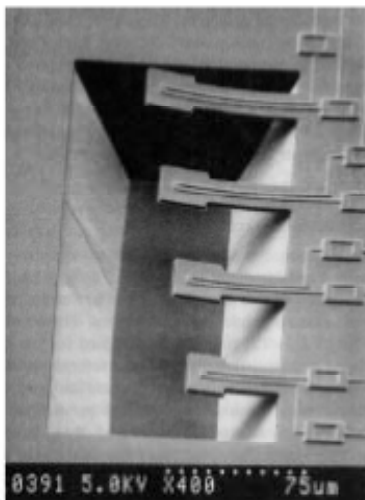
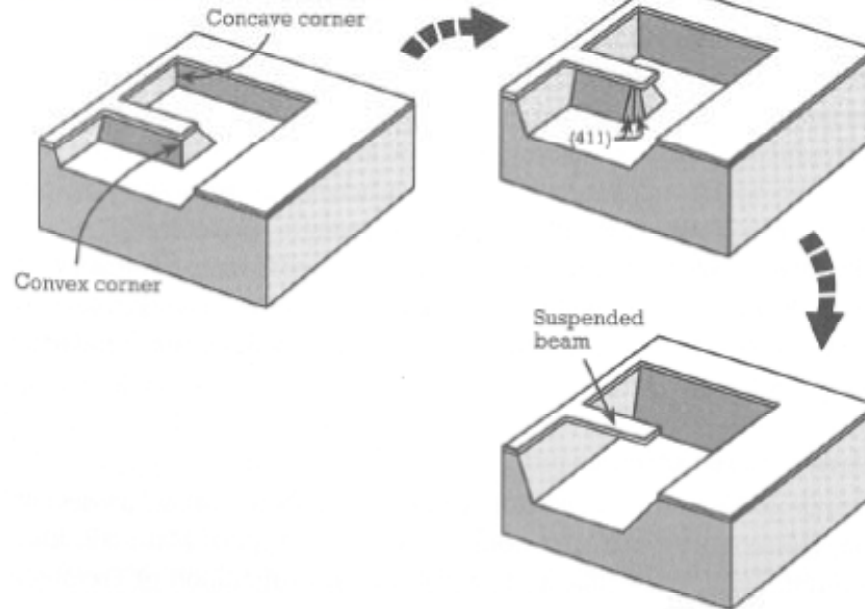
B

FIGURE 4.18 Measuring on a membrane with piezoresistors. (A) Wheatstone-bridge configuration of four in-diffused piezoresistors. The arrows indicate resistance changes when the membrane is bent downward. (B) Maximizing the piezoresistive effect with p-type resistors. The A resistors are stressed longitudinally and the B resistors are stressed transversally. (From Peeters, E., Ph.D. Thesis, KUL, Belgium, 1994. With permission.)

Piezoresistive Sensors

Undercutting

- Convex corners bounded by $\{111\}$ planes are attacked



Ristic

Capacitive Sensors

Capacitor two electrodes separated by a dielectric



Electronic should be close to the sensor, minimising the stray capacitance

$\Delta d \ll d$ result in

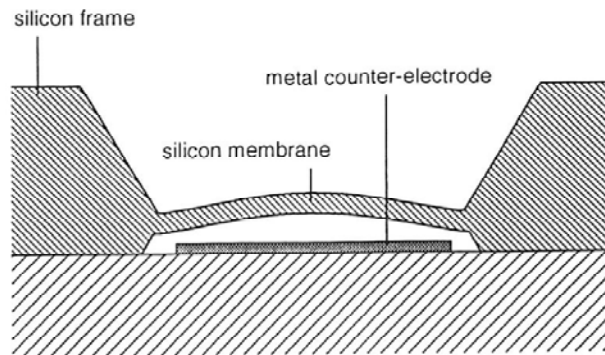
High sensitivity means Large area S and a small distance d

$$C_0 = \epsilon \frac{S}{d}$$

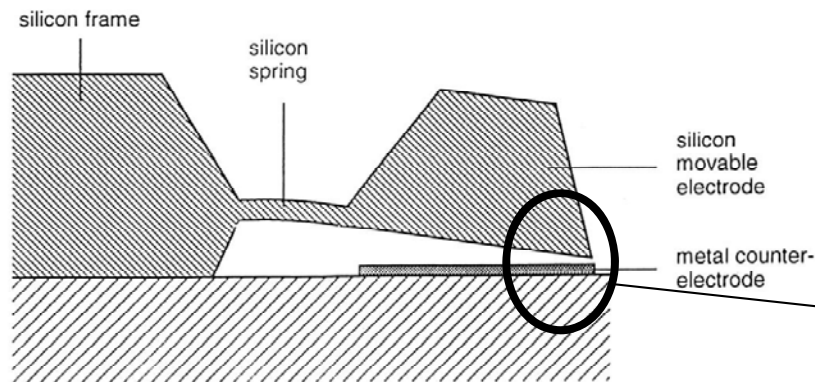
$$\frac{\Delta C}{\Delta d} = -\epsilon \frac{S}{d^2}$$

Advantage with capacitive sensors no direct sensitivity to temperature

Capacitive Sensors



(a)

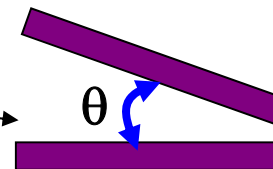


(b)

- a) Pressure sensor
- b) Accelerometer

$$C(\theta) = \frac{\epsilon S}{l\theta} \ln \left(\frac{2d + l\theta}{2d - l\theta} \right)$$

?

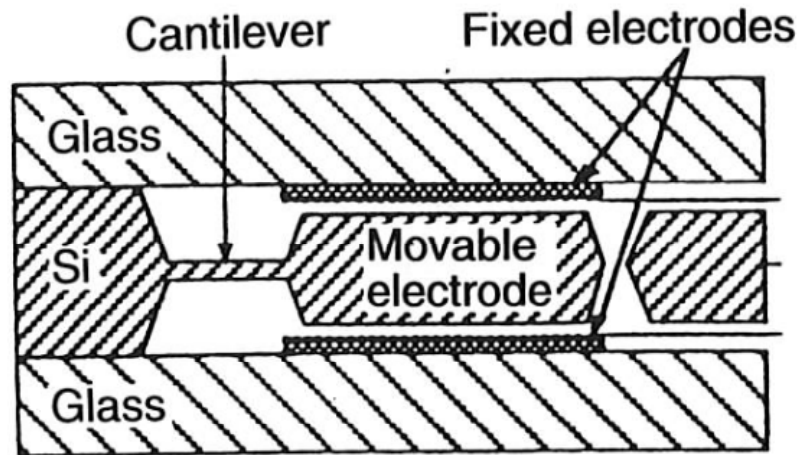


Comparison of different technologies

TABLE 3 Comparison of Some Characteristics of Three Common Sensing Technologies for Accelerometers (Adapted from Ref. 67)

	Capacitive	Piezoelectric	Piezoresistive
Impedance	High	High	Low
Size	Medium	Small	Medium
Temperature range	Very wide	Wide	Medium
Linearity error (sensor only)	High	Medium	Low
DC response	Yes	No	Yes
AC response	Wide	Wide	Medium
Damping available	Yes	No	Yes
Sensitivity	High	Medium	Medium
Zero shifts due to shock	No	Yes	No
Electronics required	Yes	Yes	No
Cost	Medium	High	Low

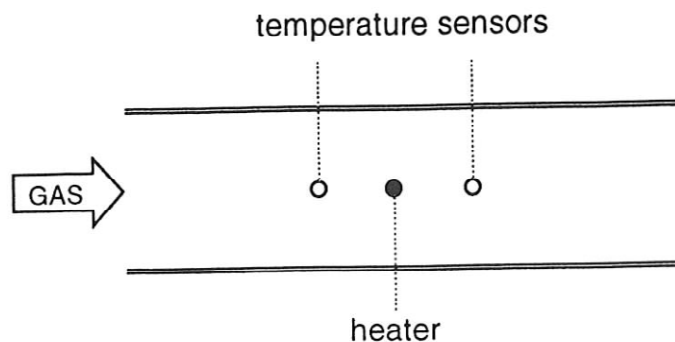
Applications



Symmetric capacitive accelerometer with low thermal sensitivity

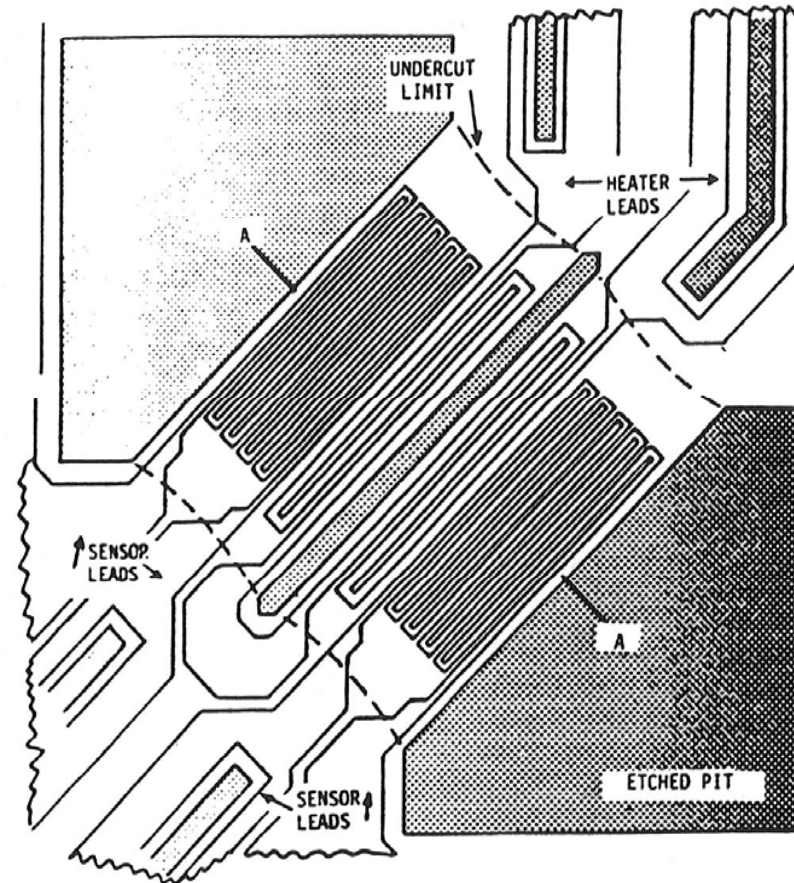
In some cases the movable electrode must be damped to avoid serious oscillations. A small cavity with a viscous liquid or gas can fulfil the requirements

Flow sensors (gas)

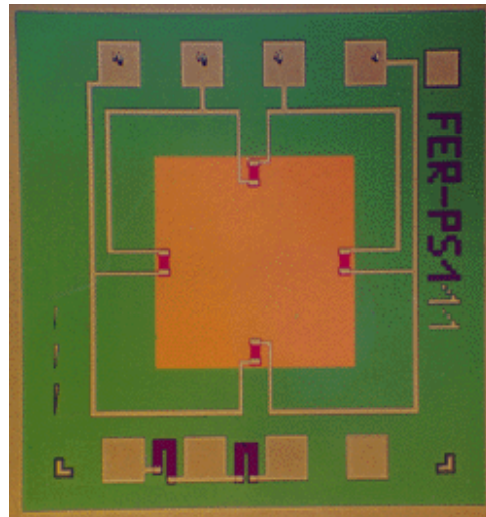


With no flow the two sensors display the same temperature

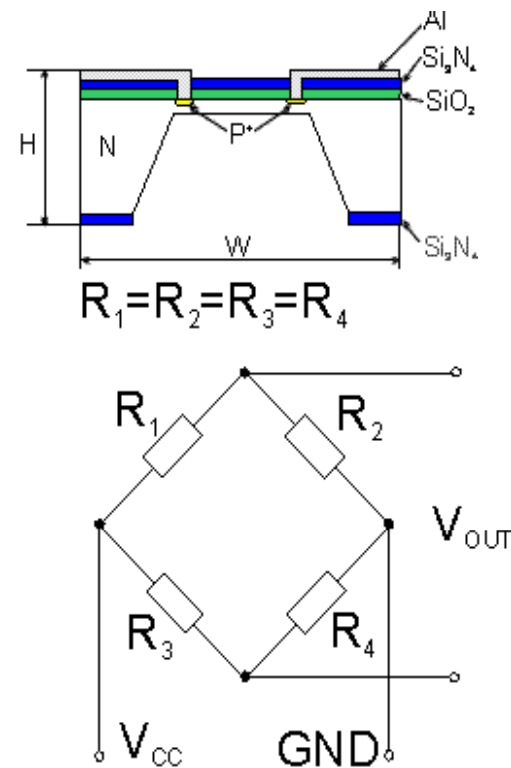
With gas flow the first sensor is cooled , while the second is heated up by the gas



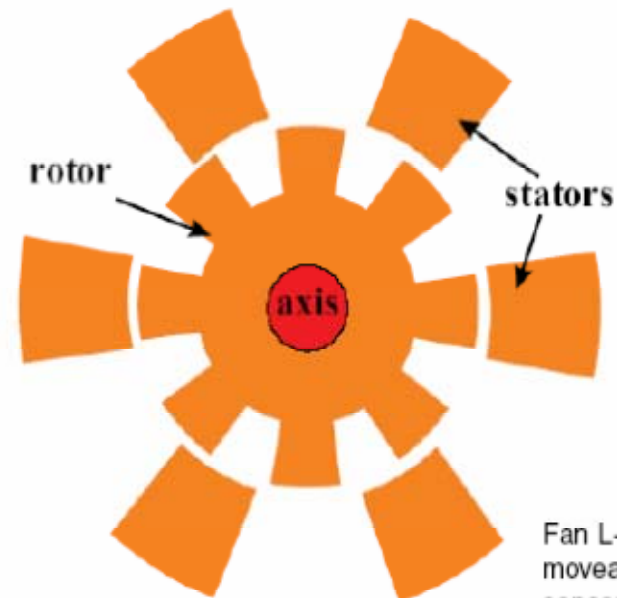
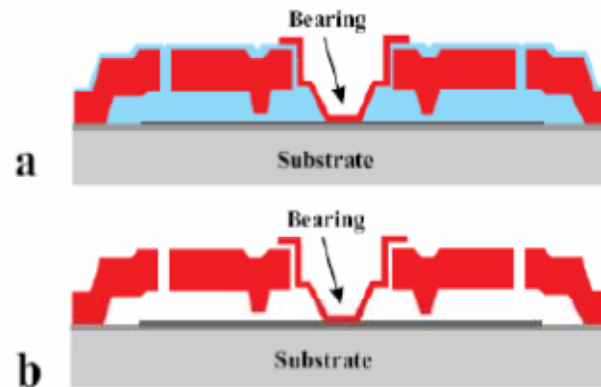
Some examples, Bulk micro-machined piezoresistive sensor



Laboratory for Electron Devices
Ljubljana SLOVENIA

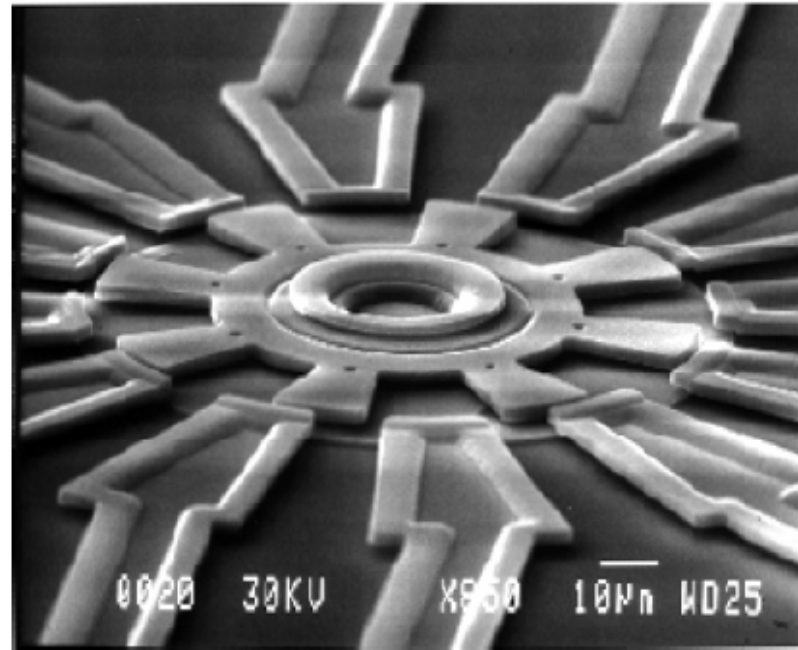


Surface Micromachined Motor



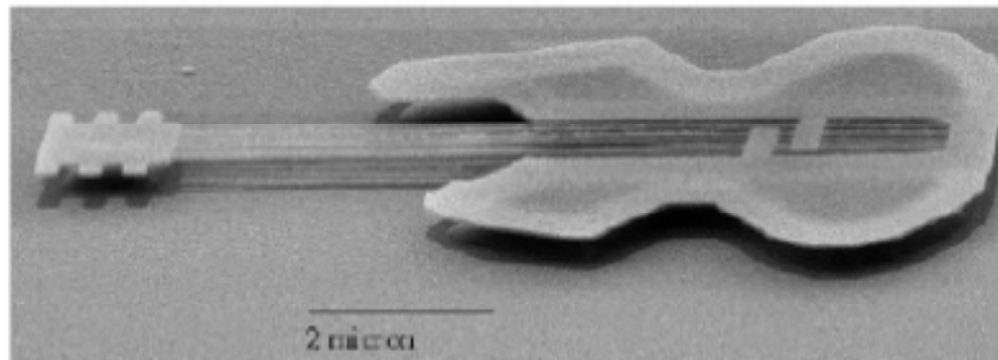
Fan L-S, Tai Y-C and Muller R S 1988 Integrated moveable micromechanical structures for sensors and actuators *IEEE Trans. Electron Devices* **ED-35** 724–30

Rotary Electrostatic Micromotor



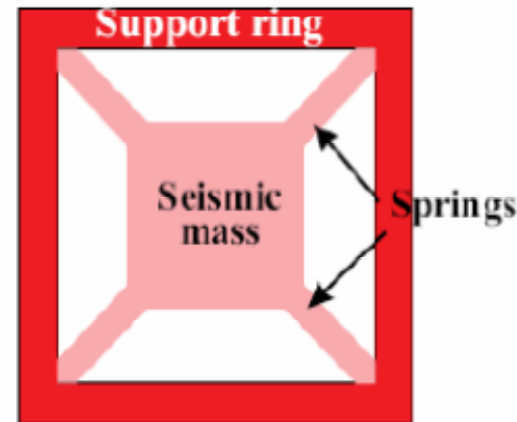
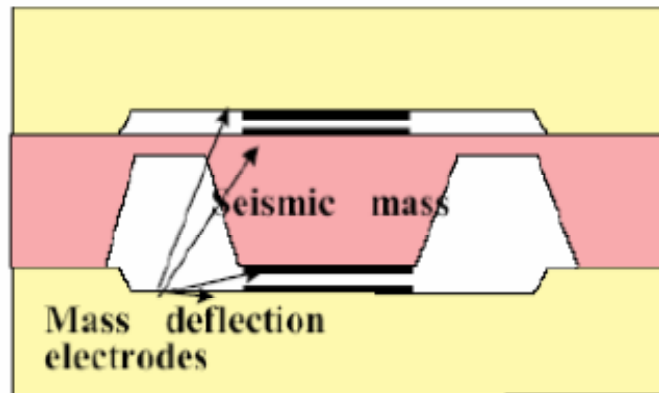
Fan Long-Shen, Tai Yu-Chong and
Muller R S 1989 IC-processed
electrostatic micromotors *Sensors
Actuators* 20 41–7

World's Smallest Guitar



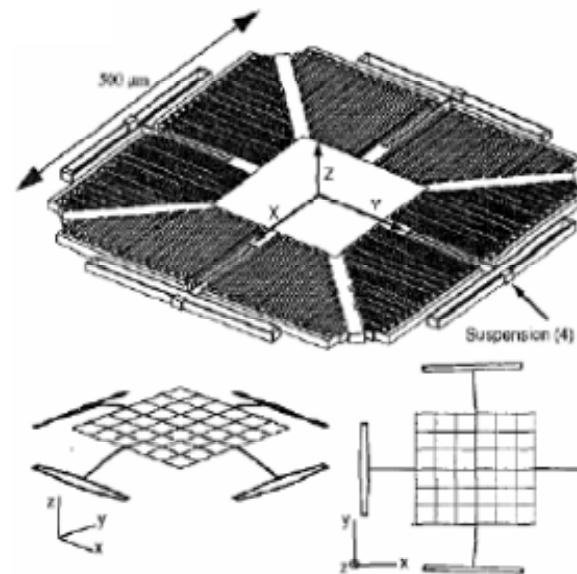
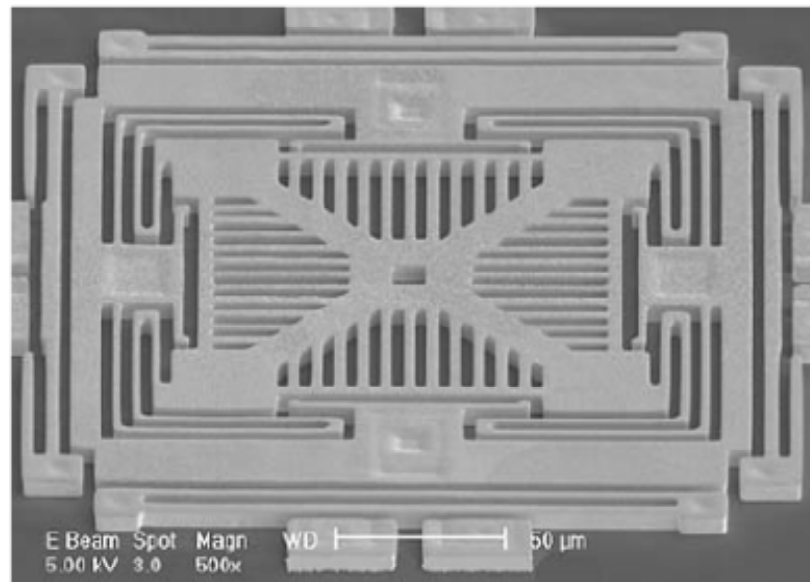
A 10 μm long Si guitar (same size as a single cell) with six strings, each ~ 50 nm (100 atoms) wide. (Cornell University)

Automotive Airbag Accelerometer



Bulk micromachined proof mass suspended between mass deflection electrodes attached by wafer bonding

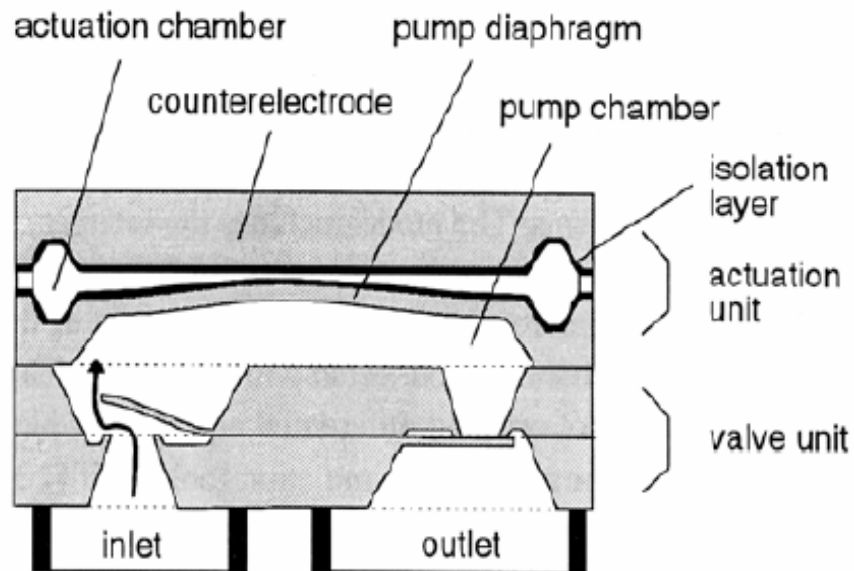
Automotive Airbag Accelerometer



A 3-Axis Force Balanced Accelerometer Using a Single Proof-Mass

Mark A. Lemkin, Bernhard E. Boser, David Auslander*, Jim H. Smith**

Fluidic MEMS



Electrostatic micropump with two one-way check valves

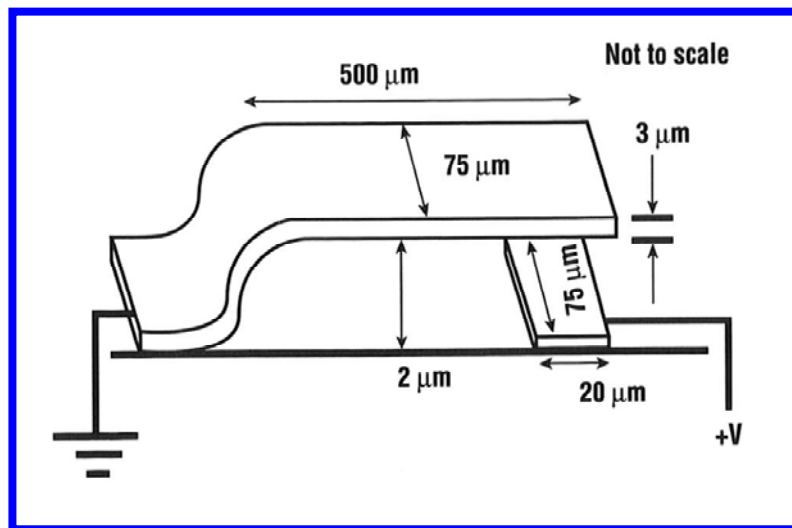
Zengerle R, Ulrich J, Kluge S, Richter M and Richter A
1995, A bi-directional silicon micropump *Sensors
Actuators A50* 81–6

EXERCISES

- 1) Calculate the maximum deflection and maximum stresses for a square silicon membrane of thickness $10\text{ }\mu\text{m}$ and side length 2 mm for an applied pressure of 1000 Pa .
For silicon, $E = 190\text{ GPa}$ and $\nu = 0.28$.
- 2) A silicon cantilever beam with a piezoresistor located at the point of maximum stress is subjected to a point load Q at the end of the beam. Q is $10\text{ }\mu\text{N}$, the length of the beam is $1000\text{ }\mu\text{m}$, and the beam thickness is $3\text{ }\mu\text{m}$. Calculate the beam width that results in a 3% resistance change for the piezoresistor due to the load Q . Assume the beam lies perpendicular to the silicon $\langle 110 \rangle$ lattice direction.
- 3) What is the resonant frequency F_o for a silicon cantilever beam $1000\text{ }\mu\text{m}$ long, $100\text{ }\mu\text{m}$ wide, and $3\text{ }\mu\text{m}$ thick? The density of silicon is 2.3 g/cm^3 .

EXERCISES

- 4) A polysilicon cantilever fabricated using surface micromachining is 500 μm long, 75 μm wide, and 3 μm thick. The sacrificial layer thickness for the process was 2.0 μm . The electrically grounded beam is electrostatically actuated using a positive voltage V applied to a conducting bottom electrode (length 20 μm , width 75 μm) under the end of the beam. Neglecting fringing effects, estimate the voltage V required to deflect the beam by 0.2 μm .



Force Q between two parallel plates of area A , separation d and applied voltage V

$$Q = 0.5 \epsilon_0 \epsilon_r \frac{AV^2}{d^2}$$

EXERCISES

ANSWER:

- 1) $W_{\max}=1.17\mu\text{m}$,
 $\sigma_l=12.3\text{ MPa}$,
 $\sigma_t=3.45\text{ MPa}$
- 2) $\sigma_{\max}=4.3\cdot 10^7\text{ Pa}$,
 $a=150\text{ }\mu\text{m}$
- 3) $f_0=4.39\text{ kHz}$
- 4) $V=9.6\text{ V}$

