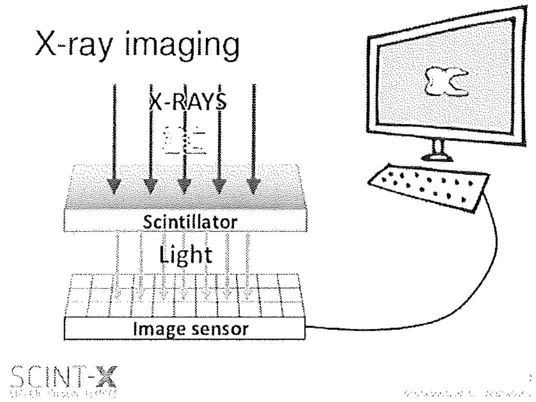
Department of electronics design
Börje Norlin

EL004A Sensor devices
2016-04-21

Written exam in Sensor Devices

Tentamen i Sensorkomponenter

Hjälpmedel: Miniräknare - *Aid*: Calculator



Basic part (50 % result compulsory for pass grade)

1 Describe IV and CV measurements of semiconductor sensor samples. (4 p)

2 How do you terminate dangling bonds in a silicon surface? (3 p)

3 In fabrication of thermal detectors, thin membranes are often used. Why? (3 p)

4 In lithographical processing equipment, the wavelength of exposure light has a tendency to decrease during the year. Why is it so? Advantage and disadvantage! (4 p)

Calculation part

5 A p-type silicon cantilever beam with a piezoresistor located at the point of maximum stress is subjected to a point load Q at the end of the beam. Q is 20 µN, the length of the beam is 800 µm and the beam thickness is 3 µm. Calculate the beam with that results in a 5 % resistance change for the piezoresistor due to the load Q. Assume the beam lies perpendicular to the silicon <110> lattice direction. You can neglect the influence of transverse stress.

(8 p)

6 Two radioactive elements are mixed, they emit gamma rays with one specific energy each, 10 keV and 4.0 MeV. You want to build a sensor system to measure the respective count rates, so that you can estimate the concentration of each element. You have two detector principles to evaluate, direct absorption in Silicon or conversion to visible photons in a scintillator.

a) Direct absorption uses a fully depleted 250 μm thick silicon sensor. What fraction of the incident radiation is absorbed in the detector? What charge/photon is produced in the sensor for each X-ray photon? The conversion factor for Si in room temperature is 3.62.

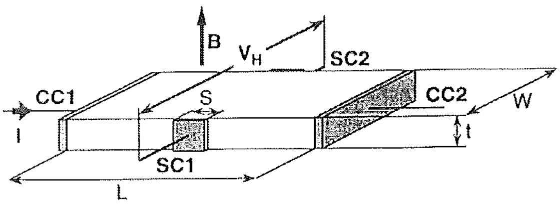
b) A 10 mm thick CsI scintillator crystal is used in front of the silicon sensor. The conversion factor for Tl doped CsI is 65 photons/keV. The photons are in the visible range with a wavelength of 540 nm. Since they are just above the band gap energy for Silicon each photon generates one electron-hole-pair (no conversion factor). You can assume a wide geometry and uniform radiation meaning that 50% of the photons will reach the detector (while the other 50% shines back towards the radioactive source). What charge/photon is produced in the sensor for each X-ray photon?

c) Compare the two principles and argue which principle to use for which radioactive element.

(9 p)

7 *Hall sensor*

A Hall-structure experiences an orthogonal magnetic field $B = 1.9 \cdot 10^{-5}$ Wb/cm^2. For a current $I = 2.2$ mA the Hall voltage becomes $V_{sc12} = -2.8$ mV. The geometry of the structure is a rectangular silicon sensor with dimensions length $L = 5.0$ mm, width $w = 0.30$ mm and thickness $t = 25$ μm. The majority carrier type is electrons. Calculate the concentration and mobility of the majority carriers. The scattering factor for electrons can be neglected (i.e r_n=1).



(7 p)

GOOD LUCK!

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Appendix to exam in sensor devices
Semiconductor Sensor Technologies

Reactive growth

Result of the Deal-Grove model for growth of silicon dioxide

$$\tau = \frac{x_i^2 + Ax_i}{B} \qquad x_i \approx 0.02\mu m\,(200\AA)$$

$$X_0 = \frac{A}{2}\left[\sqrt{1+\frac{t+\tau}{A^2/4B}}-1\right]$$

$$B = C_1 e^{-E_1/kT} \quad \textit{parabolic growth}$$

$$\frac{B}{A} = C_2 e^{-E_2/kT} \quad \textit{linear growth}$$

Boltzmann's constant: k = $8.61739 \cdot 10^{-5}$ eV/K

Table 6–2 Rate constants describing (111) silicon oxidation kinetics at 1 Atm total pressure. For the corresponding values for (100) silicon, all C_2 values should be divided by 1.68.

Ambient	*B*	*B/A*
Dry O_2	$C_1 = 7.72 \times 10^2\ \mu m^2\ hr^{-1}$	$C_2 = 6.23 \times 10^6\ \mu m\ hr^{-1}$
	$E_1 = 1.23$ eV	$E_2 = 2.0$ eV
Wet O_2	$C_1 = 2.14 \times 10^2\ \mu m^2\ hr^{-1}$	$C_2 = 8.95 \times 10^7\ \mu m\ hr^{-1}$
	$E_1 = 0.71$ eV	$E_2 = 2.05$ eV
H_2O	$C_1 = 3.86 \times 10^2\ \mu m^2\ hr^{-1}$	$C_2 = 1.63 \times 10^8\ \mu m\ hr^{-1}$
	$E_1 = 0.78$ eV	$E_2 = 2.05$ eV

Mechanical sensors

Table 19.1 Properties of materials

	Yield Strength (10^9 Pa)	Young's Modulus (10^9 Pa)	Density (g/cm^3)	Thermal Conductivity (W/cm °C)	Thermal Expansion (10^{-6}/°C)
Diamond (single crystal)	53.0	1035.0	3.5	20.0	1.0
SiC (single crystal)	21.0	700.0	3.2	3.5	3.3
Si (single crystal)	7.0	190.0	2.3	1.6	2.3
Al_2O_3	15.4	530.0	4.0	0.5	5.4
Si_3N_4 (single crystal)	14.0	385.0	3.1	0.2	0.8
Gold	—	80.0	19.4	3.2	14.3
Nickel	—	210.0	9.0	0.9	12.8
Steel	4.2	210.0	7.9	1.0	12.0
Aluminum	0.2	70.0	2.7	2.4	25.0

E: Young's Modulus
ν: Poisson's ratio (= 0.28 for Si, ≈ 0.3 for most metals)

Square membranes

Max deflection $W_{max} = 0.001265Pa^4/D$

Max longitudinal stress $\sigma_l = 0.3081P(a/t)^2$

Max transverse stress $\sigma_t = \nu\sigma_l$

Resonant frequency $F_o = \frac{1.654t}{a^2}\left[\frac{E}{\rho(1-\nu^2)}\right]^{1/2}$

D, measure the stiffness of the membrane

$$D = \frac{Et^3}{12(1-\nu^2)}$$

Cantilever beams

Cantilever beam uniform distributed load

Uniform distributed load P (F is uniformed distributed over L)

$$P = F/a$$

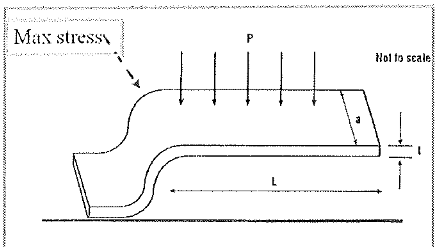


Figure 19.8 Cantilever beam fixed at one end, subjected to downward force per unit length P.

I, bending of inertia "bending resistance"

$$I = \frac{at^3}{12}$$

Cantilever beam point load at the end

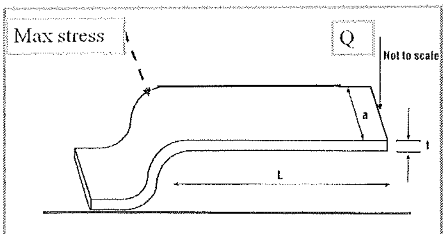


Figure 19.8 Cantilever beam fixed at one end, subjected to downward force per unit length P.

Deflection $W(P, x) = \frac{Qx^2}{6EI}(3L-x)$

Max stress $\sigma = QLt/2I$

Fundamental mode resonant frequency

$$F_o = 0.161\frac{t}{L^2}\left(\frac{E}{\rho}\right)^{1/2}$$

Cantilever beam mass M

$$F_o = 0.161\frac{t}{L}\left(\frac{Eta}{ML}\right)^{1/2}$$

Piezoresistivity

Resistance change due to longitudinal and transverse stresses:
Longitudinal stress σ_l and longitudinal piezoresistance coefficient π_l
Transverse stress σ_t and transverse piezoresistance coefficient π_t
Stress: σ=Force/Area

$$\pi_l = \frac{1}{2}\left(\pi_{11} + \pi_{12} + \pi_{44}\right)$$

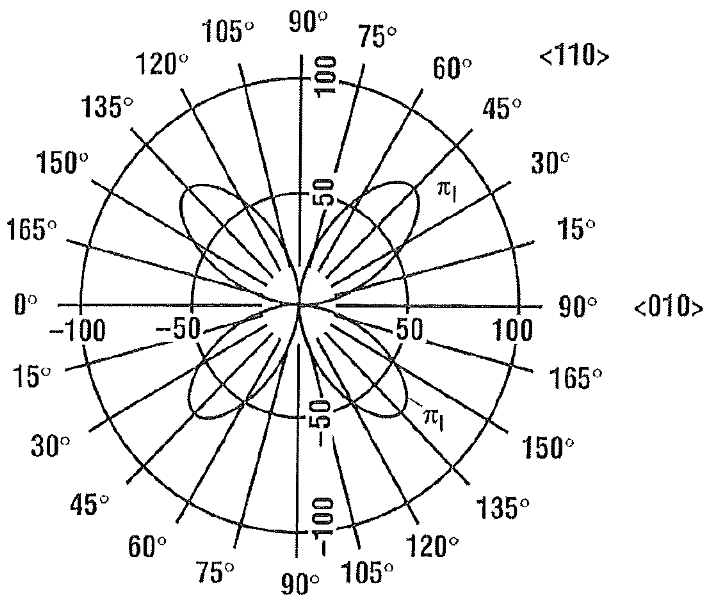
$$\pi_t = \frac{1}{2}\left(\pi_{11} + \pi_{12} - \pi_{44}\right)$$

$$\frac{\Delta R}{R} = \pi_l \sigma_l + \pi_t \sigma_t$$

$$\approx \frac{\pi_{44}}{2}(\sigma_l - \sigma_t)$$

Table 18.1. Typical room-temperature piezoresistance coefficients for n- and p-type silicon [98].

Type	Resistivity	π_{11}	π_{12}	π_{44}
Units	Ω-cm	10^{-11} Pa^{-1}	10^{-11} Pa^{-1}	10^{-11} Pa^{-1}
n-type	11.7	-102.2	53.4	-13.6
p-type	7.8	6.6	-1.1	138.1



Piezoresistance coefficients in p-type silicon [10^{-11} /Pa]

Resistors along <110> direction in (100) wafers is common for bulk micromachining of sensors.

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Magnetic sensors

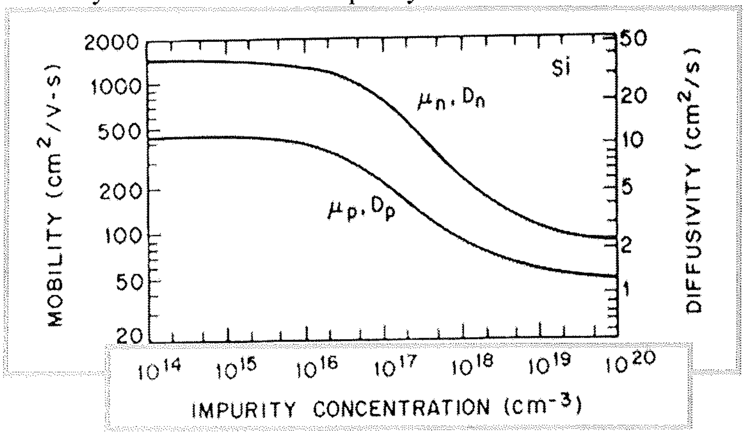
Hall voltage

$$V_H = R_H I B / t.$$

The hall coefficient R_H depends on the scattering factor r_n (denoted for electrons). The approximation $r_n = 1$ is often used.

$$R_H = -\mu_n^* / \sigma_n = -r_n / qn$$

Mobility as a function of impurity concentration



The SI-unit for the magnetic field is Tesla (T). A common unit is Gauss (G). 1 G = $1 \cdot 10^{-4}$ Tesla. Magnetic flux is measured in Weber (Wb). 1 Wb/m^2 = 1 T.

Radiation sensors

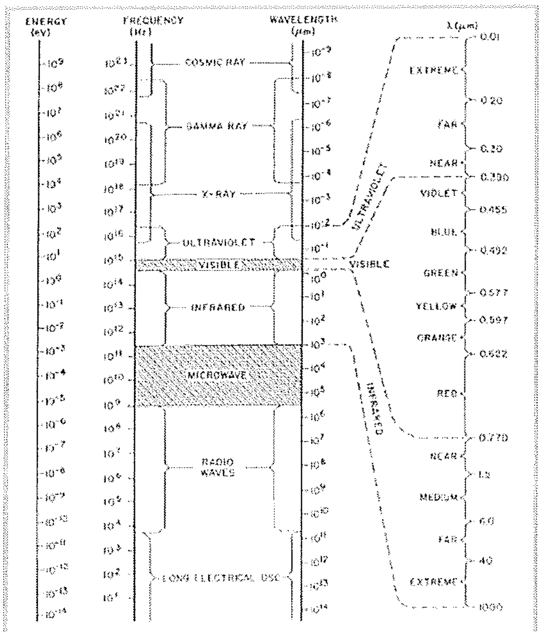


Fig. 1 Chart of electromagnetic spectrum. (After Ref. 4)

$$E = h\nu.$$

$$h = 6.626 \times 10^{-34} \text{ J-s.}$$

E=energy

ν=frequency

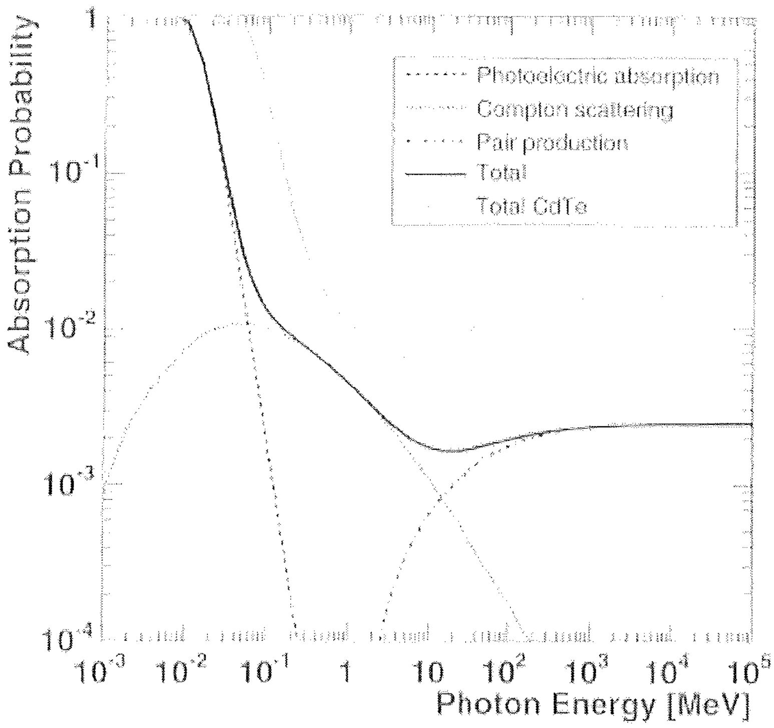


Fig. 2.4. Probability of photon absorption for 300μm silicon as function of the photon energy. Contributions from different processes are indicated. The total absorption probability for 300μm CdTe is also given for comparison (Data from [52])

The conversion factor for Si in room temperature is 3.62.
Electron charge: $q = 1.602 \cdot 10^{-19}$ C

Radiation sensors

6.2.3 Interactions of Electromagnetic and Nuclear-Particle Radiation with Semiconductors

Electromagnetic radiation interacts with semiconductors primarily through absorption processes. However, interference, diffraction, reflection, polarization, transmission, and refraction all play a role relative to the electromagnetic radiation's propagation through the various media separating the semiconductor and the radiation source. Absorption is defined as the relative decrease of the irradiance, Φ, per unit path length, $\delta\Phi(x)/\Phi = \alpha\,\delta x$, which has the solution

$$\Phi(x) = \Phi_0 \exp(-\alpha x) \qquad (6)$$

where Φ_0 is the incident irradiance, α is the absorption coefficient and x is a path-length variable. The absorbed photon density, $1 - \Phi(x)$, reaches 63% of the incident value within one absorption length, $1/\alpha$. The total absorption coefficient is the sum of three mechanisms: the photoelectric effect, Compton scattering, and pair production. We shall consider these mechanisms in the following paragraphs.

The photoelectric effect dominates for low to moderate radiant energies, $h\nu < 50$ keV. Absorption by the photoelectric effect results in a complete transfer of the electromagnetic radiant energy from an incident photon to the interacting atom, which ejects a photoelectron. The minimum photon energy required is approximately equal to the semiconductor bandgap, for example the binding energy of a valence electron involved in an atomic bond. Photons with energy below this threshold may be absorbed by carriers already within the conduction band, which is called free-carrier absorption. However, the probability for this absorption mechanism is low in semiconductors because of small concentration of conduction-band electrons relative to the concentration in the valence band.

Absorption data

Material	Mass absorption coefficient (cm^2/g)				Density (g/cm^3)
	10 keV	20 keV	50 keV	4.0 MeV	
Si	33.89	4.464	0.4385	0.0324	2.3290
In	132.1	20.44	10.30	0.03582	7.310
Au	118.1	78.83	7.256	0.04166	19.32
Pb	130.6	86.36	8.041	0.04197	11.35
CdTe	138.1	21.44	10.67	0.03525	6.200
CsI	171.1	26.86	12.87	0.03616	4.510

Table: Material data for calculation of α, the linear attenuation coefficient (cm^{-1})

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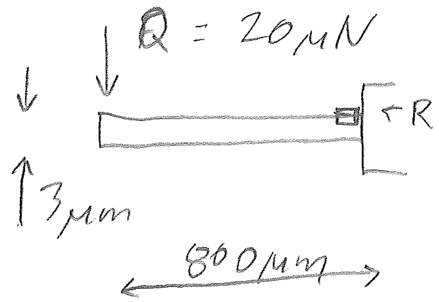

In the <110> direction $\pi_{44}^{L} = 138{,}1 \cdot 10^{-11}$ (p-type)

The relative resistance change $\frac{\Delta R}{R} = 5\% = 0{,}05$

$$\frac{\Delta R}{R} = \frac{\pi_{44}}{2}(\sigma_l - \sigma_t) \approx \frac{\pi_{44}}{2}\sigma_l = 69{,}05 \cdot 10^{-11}\,\sigma_l$$

$$\Rightarrow \sigma_l = \frac{0{,}05}{69{,}05 \cdot 10^{-11}} = 72{,}4 \text{ MPa}$$

Max beam stress $\sigma = QLt/2I$

$$= \frac{QLt}{2} \cdot \frac{12}{at^3} = \frac{6QL}{at^2}$$

$$a = \sigma = \frac{6QL}{\sigma t^2} = \frac{6 \cdot 20\mu \cdot 800\mu}{72{,}4\,M \cdot (3\mu)^2} = \underline{147\ \mu m}$$

147 μm beam width gives 5% resistance change.

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6 a) 10 keV

$\alpha_{Si} = 33{,}89 \cdot 2{,}329 = 78{,}29 \text{ cm}^{-1}$

$\alpha_{Si} \cdot x_{Si} = 78{,}29 \cdot 0{,}025 = 1{,}97$

$\Phi_{Si}(250 \mu m) = 1 - e^{-\alpha_{Si} x_{Si}} = 0{,}861$

Charge/photon $Q = \frac{0{,}861}{3{,}62} \cdot 1{,}602 \cdot 10^{-19} \cdot 10 \cdot 10^{3}$
$= 3{,}810 \cdot 10^{-16} \text{ C}$

4 MeV

$\alpha_{Si} = 0{,}0754 \text{ cm}^{-1}$

$\alpha_{Si} x_{Si} = 0{,}001886$

$\Phi_{Si}(250 \mu m) = 1 - e^{-\alpha_{Si} x_{Si}} = 0{,}001884$

Charge/photon $Q = 3{,}335 \cdot 10^{-16} \text{ C}$

b) 10 keV $\alpha_{CsI} = 171{,}1 \cdot 4{,}51 = 771{,}6 \text{ cm}^{-1}$

$\alpha_{CsI} \cdot x_{CsI} = 771{,}6 \cdot 1{,}0 = 771{,}6$

$\Phi(10 \text{ mm}) = 1 - e^{-771{,}6} = 1$

$Q = \Phi \cdot 50\% \cdot 1{,}602 \cdot 10^{-19} \cdot 65 \cdot 10 = 0{,}52 \cdot 10^{-16}$

4 MeV

$\alpha_{CsI} x_{CsI} = 0{,}163$

$\Phi(10 \text{ mm}) = 1 - e^{-0{,}163} = 0{,}150$

$Q = 0{,}150 \cdot 0{,}5 \cdot 1{,}602 \cdot 10^{-19} \cdot 65 \cdot 4 \cdot 10^{3} = 31{,}3 \cdot 10^{-16}$

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$B = 1{,}9 \cdot 10^{-5}$ Wb/cm^2 $= 1{,}9 \cdot 10^{-1}$ T

$$R_H = \frac{V_H \cdot t}{I \cdot B} = \frac{2{,}8 \cdot 10^{-3} \cdot 25 \cdot 10^{-6}}{2{,}2 \cdot 10^{-3} \cdot 0{,}19} = 1{,}675 \cdot 10^{-4} \ m^3/C$$

$$n = \frac{1}{q R_H} = \frac{1}{1{,}602 \cdot 10^{-19} \cdot 1{,}675 \cdot 10^{-4}} = 3{,}72 \cdot 10^{+22} \ /m^3$$

$$= 3{,}72 \cdot 10^{+16} \ cm^{-3}$$

Read from table $\mu_n \approx 550$ cm^2/Vs