over the entire conduction band, as in Eq. (3–12).[5] The function $N(E)$ can be calculated by using quantum mechanics and the Pauli exclusion principle (Appendix IV).

It is shown in Appendix IV that $N(E)$ is proportional to $E^{1/2}$, so the density of states in the conduction band increases with electron energy. On the other hand, the Fermi function becomes extremely small for large energies. The result is that the product $f(E)N(E)$ decreases rapidly above E_c, and very few electrons occupy energy states far above the conduction band edge. Similarly, the probability of finding an empty state (hole) in the valence band $[1 - f(E)]$ decreases rapidly below E_v, and most holes occupy states near the top of the valence band. This effect is demonstrated in Fig. 3–16, which shows the density of available states, the Fermi function, and the resulting number of electrons and holes occupying available energy states in the conduction and valence bands at thermal equilibrium (i.e., with no excitations except thermal energy). For holes, increasing energy points down in Fig. 3–16, since the E scale refers to electron energy.

The result of the integration of Eq. (3–12) is the same as that obtained if we represent all of the distributed electron states in the conduction band by an *effective density of states* N_c located at the conduction band edge E_c. Therefore, the conduction band electron concentration is simply the effective density of states at E_c times the probability of occupancy at E_c[6]

$$n_0 = N_c f(E_c) \qquad (3\text{–}13)$$

In this expression we assume the Fermi level E_F lies at least several kT below the conduction band. Then the exponential term is large compared with unity, and the Fermi function $f(E_c)$ can be simplified as

$$f(E_c) = \frac{1}{1 + e^{(E_c - E_F)/kT}} \simeq e^{-(E_c - E_F)/kT} \qquad (3\text{–}14)$$

Since kT at room temperature is only 0.026 eV, this is generally a good approximation. For this condition the concentration of electrons in the conduction band is

$$\boxed{n_0 = N_c e^{-(E_c - E_F)/kT}} \qquad (3\text{–}15)$$

The effective density of states N_c is shown in Appendix IV to be

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \qquad (3\text{–}16a)$$

---

[5] The upper limit is actually improper in Eq. (3–12), since the conduction band does not extend to infinite energy. This is unimportant in the calculation of n_0, however, since $f(E)$ becomes negligibly small for large values of E. Most electrons occupy states near the bottom of the conduction band at equilibrium.

[6] The simple expression for n_0 obtained in Eq. (3–13) is the direct result of integrating Eq. (3–12), as in Appendix IV. Equations (3–15) and (3–19) properly include the effects of the conduction and valence bands through the density-of-states terms.

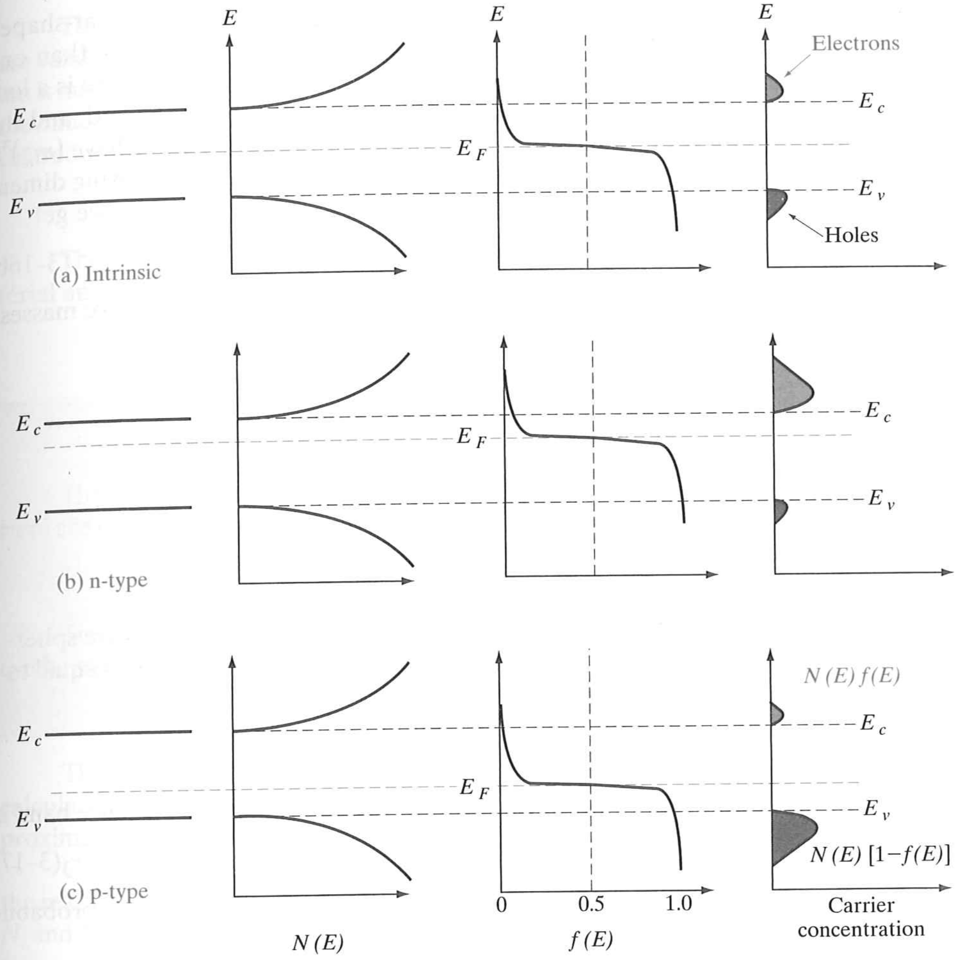


Figure 3–16
Schematic band diagram, density of states, Fermi–Dirac distribution, and the carrier concentrations for (a) intrinsic, (b) n-type, and (c) p-type semiconductors at thermal equilibrium.

Since the quantities in Eq. (3–16a) are known, values of N_c can be tabulated as a function of temperature. As Eq. (3–15) indicates, the electron concentration increases as E_F moves closer to the conduction band. This is the result we would predict from Fig. 3–15b.

In Eq. (3–16a), m_n^* is the density-of-states effective mass for electrons. To illustrate how it is obtained from the band curvature effective masses mentioned in Section 3.2.2, let us consider the 6 equivalent conduction band minima along the X-directions for Si. Looking at the cigar-shaped equi-energy surfaces in Fig. 3-10b, we find that we have more than one band curvature to deal with in calculating effective masses. There is a longitudinal effective mass m_l along the major axis of the ellipsoid, and the transverse effective mass m_t along the two minor axes. Since we have $(m_n^*)^{3/2}$ appearing in the density-of-states expression Eq. (3-16a), by using dimensional equivalence and adding contributions from all 6 valleys, we get

$$(m_n^*)^{3/2} = 6(m_l m_t^2)^{1/2} \tag{3-16b}$$

It can be seen that this is the geometric mean of the effective masses.

EXAMPLE 3–4 Calculate the density-of-states effective mass of electrons in Si.

SOLUTION For Si, $m_l = 0.98\ m_0$; $m_t = 0.19\ m_0$ from Appendix III.
There are six equivalent X valleys in the conduction band.

$$m_n^* = 6^{2/3}[0.98(0.19)^2]^{1/3}\, m_0 = 1.1\ m_0$$

Note: For GaAs, the conduction band equi-energy surfaces are spherical. So there is only one band curvature effective mass, and it is equal to the density-of-states effective mass ($= 0.067\ m_0$).

By similar arguments, the concentration of holes in the valence band is

$$p_0 = N_v[1 - f(E_v)] \tag{3-17}$$

where N_v is the effective density of states in the valence band. The probability of finding an empty state at E_v is

$$1 - f(E_v) = 1 - \frac{1}{1 + e^{(E_v - E_F)/kT}} \simeq e^{-(E_F - E_v)/kT} \tag{3-18}$$

for E_F larger than E_v by several kT. From these equations, the concentration of holes in the valence band is

$$\boxed{p_0 = N_v e^{-(E_F - E_v)/kT}} \tag{3-19}$$

The effective density of states in the valence band reduced to the band edge is

$$N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2} \tag{3-20}$$

As expected from Fig. 3-15c, Eq. (3-19) predicts that the hole concentration increases as E_F moves closer to the valence band.

The electron and hole concentrations predicted by Eqs. (3-15) and (3-19) are valid whether the material is intrinsic or doped, provided thermal equilibrium is maintained. Thus *for intrinsic material, E_F lies at some intrinsic level E_i* near the middle of the band gap (Fig. 3-15a), and the intrinsic electron and hole concentrations are

$$n_i = N_c e^{-(E_c - E_i)/kT}, \quad p_i = N_v e^{-(E_i - E_v)/kT} \tag{3-21}$$

The product of n_0 and p_0 at equilibrium is a constant for a particular material and temperature, even if the doping is varied:

$$n_0 p_0 = (N_c e^{-(E_c - E_F)/kT})(N_v e^{-(E_F - E_v)/kT}) = N_c N_v e^{-(E_c - E_v)/kT} \tag{3-22a}$$

$$= N_c N_v e^{-E_g/kT}$$

$$n_i p_i = (N_c e^{-(E_c - E_i)/kT})(N_v e^{-(E_i - E_v)/kT}) = N_c N_v e^{-E_g/kT} \tag{3-22b}$$

The intrinsic electron and hole concentrations are equal (since the carriers are created in pairs), $n_i = p_i$; thus the intrinsic concentration is

$$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT} \tag{3-23}$$

The constant product of electron and hole concentrations in Eq. (3-22) can be written conveniently as

$$\boxed{n_0 p_0 = n_i^2} \tag{3-24}$$

This is an important relation, and we shall use it extensively in later calculations. The intrinsic concentration for Si at room temperature is approximately $\overline{n}_i = 1.5 \times 10^{10}\ cm^{-3}$.

Comparing Eqs. (3-21) and (3-23), we note that the intrinsic level E_i is the middle of the band gap ($E_c - E_i = E_g/2$), if the effective densities of states N_c and N_v are equal. There is usually some difference in effective mass for electrons and holes, however, and N_c and N_v are slightly different as Eqs. (3-16) and (3-20) indicate. The intrinsic level E_i is displaced from the middle of the band gap, more for GaAs than for Ge or Si.

Another convenient way of writing Eqs. (3-15) and (3-19) is

$$\boxed{\begin{aligned} n_0 &= n_i e^{(E_F - E_i)/kT} \\ p_0 &= n_i e^{(E_i - E_F)/kT} \end{aligned}} \quad \begin{aligned} &(3\text{-}25a) \\ &(3\text{-}25b) \end{aligned}$$

obtained by the application of Eq. (3-21). This form of the equations indicates directly that the electron concentrations is n_i when E_F is at the intrinsic level E_i, and that n_0 increases exponentially as the Fermi level moves away from E_i toward the conduction band. Similarly, the hole concentration p_0 varies from n_i to larger values as E_F moves from E_i toward the valence band. Since these equations reveal the qualitative features of carrier concentration so directly, they are particularly convenient to remember.

EXAMPLE 3–5 A Si sample is doped with 10^{17} As atoms/cm^3. What is the equilibrium hole concentration p_0 at 300 K? Where is E_F relative to E_i?

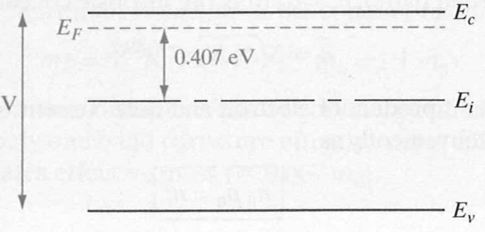
SOLUTION Since $N_d \gg n_i$, we can approximate $n_0 = N_d$ and

$$p_0 = \frac{n_i^2}{n_0} = \frac{2.25 \times 10^{20}}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

From Eq. (3-25a), we have

$$E_F - E_i = kT \ln \frac{n_0}{n_i} = 0.0259 \ln \frac{10^{17}}{1.5 \times 10^{10}} = 0.407 \text{ eV}$$

The resulting band diagram is:



3.3.3 Temperature Dependence of Carrier Concentrations

The variation of carrier concentration with temperature is indicated by Eq. (3-25). Initially, the variation of n_0 and p_0 with T seems relatively straightforward in these relations. The problem is complicated, however, by the fact that n_i has a strong temperature dependence [Eq. (3-23)] and that E_F can also vary with temperature. Let us begin by examining the intrinsic carrier concentration. By combining Eqs. (3-23), (3-16a), and (3-20) we obtain

$$n_i(T) = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_n^* m_p^*)^{3/4} e^{-E_g/2kT} \tag{3-26}$$

The exponential temperature dependence dominates $n_i(T)$, and a plot of ln n_i vs. $10^3/T$ appears linear (Fig. 3-17).[7] In this figure we neglect variations due to the $T^{3/2}$ dependence of the density-of-states function and the fact that E_g varies somewhat with temperature.[8] The value of n_i at any temperature is a definite number for a given semiconductor, and is known for most materials. Thus we can take n_i as given in calculating n_0 or p_0 from Eq. (3-25).[9]

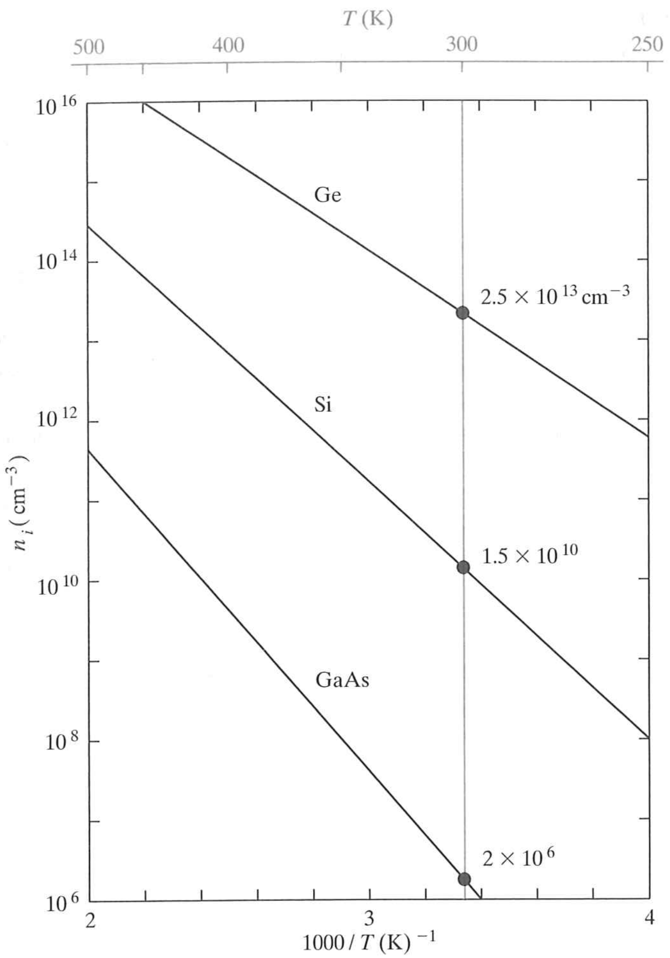


Figure 3–17 Intrinsic carrier concentration for Ge, Si, and GaAs as a function of inverse temperature. The room temperature values are marked for reference.

With n_i and T given, the unknowns in Eq. (3-25) are the carrier concentrations and the Fermi level position relative to E_i. One of these two

---

[7] When plotting quantities such as carrier concentration, which involve a Boltzmann factor, it is common to use an inverse temperature scale. This allows terms which are exponential in $1/T$ to appear linear in the semilogarithmic plot. When reading such graphs, remember that temperature increases from right to left.

[8] For Si the band gap E_g varies from about 1.11 eV at 300 K to about 1.16 eV at 0 K.

[9] Care must be taken to use consistent units in these calculations. For example, if an energy such as E_g is expressed in electron volts (eV), it should be multiplied by q (1.6×10^{-19} C) to convert to joules if k is in J/K; alternatively, E_g can be kept in eV and the value of k in eV/K can be used. At 300 K we can use $kT = 0.0259$ eV and E_g in eV.

quantities must be given if the other is to be found. If the carrier concentration is held at a certain value, as in heavily doped extrinsic material, E_F can be obtained from Eq. (3–25). The temperature dependence of electron concentration in a doped semiconductor can be visualized as shown in Fig. 3–18. In this example, Si is doped n-type with a donor concentration N_d of 10^{15} cm^{-3}. At very low temperatures (large $1/T$), negligible intrinsic EHPs exist, and the donor electrons are bound to the donor atoms. As the temperature is raised, these electrons are donated to the conduction band, and at about 100 K ($1000/T = 10$) all the donor atoms are ionized. This temperature range is called the *ionization* region. Once the donors are ionized, the conduction band electron concentration is $n_0 \simeq N_d = 10^{15}$ cm^{-3}, since one electron is obtained for each donor atom. When every available extrinsic electron has been transferred to the conduction band, n_0 is virtually constant with temperature until the concentration of intrinsic carriers n_i becomes comparable to the extrinsic concentration N_d. Finally, at higher temperatures n_i is much greater than N_d, and the intrinsic carriers dominate. In most devices it is desirable to control the carrier concentration by doping rather than by thermal EHP generation. Thus one usually dopes the material such that the extrinsic range extends beyond the highest temperature at which the device is to be used.

3.3.4 Compensation and Space Charge Neutrality

When the concept of doping was introduced, we assumed the material contained either N_d donors or N_a acceptors, so that the extrinsic majority carrier concentrations were $n_0 \simeq N_d$ or $p_0 \simeq N_a$, respectively, for the n-type or p-type material. It often happens, however, that a semiconductor contains both donors and acceptors. For example, Fig. 3–19 illustrates a semiconductor for which both donors and acceptors are present, but $N_d > N_a$. The predominance of donors makes the material n-type, and the Fermi level is therefore in the upper part of the band gap. Since E_F is well above the acceptor level E_a, this level is essentially filled with electrons. However, with E_F above E_i, we cannot expect a hole concentration in the valence band commensurate with the acceptor concentration. In fact, the filling of the E_a states occurs at the expense of the donated conduction band electrons. The mechanism can be visualized as follows: Assume an acceptor state is filled with a valence band electron as described in Fig. 3–12b, with a hole resulting in the valence band. This hole is then filled by recombination with one of the conduction band electrons. Extending this logic to all the acceptor atoms, we expect the resultant concentration of electrons in the conduction band to be $N_d - N_a$ instead of the total N_d. This process is called *compensation*. By this process it is possible to begin with an *n*-type semiconductor and add acceptors until $N_a = N_d$ and no donated electrons remain in the conduction band. In such compensated material, $n_0 = n_i = p_0$ and intrinsic conduction is obtained. With further acceptor doping the semiconductor becomes p-type with a hole concentration of essentially $N_a - N_d$.

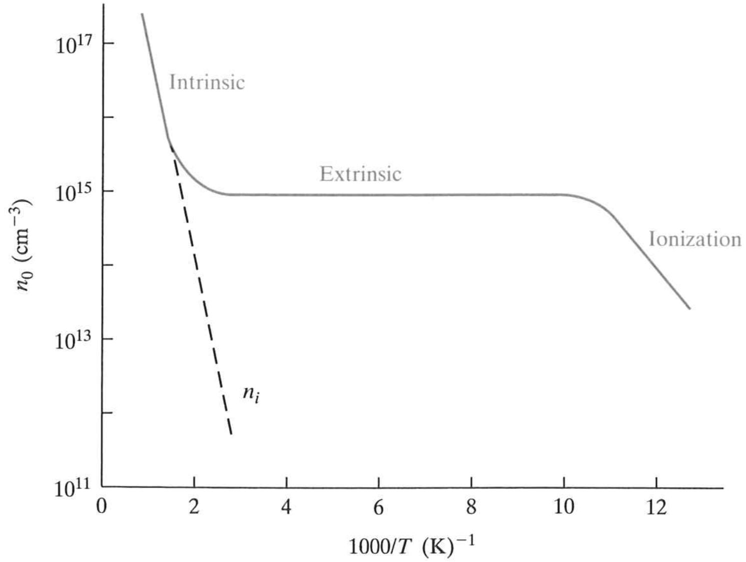


Figure 3–18 Carrier concentration vs. inverse temperature for Si doped with 10^{15} donors/cm^3.

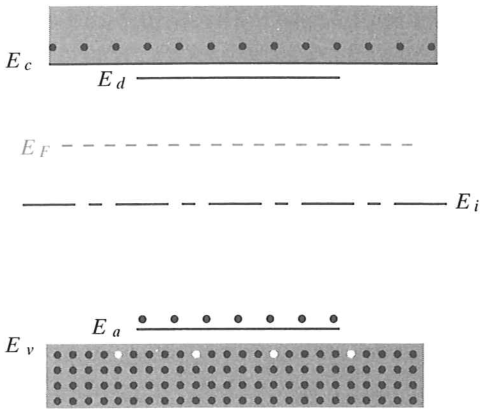


Figure 3–19 Compensation in an n-type semiconductor ($N_d > N_a$).

The exact relationship among the electron, hole, donor, and acceptor concentrations can be obtained by considering the requirements for *space charge neutrality*. If the material is to remain electrostatically neutral, the sum of the positive charges (holes and ionized donor atoms) must balance the sum of the negative charges (electrons and ionized acceptor atoms):

$$p_0 + N_d^+ = n_0 + N_a^- \tag{3-27}$$

Thus in Fig. 3–19 the net electron concentration in the conduction band is

$$n_0 = p_0 + (N_d^+ - N_a^-) \tag{3-28}$$

If the material is doped n-type ($n_0 \gg p_0$) and all the impurities are ionized, we can approximate Eq. (3-28) by $n_0 \simeq N_d - N_a$.

Since the intrinsic semiconductor itself is electrostatically neutral and the doping atoms we add are also neutral, the requirement of Eq. (3-27) must be maintained at equilibrium. The electron and hole concentrations and the Fermi level adjust such that Eqs. (3-27) and (3-25) are satisfied.

3.4 DRIFT OF CARRIERS IN ELECTRIC AND MAGNETIC FIELDS

Knowledge of carrier concentrations in a solid is necessary for calculating current flow in the presence of electric or magnetic fields. In addition to the values of n and p, we must be able to take into account the collisions of the charge carriers with the lattice and with the impurities. These processes will affect the ease with which electrons and holes can flow through the crystal, that is, their *mobility* within the solid. As should be expected, these collision and scattering processes depend on temperature, which affects the thermal motion of the lattice atoms and the velocity of the carriers.

3.4.1 Conductivity and Mobility

The charge carriers in a solid are in constant motion, even at thermal equilibrium. At room temperature, for example, the thermal motion of an individual electron may be visualized as random scattering from lattice vibrations, impurities, other electrons, and defects (Fig. 3-20a). Since the scattering is random, there is no net motion of the group of n electrons/cm^3 over any period of time. This is not true of an individual electron, of course. The probability of the electron in Fig. 3-20a returning to its starting point after some time t is negligibly small. However, if a large number of electrons is considered (e.g., 10^{16} cm^{-3} in an n-type semiconductor), there will be no preferred direction of motion for the group of electrons and no net current flow.

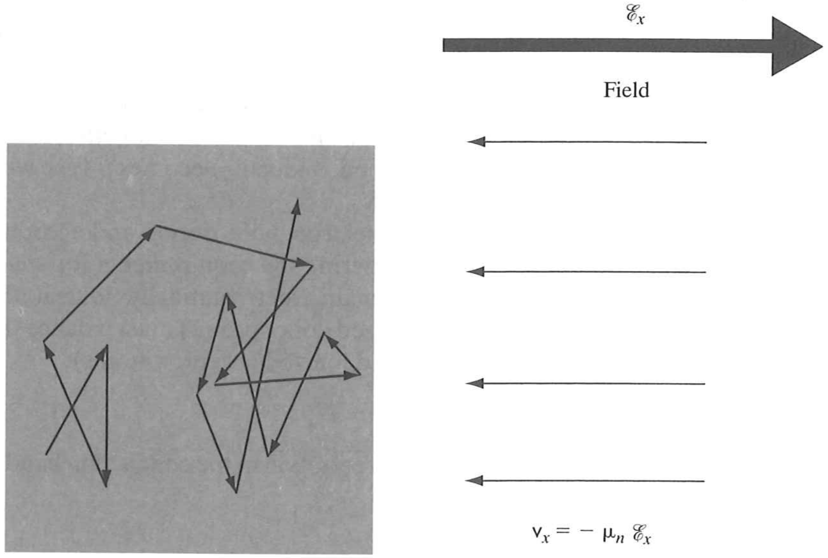


Figure 3-20
(a) Random thermal motion of an electron in a solid; (b) well-directed drift velocity with an applied electric field.

If an electric field $\mathscr{E}_x$ is applied in the x-direction, each electron experiences a net force $-q\mathscr{E}_x$ from the field. This force may be insufficient to alter appreciably the random path of an individual electron; the effect when averaged over all the electrons, however, is a net motion of the group in the $-x$-direction (Fig. 3-20b). If p_x is the x-component of the total momentum of the group, the force of the field on the n electrons/cm^3 is

$$-nq\mathscr{E}_x = \left.\frac{d\mathrm{p}_x}{dt}\right|_{\text{field}} \tag{3-29}$$

Initially, Eq. (3-29) seems to indicate a continuous acceleration of the electrons in the $-x$-direction. This is not the case, however, because the net acceleration of Eq. (3-29) is just balanced in steady state by the decelerations of the collision processes. Thus while the steady field $\mathscr{E}_x$ does produce a net momentum p_{-x}, the net rate of change of momentum when collisions are included must be zero in the case of steady state current flow.

To find the total rate of momentum change from collisions, we must investigate the collision probabilities more closely. If the collisions are truly random, there will be a constant probability of collision at any time for each electron. Let us consider a group of N_0 electrons at time $t = 0$ and define $N(t)$ as the number of electrons that *have not* undergone a collision by time t. The rate of decrease in $N(t)$ at any time t is proportional to the number left unscattered at t,

$$-\frac{dN(t)}{dt} = \frac{1}{\bar{t}}N(t) \tag{3-30}$$

where $\bar{t}^{\,-1}$ is a constant of proportionality.

The solution to Eq. (3-30) is an exponential function

$$N(t) = N_0 e^{-t/\bar{t}} \tag{3-31}$$

and $\bar{t}$ represents the mean time between scattering events,[10] called the *mean free time*. The probability that any electron has a collision in the time interval dt is $dt/\bar{t}$. Thus the differential change in p_x due to collisions in time dt is

$$d\mathrm{p}_x = -\mathrm{p}_x \frac{dt}{\bar{t}} \tag{3-32}$$

---

[10] Equations (3-30) and (3-31) are typical of events dominated by random processes, and the forms of these equations occur often in many branches of physics and engineering. For example, in the radioactive decay of unstable nuclear isotopes, N_0 nuclides decay exponentially with a mean lifetime $\bar{t}$. Other examples will be found in this text, including the absorption of light in a semiconductor and the recombination of excess EHPs.

The rate of change of p_x due to the decelerating effect of collisions is

$$\left.\frac{dp_x}{dt}\right|_{\text{collisions}} = -\frac{p_x}{\bar{t}} \tag{3-33}$$

The sum of acceleration and deceleration effects must be zero for steady state. Taking the sum of Eqs. (3-29) and (3-33), we have

$$-\frac{p_x}{\bar{t}} - nq\mathscr{E}_x = 0 \tag{3-34}$$

The average momentum per electron is

$$\langle p_x\rangle = \frac{p_x}{n} = -q\bar{t}\mathscr{E}_x \tag{3-35}$$

where the angular brackets indicate an average over the entire group of electrons. As expected for steady state, Eq. (3-35) indicates that the electrons have *on the average* a constant net velocity in the negative x-direction:

$$\langle v_x\rangle = \frac{\langle p_x\rangle}{m_n^*} = -\frac{q\bar{t}}{m_n^*}\mathscr{E}_x \tag{3-36}$$

Actually, the individual electrons move in many directions by thermal motion during a given time period, but Eq. (3-36) tells us the *net drift* of an average electron in response to the electric field. The drift speed described by Eq. (3-36) is usually much smaller than the random speed due to the thermal motion v_{th}.

The current density resulting from this net drift is just the number of electrons crossing a unit area per unit time ($n\langle v_x\rangle$) multiplied by the charge on the electron ($-q$):

$$\boxed{\begin{array}{c} J_x = -qn\langle v_x\rangle \\ \dfrac{\text{ampere}}{\text{cm}^2} = \dfrac{\text{coulomb}}{\text{electron}}\cdot\dfrac{\text{electrons}}{\text{cm}^3}\cdot\dfrac{\text{cm}}{\text{s}} \end{array}} \tag{3-37}$$

Using Eq. (3-36) for the average velocity, we obtain

$$J_x = \frac{nq^2\bar{t}}{m_n^*}\mathscr{E}_x \tag{3-38}$$

Thus the current density is proportional to the electric field, as we expect from Ohm's law:

$$J_x = \sigma\mathscr{E}_x, \quad \text{where } \sigma \equiv \frac{nq^2\bar{t}}{m_n^*} \tag{3-39}$$

The conductivity $\sigma(\Omega\text{-cm})^{-1}$ can be written

$$\sigma = qn\mu_n, \quad \text{where } \mu_n \equiv \frac{q\bar{t}}{m_n^*} \tag{3-40a}$$

The quantity μ_n, called the *electron mobility*, describes the ease with which electrons drift in the material. Mobility is a very important quantity in characterizing semiconductor materials and in device development.

Here m_n^* is the conductivity effective mass for electrons, different from the density-of-states effective mass mentioned in Eq. (3-16b). While we use the density-of-states effective mass to count the number of carriers in bands, we must use the conductivity effective mass for charge transport problems. To illustrate how it is obtained from the band curvature effective masses mentioned in Section 3.2.2, once again let us consider the 6 equivalent conduction band minima along the X-directions for Si, with the band curvature longitudinal effective mass, m_l, along the major axis of the ellipsoid, and the transverse effective mass, m_t, along the two minor axes (Fig. 3-10b). Since we have $1/m_n^*$ in the mobility expression Eq. (3-40a), by using dimensional equivalence, we can write the conductivity effective mass as the harmonic mean of the band curvature effective masses.

$$\frac{1}{m_n^*} = \frac{1}{3}\left(\frac{1}{m_l} + \frac{2}{m_t}\right) \tag{3-40b}$$

EXAMPLE 3–6

Calculate the conductivity effective mass of electrons in Si.

SOLUTION

For Si, $m_l = 0.98\ m_0$; $m_t = 0.19\ m_0$ (Appendix III).
There are 6 equivalent X valleys in the conduction band.

$$1/m_n^* = 1/3(1/m_x + 1/m_y + 1/m_z) = 1/3(1/m_l + 2/m_t)$$

$$1/m_n^* = \frac{1}{3}\left(\frac{1}{0.98\ m_0} + \frac{2}{0.19\ m_0}\right)$$

$$m_n^* = 0.26\ m_0$$

Note: For GaAs, the conduction band equi-energy surfaces are spherical. So there is only one band curvature effective mass. (The density of states effective mass and the conductivity effective mass are both $0.067\ m_0$.)

The mobility defined in Eq. (3-40a) can be expressed as the average particle drift velocity per unit electric field. Comparing Eqs. (3-36) and (3-40a), we have

$$\mu_n = -\frac{\langle v_x\rangle}{\mathscr{E}_x} \tag{3-41}$$

The units of mobility are (cm/s)/(V/cm) = cm^2/V-s, as Eq. (3-41) suggests. The minus sign in the definition results in a positive value of mobility, since electrons drift opposite to the field.

The current density can be written in terms of mobility as

$$J_x = qn\mu_n\mathscr{E}_x \tag{3-42}$$

This derivation has been based on the assumption that the current is carried primarily by electrons. For hole conduction we change n to p, $-q$ to $+q$, and μ_n to μ_p, where $\mu_p = +\langle v_x\rangle/\mathscr{E}_x$ is the mobility for holes. If both electrons and holes participate, we must modify Eq. (3-42) to

$$\boxed{J_x = q(n\mu_n + p\mu_p)\mathscr{E}_x = \sigma\mathscr{E}_x} \tag{3-43}$$

Values of μ_n and μ_p are given for many of the common semiconductor materials in Appendix III. According to Eq. (3-40), the parameters determining mobility are m^* and mean free time $\bar{t}$. Effective mass is a property of the material's band structure, as described by Eq. (3-3). Thus we expect m_n^* to be small in the strongly curved Γ minimum of the GaAs conduction band (Fig. 3-6), with the result that μ_n is very high. In a more gradually curved band, a larger m^* in the denominator of Eq. (3-40) leads to a smaller value of mobility. It is reasonable to expect that lighter particles are more mobile than heavier particles (which is satisfying, since the common-sense value of effective mass is not always apparent). The other parameter determining mobility is the mean time between scattering events, $\bar{t}$. In Section 3.4.3 we shall see that this is determined primarily by temperature and impurity concentration in the semiconductor.

3.4.2 Drift and Resistance

Let us look more closely at the drift of electrons and holes. If the semiconductor bar of Fig. 3-21 contains both types of carrier, Eq. (3-43) gives the conductivity of the material. The resistance of the bar is then

$$R = \frac{\rho L}{wt} = \frac{L}{wt}\frac{1}{\sigma} \tag{3-44}$$

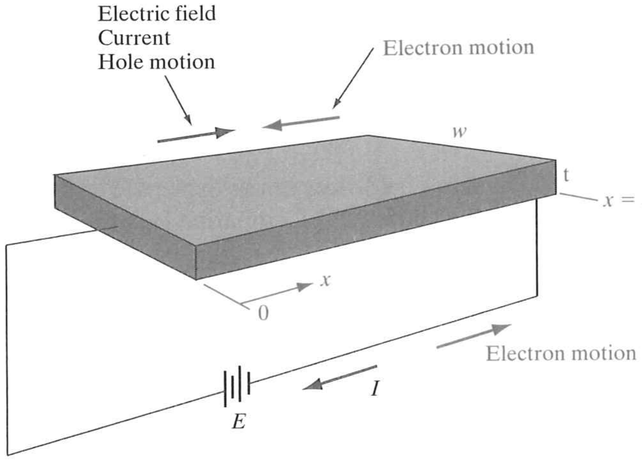


Figure 3–21 Drift of electrons and holes in a semiconductor bar.

where ρ is the resistivity (Ω-cm). The physical mechanism of carrier drift requires that the holes in the bar move as a group in the direction of the electric field and that the electrons move as a group in the opposite direction. Both the electron and the hole components of current are in the direction of the $\mathscr{E}$ field, since conventional current is positive in the direction of hole flow and opposite to the direction of electron flow. The drift current described by Eq. (3-43) is constant throughout the bar. A valid question arises, therefore, concerning the nature of the electron and hole flow at the contacts and in the external circuit. We should specify that the contacts to the bar of Fig. 3-21 are *ohmic,* meaning that they are perfect sources and sinks of both carrier types and have no special tendency to inject or collect either electrons or holes.

If we consider that current is carried around the external circuit by electrons, there is no problem in visualizing electrons flowing into the bar at one end and out at the other (always opposite to I). Thus for every electron leaving the left end ($x = 0$) of the bar in Fig. 3-21, there is a corresponding electron entering at $x = L$, so that the electron concentration in the bar remains constant at n. But what happens to the holes at the contacts? As a hole reaches the ohmic contact at $x = L$, it recombines with an electron, which must be supplied through the external circuit. As this hole disappears, a corresponding hole must appear at $x = 0$ to maintain space charge neutrality. It is reasonable to consider the source of this hole as the generation of an EHP at $x = 0$, with the hole flowing into the bar and the electron flowing into the external circuit.

3.4.3 Effects of Temperature and Doping on Mobility

The two basic types of scattering mechanisms that influence electron and hole mobility are *lattice scattering* and *impurity scattering.* In lattice scattering a carrier moving through the crystal is scattered by a vibration of the lattice, resulting from the temperature.[11] The frequency of such scattering events increases as the temperature increases, since the thermal agitation of the lattice becomes greater. Therefore, we should expect the mobility to decrease as the sample is heated (Fig. 3-22). On the other hand, scattering from crystal defects such as ionized impurities becomes the dominant mechanism at low temperatures. Since the atoms of the cooler lattice are less agitated, lattice scattering is less important; however, the thermal motion of the carriers is also slower. Since a slowly moving carrier is likely to be scattered more strongly by an interaction with a charged ion than is a carrier with greater momentum, impurity scattering events cause a decrease in mobility with decreasing temperature. As Fig. 3-22 indicates, the approximate temperature dependencies are $T^{-3/2}$ for lattice scattering and $T^{3/2}$ for impurity scattering. Since the scattering probability of Eq. (3-32) is inversely proportional to the

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[11] Collective vibrations of atoms in the crystal are called *phonons.* Thus lattice scattering is also known as *phonon scattering.*

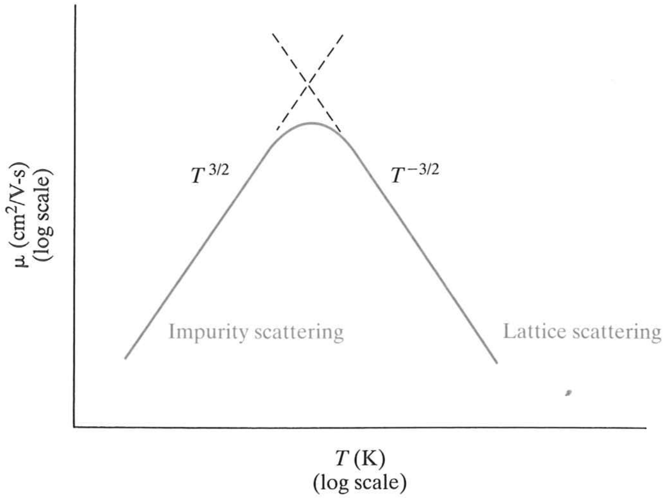

Figure 3–22
Approximate temperature dependence of mobility with both lattice and impurity scattering.

mean free time and therefore to mobility, the mobilities due to two or more scattering mechanisms add inversely:

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \ldots \tag{3-45}$$

As a result, the mechanism causing the lowest mobility value dominates, as shown in Fig. 3-22.

As the concentration of impurities increases, the effects of impurity scattering are felt at higher temperatures. For example, the electron mobility μ_n of intrinsic silicon at 300 K is 1350 $\text{cm}^2/(\text{V-s})$. With a donor doping concentration of $10^{17}\ \text{cm}^{-3}$, however, μ_n is 700 $\text{cm}^2/(\text{V-s})$. Thus, the presence of the 10^{17} ionized donors/cm^3 introduces a significant amount of impurity scattering. This effect is illustrated in Fig. 3-23, which shows the variation of mobility with doping concentration at room temperature.

EXAMPLE 3–7 A Si bar 0.1 cm long and 100 μm^2 in cross-sectional area is doped with $10^{17}\ \text{cm}^{-3}$ phosphorus. Find the current at 300 K with 10 V applied.

SOLUTION For this applied voltage, we see that the electric field is low enough that we are in the ohmic regime. From Fig. 3-23, for this doping, the $\mu_n = 700\ \text{cm}^2/\text{V-s}$.

$$\sigma = q\mu_n n_0 = 1.6 \times 10^{-19} \times 700 \times 10^{17} = 11.2(\Omega \cdot \text{cm})^{-1} = \rho^{-1}$$

$$\rho = 0.0893\ \Omega \cdot \text{cm}$$

$$R = \rho L/A = 0.0893 \times 0.1/10^{-6} = 8.93 \times 10^3 \Omega$$

$$I = V/R = 10/(8.93 \times 10^3) = \mathbf{1.12\ mA}$$

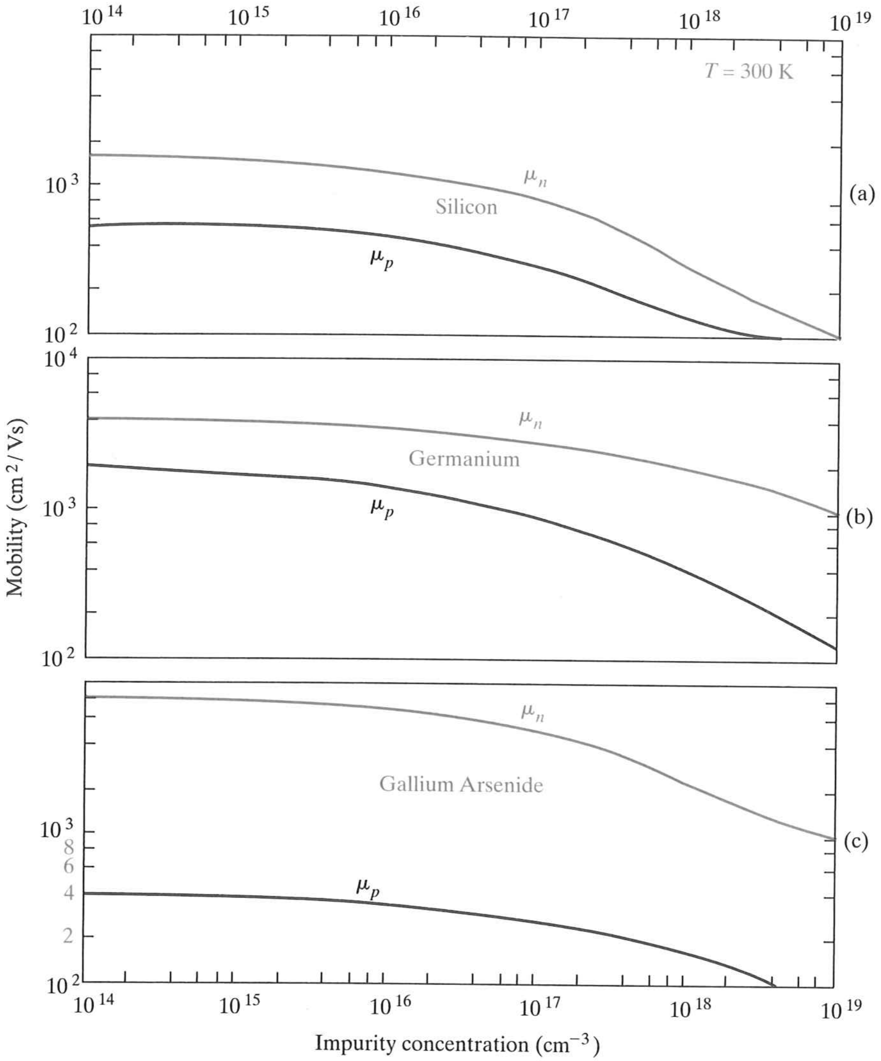

Figure 3–23
Variation of mobility with total doping impurity concentration ($N_a + N_d$) for Ge, Si, and GaAs at 300 K.

3.4.4 High-Field Effects

One assumption implied in the derivation of Eq. (3-39) was that Ohm's law is valid in the carrier drift processes. That is, it was assumed that the drift current is proportional to the electric field and that the proportionality constant (σ) is not a function of field $\mathscr{E}$. This assumption is valid over a wide range of $\mathscr{E}$. However, large electric fields ($> 10^3$V/cm) can cause the drift velocity and therefore the current $J = -qnv_d$ to exhibit a sublinear dependence on the

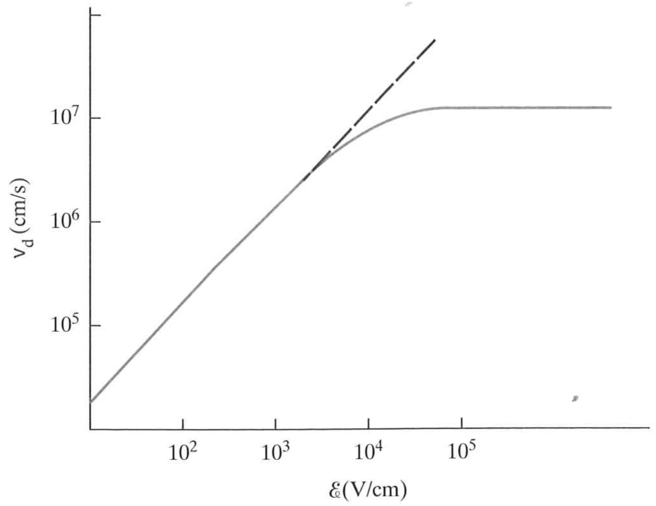

Figure 3–24 Saturation of electron drift velocity at high electric fields for Si.

electric field. This dependence of σ upon $\mathscr{E}$ is an example of a *hot carrier* effect, which implies that the carrier drift velocity v_d is comparable to the thermal velocity v_{th}.

In many cases an upper limit is reached for the carrier drift velocity in a high field (Fig. 3-24). This limit occurs near the mean thermal velocity ($\simeq 10^7$ cm/s) and represents the point at which added energy imparted by the field is transferred to the lattice rather than increasing the carrier velocity. The result of this *scattering limited velocity* is a fairly constant current at high field. This behavior is typical of Si, Ge, and some other semiconductors. However, there are other important effects in some materials; for example, in Chapter 10 we shall discuss a *decrease* in electron velocity at high fields for GaAs and certain other materials, which results in negative conductivity and current instabilities in the sample. Another important high-field effect is avalanche multiplication, which we shall discuss in Section 5.4.2.

3.4.5 The Hall Effect

If a magnetic field is applied perpendicular to the direction in which holes drift in a p-type bar, the path of the holes tends to be deflected (Fig. 3-25). Using vector notation, the total force on a single hole due to the electric and magnetic fields is

$$\mathbf{F} = q(\mathscr{E} + \mathrm{v} \times \mathscr{B}) \tag{3-46}$$

In the y-direction the force is

$$F_y = q(\mathscr{E}_y - \mathrm{v}_x\mathscr{B}_z) \tag{3-47}$$

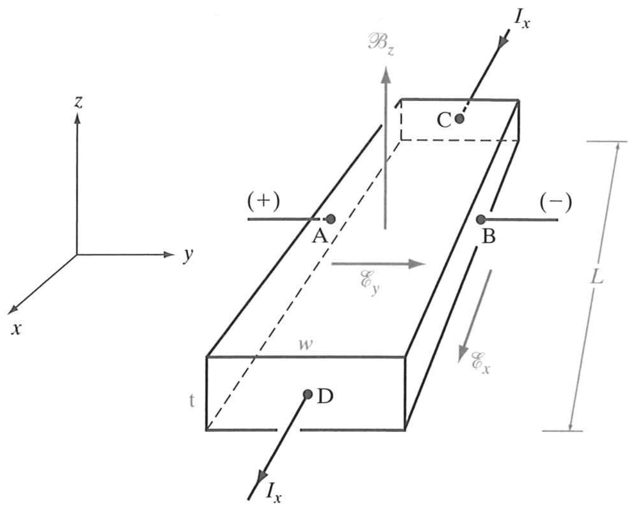

Figure 3–25 The Hall effect.

The important result of Eq. (3-47) is that unless an electric field $\mathscr{E}_y$ is established along the width of the bar, each hole will experience a net force (and therefore an acceleration) in the $-y$-direction due to the $q\mathrm{v}_x\mathscr{B}_z$ product. Therefore, to maintain a steady state flow of holes down the length of the bar, the electric field $\mathscr{E}_y$ must just balance the product $\mathrm{v}_x\mathscr{B}_z$:

$$\mathscr{E}_y = \mathrm{v}_x\mathscr{B}_z \tag{3-48}$$

so that the net force F_y is zero. Physically, this electric field is set up when the magnetic field shifts the hole distribution slightly in the $-y$-direction. Once the electric field $\mathscr{E}_y$ becomes as large as $\mathrm{v}_x\mathscr{B}_z$, no net lateral force is experienced by the holes as they drift along the bar. The establishment of the electric field $\mathscr{E}_y$ is known as the *Hall effect,* and the resulting voltage $V_{AB} = \mathscr{E}_y w$ is called the *Hall voltage.* If we use the expression derived in Eq. (3-37) for the drift velocity (using $+q$ and p_0 for holes), the field $\mathscr{E}_y$ becomes

$$\mathscr{E}_y = \frac{J_x}{qp_0}\mathscr{B}_z = R_H J_x\mathscr{B}_z, \quad R_H \equiv \frac{1}{qp_0} \tag{3-49}$$

Thus the Hall field is proportional to the product of the current density and the magnetic flux density. The proportionality constant $R_H = (qp_0)^{-1}$ is called the *Hall coefficient.* A measurement of the Hall voltage for a known current and magnetic field yields a value for the hole concentration p_0

$$p_0 = \frac{1}{qR_H} = \frac{J_x\mathscr{B}_z}{q\mathscr{E}_y} = \frac{(I_x/wt)\mathscr{B}_z}{q(V_{AB}/w)} = \frac{I_x\mathscr{B}_z}{qtV_{AB}} \tag{3-50}$$

Since all of the quantities in the right-hand side of Eq. (3-50) can be measured, the Hall effect can be used to give quite accurate values for carrier concentration.

If a measurement of resistance R is made, the sample resistivity ρ can be calculated:

$$\rho(\Omega\text{–cm}) = \frac{Rwt}{L} = \frac{V_{CD}/I_x}{L/wt} \tag{3-51}$$

Since the conductivity $\sigma = 1/\rho$ is given by $q\mu_p p_0$, the mobility is simply the ratio of the Hall coefficient and the resistivity:

$$\mu_p = \frac{\sigma}{qp_0} = \frac{1/\rho}{q(1/qR_H)} = \frac{R_H}{\rho} \tag{3-52}$$

Measurements of the Hall coefficient and the resistivity over a range of temperatures yield plots of majority carrier concentration and mobility vs. temperature. Such measurements are extremely useful in the analysis of semiconductor materials. Although the discussion here has been related to p-type material, similar results are obtained for n-type material. A negative value of q is used for electrons, and the Hall voltage V_{AB} and Hall coefficient R_H are negative. In fact, measurement of the sign of the Hall voltage is a common technique for determining if an unknown sample is p-type or n-type.

EXAMPLE 3–8 Referring to Fig. 3-25, consider a semiconductor bar with $w = 0.1$ mm, $t = 10$ μm, and $L = 5$ mm. For $\mathscr{B} = 10$ kg in the direction shown (1 kG = 10^{-5} Wb/cm^2) and a current of 1 mA, we have $V_{AB} = -2$ mV, $V_{CD} = 100$ mV. Find the type, concentration, and mobility of the majority carrier.

SOLUTION $\mathscr{B}_z = 10^{-4}$ Wb/cm^2

From the sign of V_{AB}, we can see that the majority carriers are electrons:

$$n_0 = \frac{I_x\mathscr{B}_z}{qt(-V_{AB})} = \frac{(10^{-3})(10^{-4})}{1.6\times10^{-19}(10^{-3})(2\times10^{-3})} = 3.125\times10^{17}\ \text{cm}^{-3}$$

$$\rho = \frac{R}{L/wt} = \frac{V_{CD}/I_x}{L/wt} = \frac{0.1/10^{-3}}{0.5/0.01\times10^{-3}} = 0.002\ \Omega\cdot\text{cm}$$

$$\mu_n = \frac{1}{\rho q n_0} = \frac{1}{(0.002)(1.6\times10^{-19})(3.125\times10^{17})} = \mathbf{10{,}000\ cm^2(V\cdot s)^{-1}}$$

3.5 INVARIANCE OF THE FERMI LEVEL AT EQUILIBRIUM

In this chapter we have discussed homogeneous semiconductors, without variations in doping and without junctions between dissimilar materials. In the following chapters we will be considering cases in which nonuniform doping occurs in a given semiconductor, or junctions occur between different semiconductors or a semiconductor and a metal. These cases are crucial to the various types of electronic and optoelectronic devices made in semiconductors. In anticipation of those discussions, an important concept should be established here regarding the demands of equilibrium. That concept can be summarized by noting that *no discontinuity or gradient can arise in the equilibrium Fermi level* E_F.

To demonstrate this assertion, let us consider two materials in intimate contact such that electrons can move between the two (Fig. 3-26). These may be, for example, dissimilar semiconductors, n- and p-type regions, a metal and a semiconductor, or simply two adjacent regions of a nonuniformly doped semiconductor. Each material is described by a Fermi–Dirac distribution function and some distribution of available energy states that electrons can occupy.

There is no current, and therefore no net charge transport, at thermal equilibrium. There is also no net transfer of energy. Therefore, for each energy E in Fig. 3-26 any transfer of electrons from material 1 to material 2 must be exactly balanced by the opposite transfer of electrons from 2 to 1. We will let the density of states at energy E in material 1 be called $N_1(E)$ and in material 2 we will call it $N_2(E)$. At energy E the rate of transfer of electrons from 1 to 2 is proportional to the number of filled states at E in material 1 times the number of empty states at E in material 2:

$$\text{rate from 1 to 2} \propto N_1(E)f_1(E)\cdot N_2(E)[1 - f_2(E)] \tag{3-53}$$

where $f(E)$ is the probability of a state being filled at E in each material, i.e., the Fermi–Dirac distribution function given by Eq. (3-10). Similarly,

$$\text{rate from 2 to 1} \propto N_2(E)f_2(E)\cdot N_1(E)[1 - f_1(E)] \tag{3-54}$$

At equilibrium these must be equal:

$$N_1(E)f_1(E)\cdot N_2(E)[1 - f_2(E)] = N_2(E)f_2(E)\cdot N_1(E)[1 - f_1(E)] \tag{3-55}$$

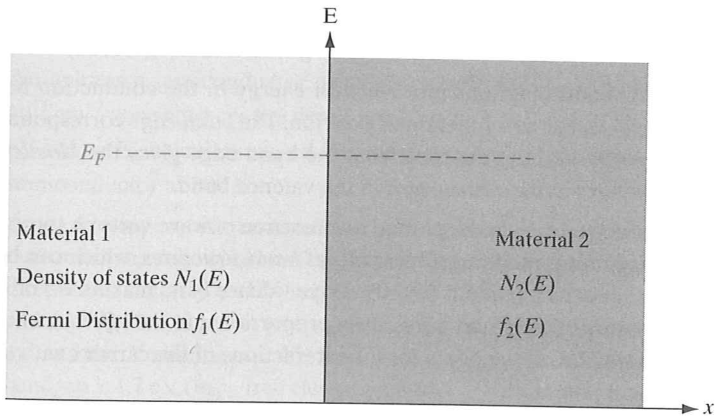

Figure 3–26
Two materials in intimate contact at equilibrium. Since the net motion of electrons is zero, the equilibrium Fermi level must be constant throughout.

Rearranging terms, we have, at energy E,

$$N_1 f_1 N_2 - N_1 f_1 N_2 f_2 = N_2 f_2 N_1 - N_2 f_2 N_1 f_1 \tag{3-56}$$

which results in

$$f_1(E) = f_2(E), \quad \text{that is,} \quad [1 + e^{(E-E_{F1})/kT}]^{-1} = [1 + e^{(E-E_{F2})/kT}]^{-1} \tag{3-57}$$

Therefore, we conclude that $E_{F1} = E_{F2}$. That is, there is no discontinuity in the equilibrium Fermi level. More generally, we can state that the Fermi level at equilibrium must be constant throughout materials in intimate contact. One way of stating this is that no gradient exists in the Fermi level at equilibrium:

$$\boxed{\frac{dE_F}{dx} = 0} \tag{3-58}$$

We will make considerable use of this result in the chapters to follow.

SUMMARY

3.1 In a diamond lattice, each Si atom (with four valence electrons) is surrounded by four Si atoms that form four *covalent* bonds consisting of shared electron pairs, thereby forming an *octet* of electrons in the valence shell. In zinc blende structures such as GaAs, electrons are partly shared (covalent bonding) and partly transferred from Ga to As (*ionic* bonding).

3.2 In crystals, electronic *wave functions overlap* to give various linear combinations of atomic orbitals (LCAO). Bonding or symmetric combinations of the wave functions of valence-shell electrons form (*almost*) *continuous allowed bands* of energies in the (*almost*) *filled valence* band, separated by an energy gap from higher energy states in an (*almost*) *empty conduction* band that correspond to the antibonding or antisymmetric LCAOs. Empty electronic states in the valence band can be considered to be positively charged carriers (*holes*), while filled states in the conduction band are negatively charged (*conduction*) *electrons*.

3.3 If the band gaps are large, we get insulators; if they are small (~1 eV), we get semiconductors; and if they are zero, we get conductors (metals).

3.4 *Simplified* band diagrams plot *electron energy* in the conduction band (*increasing upwards*) as a function of position. The *band edge* corresponds to *potential* energy, and the distance from the band edge gives the *kinetic* energy. *Hole energies increase downward* in the valence band.

3.5 Carrier energies can also be plotted as a function of wave vector **k** (proportional to velocity or momentum) to give (E, **k**) *band structures*, which can be direct (conduction band minimum directly above valence band maximum) or indirect. The *curvature* of the (E, **k**) is inversely proportional to the *effective mass* m^* of the carriers. The m^* accounts for the interactions of the carriers with the periodic crystal potential.

3.6 In a pure semiconductor, we have an *intrinsic* concentration of electrons (or holes), n_i, that result from *thermal* generation–recombination between the valence and conduction band (or bond breaking). If we replace some Si atoms (with four valence electrons) with *donor* impurities with five valence electrons, they donate conduction electrons, n (= N_d^+); similarly, *acceptors* create holes p.

3.7 The number of electrons, n, is the integral with respect to energy from the bottom to the top of the conduction band of the product of the available *density of states* (DOS) and the *Fermi–Dirac* (FD) distribution function. For parabolic band structures, we get a parabolic DOS. The FD function is the average occupancy of an electronic state. The electron concentration n can also be expressed as the product of an *effective* DOS at the band edge and the FD occupancy at E_c, and similarly for holes p. The np product in equilibrium is constant (n_i^2).

3.8 Electrons in a solid execute *random Brownian* motion with an average kinetic energy related to the thermal energy kT. In an electric field, electrons *drift* (on top of the random motion) with a velocity equal to *mobility* times field in the *ohmic* regime and *saturation velocity* at *high fields*. The drift current is proportional to carrier concentration times drift velocity. Negatively charged electrons drift opposite to the electric field, and the current is opposite to the motion. Positively charged holes drift in the direction of the electric field, and the current goes in the same direction as the hole flow.

3.9 Carrier mobility is determined by *scattering*, caused by *deviations* from a periodic lattice potential, such as lattice vibrations (phonons) or ionized impurities. Carrier mobility and concentration can be obtained by the Hall effect and resistivity measurements.

PROBLEMS

3.1 Calculate the approximate donor binding energy for GaAs ($\epsilon_r = 13.2$, $m_n^* = 0.067\ m_0$).

3.2 Calculate values for the Fermi function $f(C)$ at 300 K and plot vs. energy in eV as in Fig. 3–14. Choose $E_F = 1$ eV and make the calculated points closer together near the Fermi level to obtain a smooth curve. Notice that $f(E)$ varies quite rapidly within a few kT of E_F. Show that the probability that a state ΔE above E_F is occupied is the same as the probability that the state ΔE below E_F is empty.

3.3 An unknown semiconductor has $E_g = 1.1$ eV and $N_c = N_v$. It is doped with 10^{15} cm^{-3} donors, where the donor level is 0.2 eV below E_c. Given that E_F is 0.25 eV below E_c, calculate n_i and the concentration of electrons and holes in the semiconductor at 300 K.

3.4 At room temperature, an unknown indirect band gap, intrinsic, cubic semiconductor has the following band structure: There are 6 X minima along the $\langle 100 \rangle$ directions. If $m_n^*(\Gamma) = 0.065 m_0$, $m_n^*(X) = 0.30\ m_0$ (for each of the X minima) and $m_p^* = 0.47 m_0$, at what temperature is the number of electrons in the Γ minima and the X minima equal if the Γ to X energy separation is 0.35 eV, and the bandgap is 1.7 eV (m_0 = free electron mass)?

3.5 Since the effective mass of electrons in a conduction band decreases with increasing curvature of the band according to Eq. (3–3), comment on the electron effective mass in the Γ valley of GaAs compared with the indirect X or L valleys. (See Fig. 3–10.) How is this effective mass difference reflected in the electron mobility for GaAs and GaP shown in Appendix III? From Fig. 3–10, what would you expect to happen to the conductivity of GaAs if Γ-valley electrons drifting in an electric field were suddenly promoted to the L valley?

3.6 Calculate the band gap of Si from Eq. (3–23) and the plot of n_i vs. $1000/T$ (Fig. 3–17). *Hint:* The slope cannot be measured directly from a semilogarithmic plot; read the values from two points on the plot and take the natural logarithm as needed for the solution.

3.7 (a) A Si sample is doped with 10^{16} cm^{-3} boron atoms and a certain number of shallow donors. The Fermi level is 0.36 eV above E_i at 300 K. What is the donor concentration N_d?

(b) A Si sample contains 10^{16} cm^{-3} In acceptor atoms and a certain number of shallow donors. The In acceptor level is 0.16 eV above E_v, and E_F is 0.26 eV above E_v at 300 K. How many (cm^{-3}) In atoms are un-ionized (i.e., neutral)?

3.8 Show that Eq. (3–25) results from Eqs. (3–15) and (3–19). If $n_0 = 10^{16}$ cm^{-3}, where is the Fermi level relative to E_i in Si at 300 K?

3.9 Derive an expression relating the intrinsic level E_i to the center of the band gap $E_g/2$. Calculate the displacement of E_i from $E_g/2$ for Si at 300 K, assuming the effective mass values for electrons and holes are $1.1m_0$ and $0.56m_0$, respectively.

3.10 A semiconductor device requires n-type material; it is to be operated at 400 K. Would Si doped with 10^{15} atoms/cm^3 of arsenic be useful in this application? Could Ge doped with 10^{15} cm^{-3} antimony be used?

3.11 A new semiconductor has $N_c = 10^{19}$ cm^{-3}, $N_v = 5 \times 10^{18}$ cm^{-3}, and $E_g = 2$ eV. If it is doped with 10^{17} donors (fully ionized), calculate the electron, hole, and intrinsic carrier concentrations at 627°C. Sketch the simplified band diagram, showing the position of E_F.

3.12 (a) Show that the minimum conductivity of a semiconductor sample occurs when $n_0 = n_i\sqrt{\mu_p/\mu_n}$. *Hint*: begin with Eq. (3–43) and apply Eq. (3–24).

(b) What is the expression for the minimum conductivity $\sigma_{\min}$?

(c) Calculate $\sigma_{\min}$ for Si at 300 K and compare with the intrinsic conductivity.

3.13 (a) A Si bar 0.1 μm long and 100 μm^2 in cross-sectional area is doped with 10^{17} cm^{-3} phosphorus. Find the current at 300 K with 10 V applied. Repeat for a Si bar 1 μm long.

(b) How long does it take an average electron to drift 1 μm in pure Si at an electric field of 100 V/cm? Repeat for 10^5 V/cm.

3.14. (a) A Si sample is doped with 10^{17} boron atoms/cm^3. What is the electron concentration n_0 at 300 K? What is the resistivity?

(b) A Ge sample is doped with 3×10^{13} Sb atoms/cm^3. Using the requirements of space charge neutrality, calculate the electron concentration n_0 at 300 K.

3.15 For a Si conductor of length 5 μm, doped n-type at 10^{15} cm^{-3}, calculate the current density for an applied voltage of 2.5 V across its length. How about for a voltage of 2500 V? The electron and hole mobilities are 1500 cm^2/V-s and 500 cm^2/V-s, respectively, in the ohmic region for electric fields below 10^4 V/cm. For higher fields, electrons and holes have a saturation velocity of 10^7 cm/s.

3.16 In a long semiconductor bar ($E_G = 2$ eV), conduction band electrons come in from the left in the positive x direction with a kinetic energy of 3 eV. They move from location A to B to C to D. Between A and B, the electric field is zero; between locations B and C, there is a linearly varying voltage increase of 4 V; between C and D, the field is again zero. Assuming no scattering, sketch a *simplified* band diagram describing the motion of these electrons. Assuming that these electrons can be described as plane waves, with a free-electron mass, write down the wave function of the electrons at D. Leave your result in terms of an arbitrary normalization constant.

3.17 Assume that a conduction electron in Si (μ_n = 1350 cm^2/V-s) has a thermal energy of kT, related to its mean thermal velocity by $E_{th} = (m_0 v_{th}^2)/2$. This electron is placed in an electric field of 100 V/cm. Show that the drift velocity of the electron in this case is small compared with its thermal velocity. Repeat for a field of 10^4 V/cm, using the same value of μ_n. Comment on the actual mobility effects at this higher value of the field.

3.18 Use Eq. (3–45) to calculate and plot the mobility vs. temperature $\mu(T)$ from 10 K to 500 K for Si doped with $N_d = 10^{14}$, 10^{16}, and 10^{18} donors cm^{-3}. Consider the mobility to be determined by impurity and phonon (lattice) scattering. Impurity scattering limited mobility can be described by

$$\mu_I = 3.29 \times 10^{15} \frac{\epsilon_r^2 T^{3/2}}{N_d^+ (m_n^*/m_0)^{1/2}\left[\ln(1+z) - \dfrac{z}{1+z}\right]}$$

where

$$z = 1.3 \times 10^{13} \epsilon_r T^2 (m_n^*/m_0)(N_d^+)^{-1}$$

Assume that the ionized impurity concentration N_d^+ is equal to N_d at all temperatures.

The conductivity effective mass m_n^* for Si is 0.26 m_0. Acoustic phonon (lattice) scattering limited mobility can be described by

$$\mu_{AC} = 1.18 \times 10^{-5} c_1 (m_n^*/m_0)^{-5/2} T^{-3/2} (E_{AC})^{-2}$$

where the stiffness (c_1) is given by

$$c_1 = 1.9 \times 10^{12} \text{ dyne cm}^{-2} \text{ for Si}$$

and the conduction band acoustic deformation potential (E_{AC}) is

$$E_{AC} = 9.5 \text{ eV for Si}$$

3.19 Rework Prob. 3.18 considering carrier freeze-out onto donors at low T. That is, consider

$$N_d^+ = \frac{N_d}{1 + \exp(E_d/kT)}$$

as the ionized impurity concentration. Consider the donor ionization energy (E_d) to be 45 meV for Si.

3.20 Hall measurements are made on a p-type semiconductor bar 500 μm wide and 20 μm thick. The Hall contacts A and B are displaced 2 μm with respect to each other in the direction of current flow of 3 mA. The voltage between A and B with a magnetic field of 10 kG (1kG = 10^{-5} Wb/cm^2) pointing out of the plane of the sample is 3.2 mV. When the magnetic field direction is reversed the voltage changes to −2.8 mV. What is the hole concentration and mobility?

3.21 In soldering wires to a sample such as that shown in Fig. 3–25, it is difficult to align the Hall probes A and B precisely. If B is displaced slightly down the length of the bar from A, an erroneous Hall voltage results. Show that the true Hall voltage V_H can be obtained from two measurements of V_{AB}, with the magnetic field first in the $+z$-direction and then in the $-z$-direction.

3.22 A sample of Si is doped with 10^{17} phosphorus atoms/cm^3. What would you expect to measure for its resistivity? What Hall voltage would you expect in a sample 100 μm thick if I_x = 1 mA and B_z = 1 kG = 10^{-5} Wb/cm^2?

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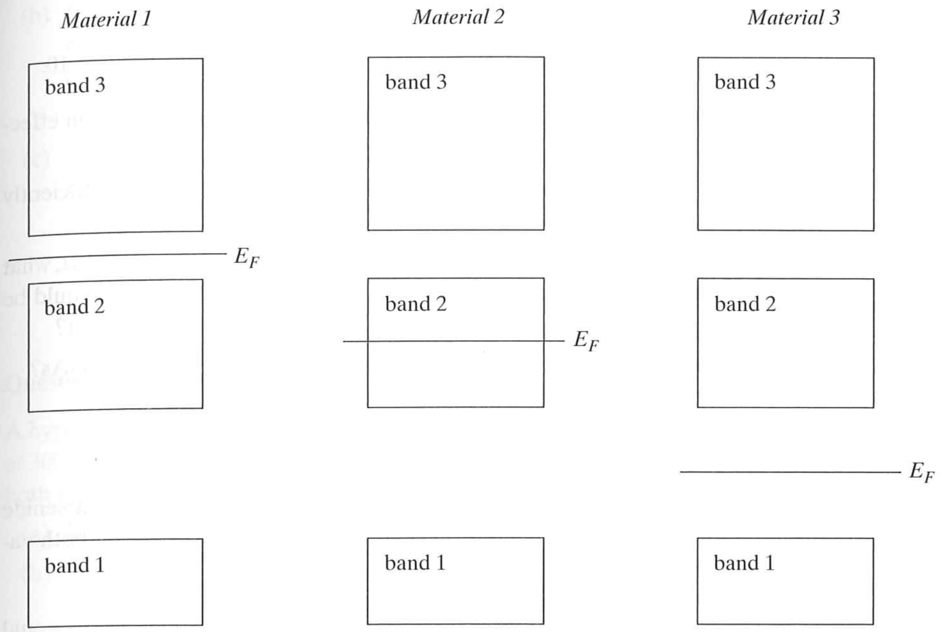
SELF QUIZ

Question 1

(a) The following three diagrams show three different energy bands of some hypothetical crystalline materials (energy varies vertically). The only difference between the three materials is the assumed Fermi level energy E_F. Characterize each material as a ***metal***, an ***insulator***, or a ***semiconductor***.

(b) Assuming you can see through one and only one of the materials of part (a) above, which one would it most likely be? ***Material 1 / Material 2 / Material 3***

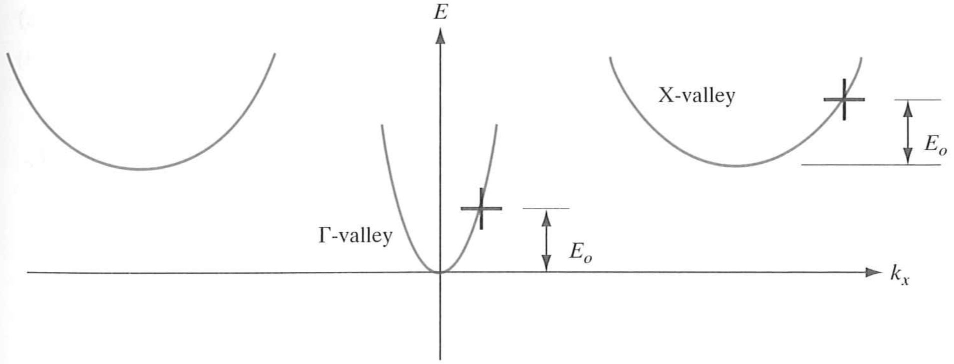


Question 2

Consider the following conduction band energy E vs. wave vector k_x dispersion relation.

(a) Which energy valley has the greater effective mass in the x-direction m_x (circle one)? **Γ-valley / X-valley**

(b) Consider two electrons, one each located at the positions of the heavy crosses. Which has the greater velocity magnitude (circle one)? **The one in the Γ-valley / The one in the X-valley**



Question 3

These questions refer to the band structures of Si and GaAs shown in Fig. 3–10.

(a) Which material appears to have the *lowest* (conduction band) electron effective mass, Si or GaAs?

(b) Which of these would you expect to produce photons (light) more efficiently through electron–hole recombination?

(c) Consistent with your answer to part (b) and making use of Appendix III, what would you expect the energy of the emitted photons to be? What would be their wavelength in μm? Would these be visible, infrared, or ultraviolet?

(d) How many equivalent conduction band minima do we have for Si? GaAs?

Question 4

Refer to Fig. 3–10, which shows the E vs. $\mathbf{k}$ dispersion relations for gallium arsenide (GaAs) and for silicon (Si) along the [111] and [100] directions, showing both valence and conduction bands.

(a) Neglecting differences in electron scattering rates in the two materials, would you expect Si or GaAs to have the greatest electron mobility μ_n?

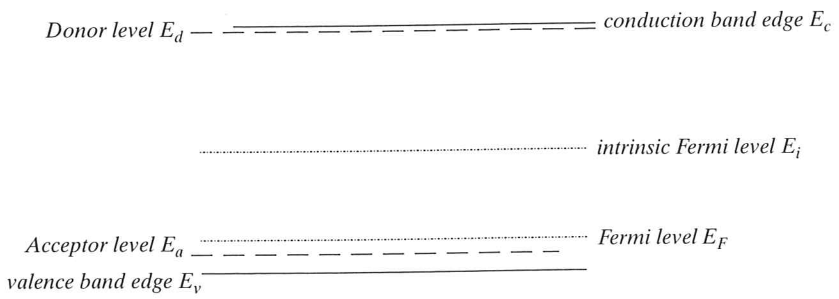
(b) If a constant force were applied in the [100] direction for a short period of time on an electron initially located at the conduction band minimum of each semiconductor and if scattering were neglected, would the magnitude of change in **k** in Si be greater, equal to, or smaller than the magnitude of the change in **k** in GaAs for the same force **F**?

greater/equal/smaller

Question 5

(a) The equilibrium band diagram for a doped direct gap semiconductor is shown below. Is it n-type, p-type, or unknown? Circle one below.

n-type/p-type/not enough information provide



(b) Based on the band diagram opposite (E_i is in the middle of the gap), would you expect that the conduction band density-of-states effective mass is greater than, equal to, or smaller than the valence band effective mass? Circle one:

Greater than/equal/smaller than

(c) What, if any, of the following conditions by themselves could lead to the above band diagram? Circle each correct answer.

(a) very high temperature

(b) very high acceptor doping

(c) very low acceptor doping

Question 6

A hypothetical semiconductor has an intrinsic carrier concentration of $1.0 \times 10^{10}/\text{cm}^3$ at 300 K, it has conduction and valence band effective densities of states N_c and N_v, both equal to $10^{19}/\text{cm}^3$.

(a) What is the band gap E_g?

(b) If the semiconductor is doped with $N_d = 1 \times 10^{16}$ donors/cm^3, what are the equilibrium electron and hole concentrations at 300K?

(c) If the same piece of semiconductor, already having $N_d = 1 \times 10^{16}$ donors/cm^3, is also doped with $N_a = 2 \times 10^{16}$ acceptors/cm^3, what are the new equilibrium electron and hole concentrations at 300 K?

(d) Consistent with your answer to part (c), what is the Fermi level position with respect to the intrinsic Fermi level, $E_F - E_i$?

Question 7

What is the difference between density of states and *effective* density of states, and why is the latter such a useful concept?

Question 8

(a) Does mobility have any meaning at very high field? Why?

(b) How do you measure mobility and carrier concentration?