Department of information technology and media
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Written exam in Sensor Devices 2010

Tentamen i Sensorkomponenter

Hjälpmedel: Miniräknare - *Aid*: Calculator

Basic part

1 How do you terminate dangling bonds in a silicon surface? (3 p)

2 The anisotropic wet-etching of silicon needs an etching mask. Which type of etching masks can be used for most types of etching solutions? What is the reason for sloped walls (54.74°) when etching 100 silicon? (3 p)

3 The change in piezo-resistivity is a measure of:........? (3 p)

4 Describe different types of immobilization schemes for biosensors? (5 p)

Calculation part

A square membrane of single-crystal silicon is used as a pressure sensor to detect pressure changes from 0 to 4000 Pa. Calculate the length a of the membrane side if the maximum deflection is 10% of the membrane thickness t = 20 μm. What is the resonant frequency of this membrane?

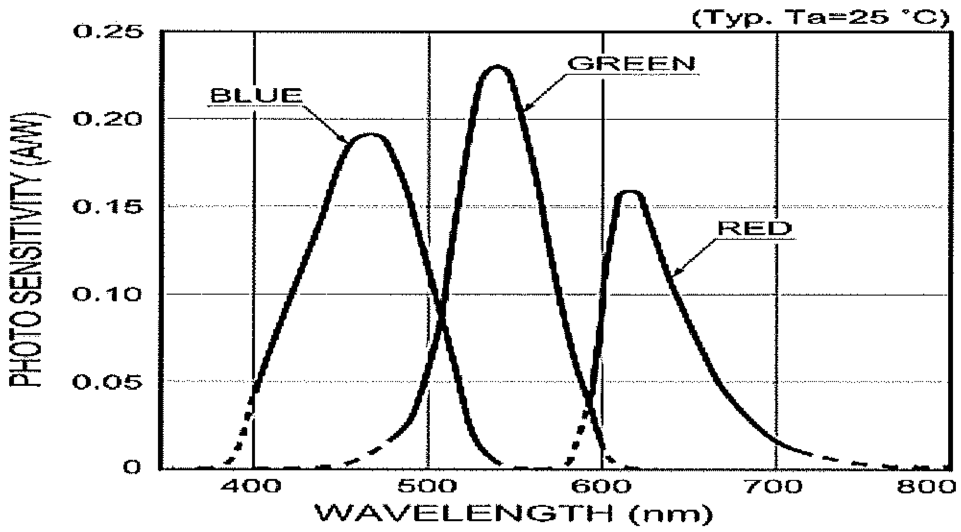
(5 points)

5 How long does it take to grow a 600 Å thick layer of silicon-dioxide on a silicon sensor. Assume <100> crystal orientation, processing temperature 1000 °C and dry O_2 oxidization? Show detailed calculations!

(4 points)

Boltzmann constant $k = 8{,}61739 \cdot 10^{-5}$ eV/K

6 An RGB sensor is used to measure the temperature of heated and melted iron in a steel factory. The sensitivity of the RGB sensor is given below. Calculate the relative sensor output for melted iron at 2000 K and solid iron at 1000 K. For the calculation the detector sensitivity can be approximated with three narrow peaks with equal width.



(5 points)

7 a) A monochrome X-ray beam with the energy 10 keV reaches a fully depleted 300 µm thick silicon detector. What fraction of the incident radiation that is absorbed in the detector? Which absorption process occurs in the silicon sensor? What current is produced in the sensor if the photon rate reaching the sensor is 10000 photos/s? The conversion factor for Si in room temperature is 3.62.

b) If the photon energy is increased to 1 MeV, how will it affect the absorption fraction and the produced current in the sensor. What absorption process is now taking place in the silicon?

(6 points)

GOOD LUCK

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Appendix to exam in sensor devices
Semiconductor Sensor Technologies

Reactive growth

Result of the Deal-Grove model for growth of silicon dioxide

$$\tau = \frac{x_i^2 + Ax_i}{B} \quad x_i \approx 0.02\mu m\,(200Å)$$

$$X_0 = \frac{A}{2}\left[\sqrt{1+\frac{t+\tau}{A^2/4B}}-1\right]$$

$$B = C_1 e^{-E_1/kT} \quad \textit{parabolic growth}$$

$$\frac{B}{A} = C_2 e^{-E_2/kT} \quad \textit{linear growth}$$

Table 6–2 Rate constants describing (111) silicon oxidation kinetics at 1 Atm total pressure. For the corresponding values for (100) silicon, all C_2 values should be divided by 1.68.

Ambient	B	B/A
Dry O_2	$C_1 = 7.72 \times 10^2\ \mu m^2\ hr^{-1}$	$C_2 = 6.23 \times 10^6\ \mu m\ hr^{-1}$
	$E_1 = 1.23$ eV	$E_2 = 2.0$ eV
Wet O_2	$C_1 = 2.14 \times 10^2\ \mu m^2\ hr^{-1}$	$C_2 = 8.95 \times 10^7\ \mu m\ hr^{-1}$
	$E_1 = 0.71$ eV	$E_2 = 2.05$ eV
H_2O	$C_1 = 3.86 \times 10^2\ \mu m^2\ hr^{-1}$	$C_2 = 1.63 \times 10^8\ \mu m\ hr^{-1}$
	$E_1 = 0.78$ eV	$E_2 = 2.05$ eV

Mechanical sensors

Table 19.1 Properties of materials

	Yield Strength (10^9 Pa)	Young's Modulus (10^9 Pa)	Density (g/cm^3)	Thermal Conductivity (W/cm°C)	Thermal Expansion (10^{-6}/°C)
Diamond (single crystal)	53.0	1035.0	3.5	20.0	1.0
SiC (single crystal)	21.0	700.0	3.2	3.5	3.3
Si (single crystal)	7.0	190.0	2.3	1.6	2.3
Al_2O_3	15.4	530.0	4.0	0.5	5.4
Si_3N_4 (single crystal)	14.0	385.0	3.1	0.2	0.8
Gold	—	80.0	19.4	3.2	14.3
Nickel	—	210.0	9.0	0.9	12.8
Steel	4.2	210.0	7.9	1.0	12.0
Aluminum	0.2	70.0	2.7	2.4	25.0

Square membranes

Max deflection $W_{max} = 0.001265Pa^4/D$

Max longitudinal stress $\sigma_l = 0.3081P(a/t)^2$

Max transverse stress $\sigma_t = \nu\sigma_l$

Resonant frequency $F_o = \frac{1.654t}{a^2}\left[\frac{E}{\rho(1-\nu^2)}\right]^{1/2}$

E: Young's Modulus

ν: Poisson's ratio (= 0.28 for Si, ≈ 0.3 for most metals)

D, measure the stiffness of the membrane

$$D = \frac{Et^3}{12(1-\nu^2)}$$

Cantilever beams

Cantilever beam uniform distributed load

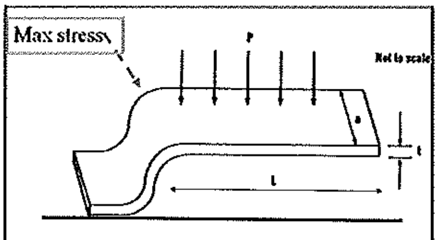

Figure 19.8 Cantilever beam fixed at one end, subjected to downward force per unit length P.

Uniform distributed load P (F is uniformed distributed over L)

$$P = F/a$$

I, bending of inertia "bending resistance"

$$I = \frac{at^3}{12}$$

Cantilever beam point load at the end

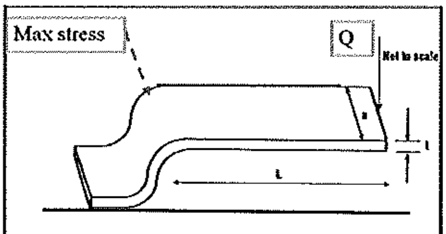

Figure 19.8 Cantilever beam fixed at one end, subjected to downward force per unit length P.

Deflection $W(P, x) = \frac{Qx^2}{6EI}(3L\text{-}x)$

Max stress $\sigma = QLt/2I$

Fundamental mode resonant frequency

$$F_o = 0.161\frac{t}{L^2}\left(\frac{E}{\rho}\right)^{1/2}$$

Cantilever beam mass M

$$F_o = 0.161\frac{t}{L}\left(\frac{Eta}{ML}\right)^{1/2}$$

Radiation sensors

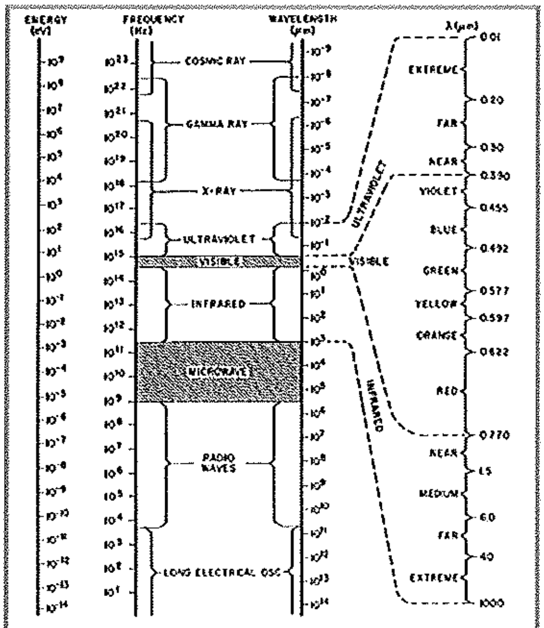

Fig. 1 Chart of electromagnetic spectrum. (After Ref. 4)

$$E = h\nu.$$

$$h = 6.626 \times 10^{-34} \text{ J-s.}$$

E=energy

ν=frequency

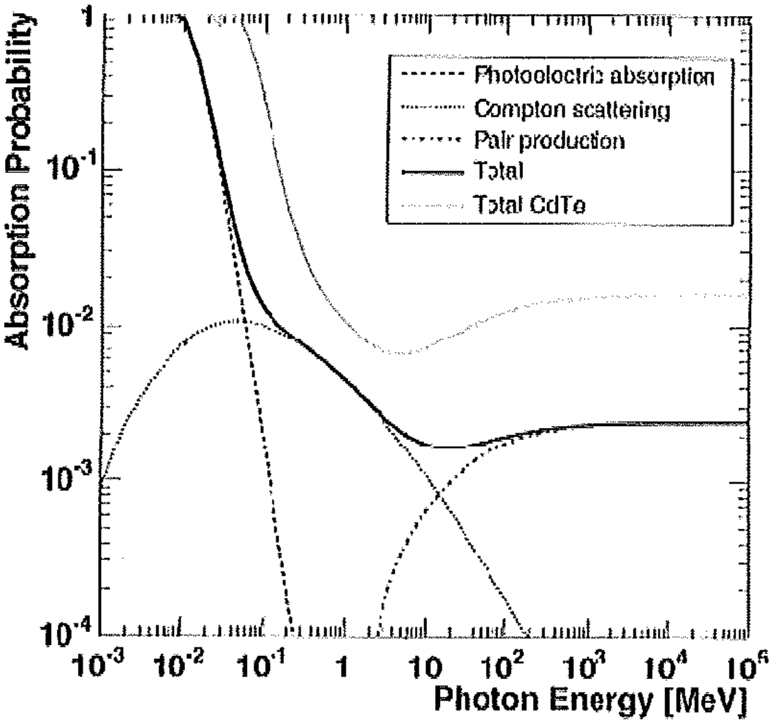

Fig. 2.4. Probability of photon absorption for 300 μm silicon as function of the photon energy. Contributions from different processes are indicated. The total absorption probability for 300 μm CdTe is also given for comparison (Data from [52])

Thermal sensors

body. According to Kirchhoff, for each wavelength, it holds that the emmissivity $\epsilon(\nu)$ is equal to the absorptivity $\alpha(\nu)$ (the fraction of radiation that is emitted, compared to that emitted by a black body is the emissivitity). The Stefan–Boltzmann law states that the power emitted by a gray or black body is equal to:[7]

$$P''_{\rm rad}(T) = \epsilon\sigma T^4 \tag{12}$$

in which $P''_{\rm rad}(T)$ is the total heat flux emitted by a body, ϵ is the emissivity of the surface (for a black body $\epsilon = 1$), and σ is the Stefan–Boltzmann constant: $\sigma = 56.7 \times 10^{-9}$ W/m^2-K^4.

For a black body, Eq. 12 is the integral (multiplied by $c/4$) over all wavelengths of the famous formula of Max Planck, that describes the energy density of black-body radiation in a cavity at temperature T. Figure 4 shows the spectral radiancy, which is the power emitted by a black body at the indicated temperature, of 1 m^2 over a solid angle of 1 sr (there are 4π sr on a full sphere) per unit of wavelength (here taken as 1 μm). The spectral radiancy is found from Planck's formula by multiplying by $c/4\pi$, to convert from radiation energy density to emitted radiation power per solid angle. The diagonal line shows

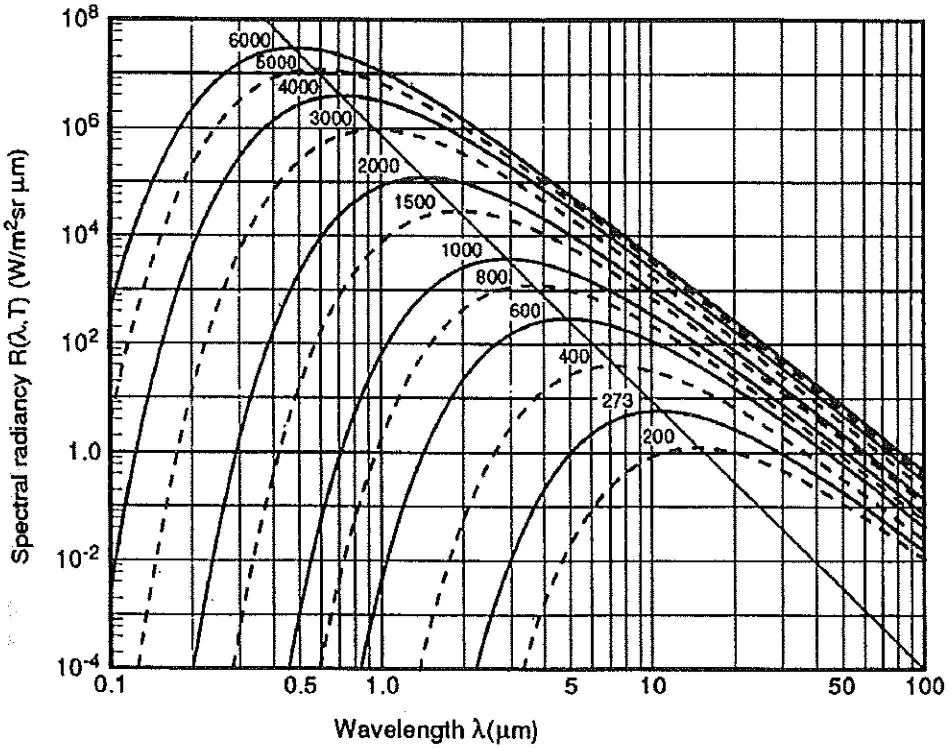

Fig. 4 Spectral radiancy (emitted power density per sr) $R(\lambda, T)$ of a black body at temperatures given in Kelvin. The diagonal line shows Wien's displacement law. (After Ref. 1)

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In the steady state, the temperature distribution is given by a constant temperature gradient equal to $\Delta T/L$, and the total flow through the rod is $\kappa A\,\Delta T/L$. Hence, for this configuration we find a thermal resistance R_{th}:

$$R_{th}=\frac{L}{\kappa A} \quad \text{(structure with length } L \text{ and uniform cross-section } A\text{).} \tag{16}$$

This expression can be used for the one-dimensional heat flow in all kinds of structure with a uniform cross-section, such as rods, plates or wires as shown in Fig. 5.

For example, the thermal resistance of a flat plate with a rectangular cross-section on length L, width W and thickness D (Fig. 5a) is equal to:

$$R_{th}=\frac{L}{W}\frac{1}{\kappa D}. \tag{17}$$

For a square plate, where the length L is equal to the width W, the thermal resistance is equal to $(\kappa D)^{-1}$, which is called the thermal sheet resistance R_{st} of the plate.

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We can use this model to calculate the transfer function and the time constant of the floating-membrane structure. In the steady-state situation the temperature rise in the sensor above the ambient that results from the power dissipation P is equal to $\Delta T_{flm}=T-T_{amb}=P/(1/R_{beam}+G_{flm}+G_{sen})$. The time response of the system when the heating power is abruptly changed from zero to a constant

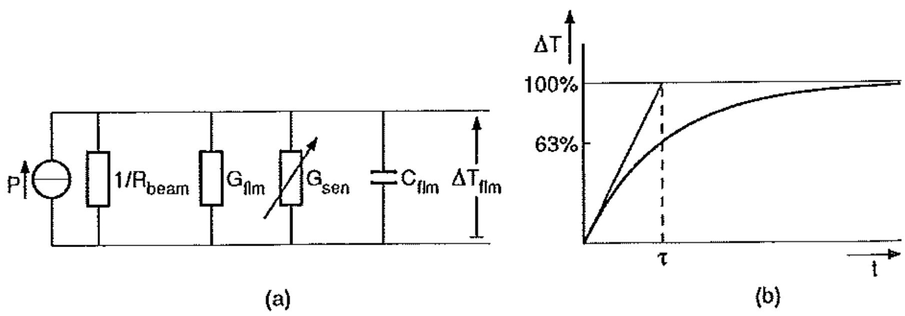


Fig. 8 Discrete-element model of the floating-membrane structure: (a) electrical circuit and (b) step response curve for the first-order circuit. (After Ref. 1)

value P_0 at time $t=0$ is given by:

$$\Delta T_{\mathrm{flm}}(t)=\frac{P}{1/R_{\mathrm{beam}}+G_{\mathrm{flm}}+G_{\mathrm{sen}}}(1-\exp(-t/\tau_{\mathrm{flm}})) \tag{20}$$

(see Fig. 8b). A similar result is found for an abrupt change in T_{amb} or in/$(1/R_{\mathrm{beam}}+G_{\mathrm{flm}}+G_{\mathrm{sen}})$. The time constant τ_{flm} follows from the total thermal conduction and thermal capacitance:

$$\tau_{\mathrm{flm}}=\frac{C_{\mathrm{flm}}}{G_{\mathrm{flm}}+G_{\mathrm{sen}}+1/R_{\mathrm{beam}}}. \tag{21}$$

For this simple RC time constant, the step response shows the time constant in several ways: the derivative of the curve at the step time crosses the final value after one time constant, and the curve reaches 63% of its final value after one time constant, as shown in Fig. 8b.

In general, the factor R_{beam} can be designed independently of the factors C_{flm}, G_{flm} and G_{sen}, which are closely related. For example, enlarging the interaction area will usually enlarge G_{sen}, G_{flm} and C_{flm} by approximately the same factor.

Solution Sensorkomponents 20100428

5A

Given:

Range of pressure =>P= 0 to 4000 Pa

Thickness of the side of membrane => t = 20 um

From the table we have the following values for single crystal silicon

Young's Modulus => E = 190 GPa

Density => ρ = 3.2 g/cm^3 = 3200 Kg/m^3

Poison's ratio => ν = 0.28

Maximum deflection = 10% of thickness

Must also be given: Boltzmanns constant k = $8.61739 \cdot 10^{-5}$ eV/K

To Find:

Length of the membrane L=?

Resonant frequency of the membrane =?

Solution:

(a)

As maximum deflection is 10% of the thickness

So W_{max} = (10 * 20um)/100

= 2.00 um

Maximum deflection in the membrane is given by

$W_{max} = (0.001265) * P * L^4 / D$ 21.1

Where D is given by the following equation

$D = Et^3 / 12 (1-\nu^2)$ 21.2

Thus on putting values we get

D = 1.37 x 10^{14} Pa(um)^3

So thus solving the above two equations for 'L' we have:

$$L = \left[\frac{Wmax * D}{0.001265 * P} \right]^{1/4}$$ 21.3

Thus we have L = 2.7 mm

(b)

Also we can calculate the resonant frequency of the silicon membrane which is given by the following equation

$$F_0 = \frac{1.654t}{\hat{L}^2} \left[\frac{E}{\rho(1-\nu^2)} \right]^{1/2}$$ 21.4

Thus putting the values we have the following result:

$$F_0 = \frac{1.654 \cdot 20 \cdot 10^{-4}}{0.27^2} \left[\frac{190 \cdot 10^9}{3200(1-0.28^2)} \right] = 364 \text{ kHz}$$

5B

Given:

Temperature = 1000 °C

Thus temperature in Kelvin = 1000+ 273 = 1273 K

Wafer Orientation = <100>

Thickness of Si O_2 Layer to be grown in 1st step = x1= 600 Å = 0.06 um

Boltzmann constant = 8.617 X 10^{-5} eV/K

We have the following values from the table.

assuming the oxidation process being due to dry oxygen.

$C_1 = 7.72 \times 10^2$ um^2/hr

E_1 = 1.23 eV

$C_2 = 3.7083 \times 10^6$ um/hr

E_2 = 2.00 eV

To Find:

Time to grow the required Layer t1 (for 1st growth) =?

Solution:

I have assumed that this is a dry oxygen oxidation process for this we have the following calculations

Time for the oxidation process if thickness of the oxide is layer is 'x' is given by the following relation

$t = \frac{x}{B/A} + \frac{\hat{x}2}{B}$ where $\hat{x}2 = x^2$ (where x is the thickness and t is the time) 17.1

To calculate time we must have B and B/A So

For the linear growth of the oxide layer we have

$B = C_1 e^{-E1/kT}$ (um^2/hr) 17.2

And for the parabolic growth of the oxide layer we have the following equation

$B/A = C_2 e^{-E2/kT}$(um/hr) 17.3

So by putting the values in the above equations we get the following values

B = 0.0104

B/A= 0.0448

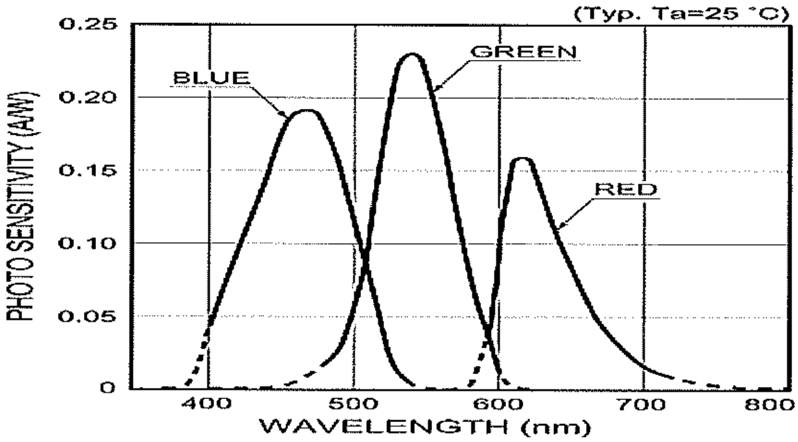
We calculate the value of time for the total growth of the oxide layer which is 600 Å = 0.06 um

Now putting the values in the equation 17.1 we have

T total = 1.685 Hours

Which means 1 hours, 41 minutes and 8 seconds (rounded to nearest second)

6



Read from above graph

Color	Wavelength peak (nm)	Photo sensitivity (A/W)
Blue	466	0.191
Green	541	0.227
Red	616	0.157

Read from spectral radiancy graph

Color	Radiancy (W/m^2sr)	Current (A/m^2sr)	Radiancy (W/m^2sr)	Current (A/m^2sr)
	1000 K	1000 K	2000 K	2000 K
Blue	$2 \cdot 10^{-4}$	$3.82 \cdot 10^{-5}$	10^3	$1.91 \cdot 10^2$
Green	10^{-2}	$2.27 \cdot 10^{-3}$	$4 \cdot 10^3$	$9.08 \cdot 10^2$
Red	10^{-1}	$1.57 \cdot 10^{-2}$	$1.1 \cdot 10^4$	$1.73 \cdot 10^3$

Relative signal

Color	Current (%)	Current (%)	Signal increase
	1000 K	2000 K	1000 to 2000 K
Blue	0.24	11	$5 \cdot 10^6$
Green	14	52	$4 \cdot 10^5$
Red	100	100	$1.1 \cdot 10^5$

7

Photon absorption probability in 300 μm Si for 10 keV is $9 \cdot 10^{-1}$. The fraction absorbed is 90 %.

The most likely absorption process is photoelectric absorption (0.9), with about 100 times higher probability than Compton scattering ($7.0 \cdot 10^{-3}$).

10000 photons/s gives a current of
$I = 10000 \text{ (photons/s)} \cdot 10000 \text{ (eV)} \cdot 0.9 \cdot 1.6 \cdot 10^{-19} / 3.62 = 4.0 \cdot 10^{-12}$ A

For 1 MeV the total aborption fraction is $4 \cdot 10^{-3}$. The current becomes = $1.8 \cdot 10^{-14}$ A. The only significant aborption process is Compton scattering.