Egmont Porten
Mittuniversitetet
DMA

Höst 2013/2014

Lösning till övning 4
Flervariabelanalys

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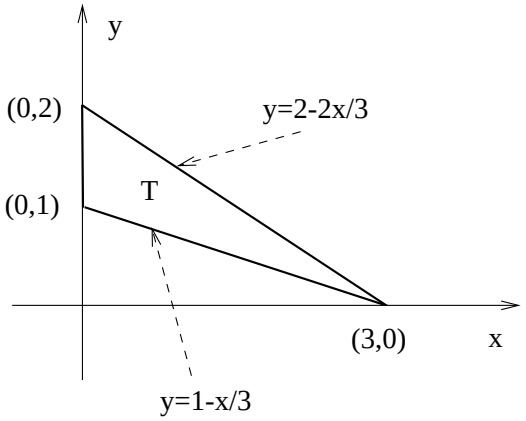
1. a) T som y-enkelt omtåde:

$$0 \le x \le 3$$
$$1 - \frac{x}{3} \le y \le 2 - \frac{2x}{3}$$



T som x-enkelt omtåde:

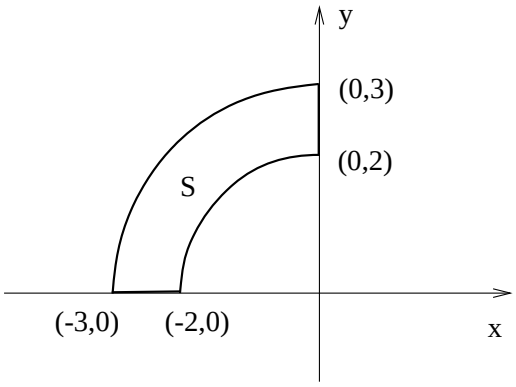
$$0 \le y \le 2$$
$$g(y) \le x \le 3 - \frac{3y}{2}$$
$$\text{med } g(y) = \begin{cases} 3 - 3y & \text{om } 0 \le y \le 1 \\ 0 & \text{om } 1 \le y \le 2 \end{cases}$$

b)

$$\begin{aligned}
\iint_T xy\,dA &= \int_0^3 dx \int_{1-\frac{x}{3}}^{2-\frac{2x}{3}} xy\,dy \\
&= \int_0^3 x\,dx \int_{1-\frac{x}{3}}^{2-\frac{2x}{3}} y\,dy \\
&= \int_0^3 x \left[\frac{y^2}{2}\right]_{y=1-\frac{x}{3}}^{y=2-\frac{2x}{3}} dx \\
&= \frac{1}{2}\int_0^3 x(3 - 2x + \frac{x^2}{3})dx \\
&= \frac{1}{2}\left[\frac{3x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{12}\right]_{x=0}^{x=3} \\
&= \frac{9}{8}
\end{aligned}$$

2. I polära koordinater beskrivs S genom

$2 \le r \le 3$
$\frac{\pi}{2} \le \theta \le \pi$



Genom att använda $x^2 + y^2 = r^2$, $dA = r\,dr\,d\theta$ får vi

$$\iint\limits_S \frac{dA}{x^2+y^2} \;=\; \int_2^3 \frac{dr}{r} \int_{\frac{\pi}{2}}^{\pi} d\theta \;=\; \frac{\pi}{2} \ln \frac{3}{2}.$$

3. Vi gör variabelbytet $x = au$, $y = bv$

$$\begin{aligned} &\Longrightarrow \quad \frac{\partial(x,y)}{\partial(u,v)} = \det \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = ab \\ &\Longrightarrow \quad dx\,dy = ab\,du\,dv. \end{aligned}$$

Eftersom $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = u^2 + v^2$ beskrivs E med avseende på koordinaterna u, v genom $u^2 + v^2 \le 1$.

$$\Longrightarrow \quad \iint\limits_E y^2\,dx\,dy \;=\; ab \iint\limits_{u^2+v^2\le 1} (bv)^2\,du\,dv \;=\; ab^3 \iint\limits_{u^2+v^2\le 1} v^2\,du\,dv.$$

Vi inför polära koordinater $u = r\cos(\theta)$, $v = r\sin(\theta)$. Disken $u^2 + v^2 \le 1$ beskrivs genom $0 \le r \le 1$, $0 \le \theta \le 2\pi$. Observera $du\,dv = r\,dr\,d\theta$.

$$\begin{aligned} \Longrightarrow \iint\limits_{u^2+v^2\le 1} v^2\,du\,dv &= \int_0^1 r^3\,dr \int_0^{2\pi} \sin^2(\theta)\,d\theta \\ &= \left[\frac{r^4}{4}\right]_{r=0}^{r=1} \int_0^{2\pi} \frac{1}{2}(1-\cos(2\theta))d\theta \\ &= \frac{1}{8}\left(2\pi - \left[\frac{1}{2}\sin(2\theta)\right]_{\theta=0}^{\theta=2\pi}\right) \\ &= \frac{\pi}{4}. \end{aligned}$$

4. I sfäriska koordinater beskrivs S genom $1 \leq \rho \leq 2$, $\frac{\pi}{2} \leq \varphi \leq \pi$, $\frac{3\pi}{2} \leq \theta \leq 2\pi$

$$
\begin{aligned}
\Longrightarrow \qquad \mathrm{Vol}(S) &= \iiint_S dx\, dy\, dz \\
&= \int_1^2 \rho^2\, d\rho \int_{\frac{\pi}{2}}^{\pi} \sin(\varphi)\, d\varphi \int_{\frac{3\pi}{2}}^{2\pi} d\theta \\
&= \frac{7}{3} \cdot \frac{\pi}{2} \left[-\cos(\varphi) \right]_{\varphi=\frac{\pi}{2}}^{\varphi=\pi} \\
&= \frac{7\pi}{6}.
\end{aligned}
$$