An important result of Eqs. (4–23) is that minority carriers can contribute significantly to the current through diffusion. Since the drift terms are proportional to carrier concentration, minority carriers seldom provide much drift current. On the other hand, diffusion current is proportional to the *gradient* of concentration. For example, in n-type material the minority hole concentration p may be many orders of magnitude smaller than the electron concentration n, but the gradient dp/dx may be significant. As a result, minority carrier currents through diffusion can sometimes be as large as majority carrier currents.

In discussing the motion of carriers in an electric field, we should indicate the influence of the field on the energies of electrons in the band diagrams. Assuming an electric field $\mathscr{E}(x)$ in the x-direction, we can draw the energy bands as in Fig. 4–15, to include the change in potential energy of electrons in the field. Since electrons drift in a direction opposite to the field, we expect the potential energy for electrons to increase in the direction of the field, as in Fig. 4–15. The electrostatic potential $\mathscr{V}(x)$ varies in the opposite direction, since it is defined in terms of positive charges and is therefore related to the electron potential energy $E(x)$ displayed in the figure by $\mathscr{V}(x) = E(x)/(-q)$.

From the definition of electric field,

$$\mathscr{E}(x) = -\frac{d\mathscr{V}(x)}{dx} \tag{4–25}$$

we can relate $\mathscr{E}(x)$ to the electron potential energy in the band diagram by choosing some reference in the band for the electrostatic potential. We are interested only in the spatial variation $\mathscr{V}(x)$ for Eq. (4–25). Choosing E_i as a convenient reference, we can relate the electric field to this reference by

$$\boxed{\mathscr{E}(x) = -\frac{d\mathscr{V}(x)}{dx} = -\frac{d}{dx}\left[\frac{E_i}{(-q)}\right] = \frac{1}{q}\frac{dE_i}{dx}} \tag{4–26}$$

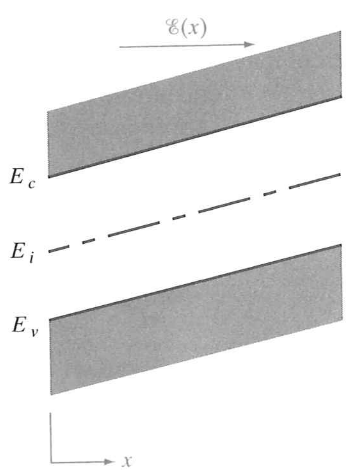

Figure 4–15
Energy band diagram of a semiconductor in an electric field $\mathscr{E}(x)$.

Therefore, the variation of band energies with $\mathscr{E}(x)$ as drawn in Fig. 4–15 is correct. The direction of the slope in the bands relative to $\mathscr{E}$ is simple to remember: Since the diagram indicates electron energies, we know the slope in the bands must be such that electrons drift "downhill" in the field. Therefore, $\mathscr{E}$ points "uphill" in the band diagram.

At equilibrium, no net current flows in a semiconductor. Thus any fluctuation which would begin a diffusion current also sets up an electric field which redistributes carriers by drift. An examination of the requirements for equilibrium indicates that the diffusion coefficient and mobility must be related. Setting Eq. (4–23b) equal to zero for equilibrium, we have

$$\mathscr{E}(x) = \frac{D_p}{\mu_p}\frac{1}{p(x)}\frac{dp(x)}{dx} \tag{4–27}$$

Using Eq. (3–25b) for $p(x)$,

$$\mathscr{E}(x) = \frac{D_p}{\mu_p}\frac{1}{kT}\left(\frac{dE_i}{dx} - \frac{dE_F}{dx}\right) \tag{4–28}$$

The equilibrium Fermi level does not vary with x, and the derivative of E_i is given by Eq. (4–26). Thus Eq. (4–28) reduces to

$$\boxed{\frac{D}{\mu} = \frac{kT}{q}} \tag{4–29}$$

This result is obtained for either carrier type. This important equation is called the *Einstein relation.* It allows us to calculate either D or μ from a measurement of the other. Table 4–1 lists typical values of D and μ for several semiconductors at room temperature. It is clear from these values that $D/\mu \simeq 0.026$ V.

An important result of the balance of drift and diffusion at equilibrium is that *built-in* fields accompany gradients in E_i [see Eq. (4–26)]. Such gradients in the bands at equilibrium (E_F constant) can arise when the band gap varies due to changes in alloy composition. More commonly, built-in fields result from doping gradients. For example, a donor distribution $N_d(x)$ causes a gradient in $n_0(x)$, which must be balanced by a built-in electric field $\mathscr{E}(x)$.

Table 4–1 Diffusion coefficient and mobility of electrons and holes for intrinsic semiconductors at 300 K. *Note:* Use Fig. 3–23 for doped semiconductors.

	D_n (cm^2/s)	D_p (cm^2/s)	μ_n (cm^2/V-s)	μ_p (cm^2/V-s)
Ge	100	50	3900	1900
Si	35	12.5	1350	480
GaAs	220	10	8500	400

4.4.3 Diffusion and Recombination; The Continuity Equation

In the discussion of diffusion of excess carriers, we have thus far neglected the important effects of recombination. These effects must be included in a description of conduction processes, however, since recombination can cause a variation in the carrier distribution. For example, consider a differential length Δx of a semiconductor sample with area A in the yz-plane (Fig. 4-16). The hole current density leaving the volume, $J_p(x + \Delta x)$, can be larger or smaller than the current density entering, $J_p(x)$, depending on the generation and recombination of carriers taking place within the volume. The net increase in hole concentration per unit time, $\partial p/\partial t$, is the difference between the hole flux per unit volume entering and leaving, minus the recombination rate. We can convert hole current density to hole particle flux density by dividing J_p by q. The current densities are already expressed per unit area; thus dividing $J_p(x)/q$ by Δx gives the number of carriers per unit volume entering $\Delta x A$ per unit time, and $(1/q)J_p(x + \Delta x)/\Delta x$ is the number leaving per unit volume and time:

$$\left.\frac{\partial p}{\partial t}\right|_{x \to x+\Delta x} = \frac{1}{q}\,\frac{J_p(x) - J_p(x + \Delta x)}{\Delta x} - \frac{\delta p}{\tau_p} \tag{4-30}$$

Rate of hole buildup = increase of hole concentration in $\delta x A$ per unit time − recombination rate

As Δx approaches zero, we can write the current change in derivative form:

$$\frac{\partial p(x,t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p} \tag{4-31a}$$

The expression (4-31a) is called the *continuity equation* for holes. For electrons we can write

$$\frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n} \tag{4-31b}$$

since the electronic charge is negative.

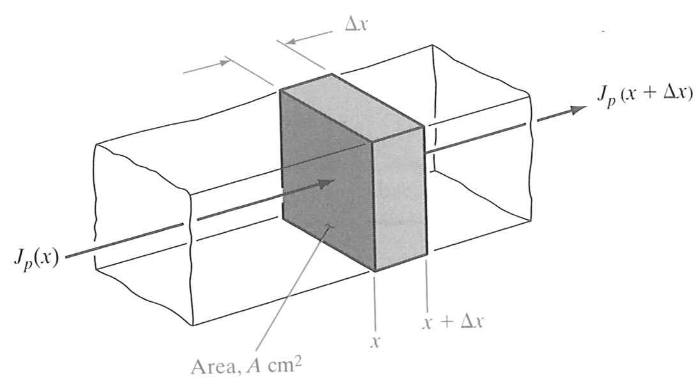

Figure 4–16 Current entering and leaving a volume $\Delta x A$.

When the current is carried strictly by diffusion (negligible drift), we can replace the currents in Eqs. (4-31) by the expressions for diffusion current; for example, for electron diffusion we have

$$J_n(\text{diff.}) = qD_n\frac{\partial \delta n}{\partial x} \tag{4-32}$$

Substituting this into Eq. (4-31b) we obtain the *diffusion equation* for electrons,

$$\boxed{\frac{\partial \delta n}{\partial t} = D_n\frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{\tau_n}} \tag{4-33a}$$

and similarly for holes,

$$\boxed{\frac{\partial \delta p}{\partial t} = D_p\frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_n}} \tag{4-33b}$$

These equations are useful in solving transient problems of diffusion with recombination. For example, a pulse of electrons in a semiconductor (Fig. 4-12) spreads out by diffusion and disappears by recombination. To solve for the electron distribution in time, $n(x, t)$, we would begin with the diffusion equation, Eq. (4-33a).

4.4.4 Steady State Carrier Injection; Diffusion Length

In many problems a steady state distribution of excess carriers is maintained, such that the time derivatives in Eqs. (4-33) are zero. In the steady state case the diffusion equations become

$$\frac{d^2\delta n}{dx^2} = \frac{\delta n}{D_n\tau_n} \equiv \frac{\delta n}{L_n^2} \tag{4-34a}$$

$$\frac{d^2\delta p}{dx^2} = \frac{\delta p}{D_p\tau_p} \equiv \frac{\delta p}{L_p^2} \tag{4-34b}$$

(steady state)

where $L_n \equiv \sqrt{D_n\tau_n}$ is called the electron *diffusion length* and L_p is the diffusion length for holes. We no longer need partial derivatives, since the time variation is zero for steady state.

The physical significance of the diffusion length can be understood best by an example. Let us assume that excess holes are somehow injected into a semi-infinite semiconductor bar at $x = 0$, and the steady state hole injection maintains a constant excess hole concentration at the injection point $\delta p(x = 0) = \Delta p$. The injected holes diffuse along the bar, recombining with

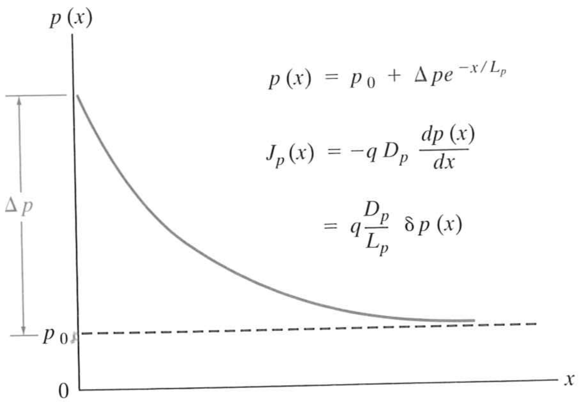

Figure 4–17 Injection of holes at $x = 0$, giving a steady state hole distribution $p(x)$ and a resulting diffusion current density $J_p(x)$.

a characteristic lifetime τ_p. In steady state we expect the distribution of excess holes to decay to zero for large values of x, because of the recombination (Fig. 4–17). For this problem we use the steady state diffusion equation for holes, Eq. (4–34b). The solution to this equation has the form

$$\delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p} \tag{4-35}$$

We can evaluate C_1 and C_2 from the boundary conditions. Since recombination must reduce $\delta p(x)$ to zero for large values of x, $\delta p = 0$ at $x = \infty$ and therefore $C_1 = 0$. Similarly, the condition $\delta p = \Delta p$ at $x = 0$ gives $C_2 = \Delta p$, and the solution is

$$\boxed{\delta p(x) = \Delta p e^{-x/L_p}} \tag{4-36}$$

The injected excess hole concentration dies out exponentially in x due to recombination, and the diffusion length L_p represents the distance at which the excess hole distribution is reduced to $1/e$ of its value at the point of injection. We can show that L_p *is the average distance a hole diffuses before recombining*. To calculate an average diffusion length, we must obtain an expression for the probability that an injected hole recombines in a particular interval dx. The probability that a hole injected at $x = 0$ *survives* to x without recombination is $\delta p(x)/\Delta p = \exp(-x/L_p)$, the ratio of the steady state concentrations at x and 0. On the other hand, the probability that a hole at x will *recombine* in the subsequent interval dx is

$$\frac{\delta p(x) - \delta p(x + dx)}{\delta p(x)} = \frac{-(d\delta p(x)/dx)dx}{\delta p(x)} = \frac{1}{L_p}dx \tag{4-37}$$

Thus the total probability that a hole injected at $x = 0$ will recombine in a given dx is the product of the two probabilities:

$$(e^{-x/L_p})\left(\frac{1}{L_p}dx\right) = \frac{1}{L_p}e^{-x/L_p}dx \tag{4-38}$$

Then, using the usual averaging techniques described by Eq. (2–21), the average distance a hole diffuses before recombining is

$$\langle x \rangle = \int_0^\infty x \frac{e^{-x/L_p}}{L_p} dx = L_p \tag{4-39}$$

The steady state distribution of excess holes causes diffusion, and therefore a hole current, in the direction of decreasing concentration. From Eqs. (4–22b) and (4–36) we have

$$J_p(x) = -qD_p\frac{dp}{dx} = -qD_p\frac{d\delta p}{dx} = q\frac{D_p}{L_p}\Delta p e^{-x/L_p} = q\frac{D_p}{L_p}\delta p(x) \tag{4-40}$$

Since $p(x) = p_0 + \delta p(x)$, the space derivative involves only the excess concentration. We notice that since $\delta p(x)$ is proportional to its derivative for an exponential distribution, the diffusion current at any x is just proportional to the excess concentration δp at that position.

Although this example seems rather restricted, its usefulness will become apparent in Chapter 5 in the discussion of p-n junctions. The injection of minority carriers across a junction often leads to exponential distributions as in Eq. (4–36), with the resulting diffusion current of Eq. (4–40).

EXAMPLE 4–5 In a very long p-type Si bar with cross-sectional area = 0.5 cm^2 and $N_a = 10^{17}$ cm^{-3}, we inject holes such that the steady state *excess* hole concentration is 5×10^{16} cm^{-3} at $x = 0$. What is the steady state separation between F_p and E_c at $x = 1000$ Å? What is the hole current there? How much is the excess stored hole charge? Assume that $\mu_p = 500$ cm^2/V-s and $\tau_p = 10^{-10}$ s.

SOLUTION

$$D_p = \frac{kT}{q}\mu_p = 0.0259 \times 500 = 12.95 \text{ cm/s}$$

$$L_p = \sqrt{D_p\tau_p} = \sqrt{12.95 \times 10^{-10}} = 3.6 \times 10^{-5} \text{ cm}$$

$$p = p_0 + \Delta p e^{-\frac{x}{L_p}} = 10^{17} + 5 \times 10^{16} e^{-\frac{10^{-5}}{3.6\times10^{-5}}}$$

$$= 1.379 \times 10^{17} = n_i e^{(E_i - F_p)/kT} = (1.5 \times 10^{10} \text{ cm}^{-3})e^{(E_i - F_p)/kT}$$

$$E_i - F_p = \left(\ln\frac{1.379 \times 10^{17}}{1.5 \times 10^{10}}\right)\cdot 0.0259 = 0.415 \text{ eV}$$

$$E_c - F_p = 1.1/2 \text{ eV} + 0.415 \text{ eV} = \mathbf{0.965\ eV}$$

We can calculate the hole current from Eq. (4–40)

$$I_p = -qAD_p\frac{dp}{dx} = qA\frac{D_p}{L_p}(\Delta p)e^{-\frac{x}{L_p}}$$

$$= 1.6 \times 10^{-19} \times 0.5 \times \frac{12.95}{3.6 \times 10^{-5}} \times 5 \times 10^{16}\, e^{-\frac{10^{-5}}{3.6\times10^{-5}}}$$

$$= \mathbf{1.09 \times 10^3\ A}$$

$$Q_p = qA(\Delta p)L_p$$

$$= 1.6 \times 10^{-19}(0.5)(5 \times 10^{16})(3.6 \times 10^{-5})$$

$$= \mathbf{1.44 \times 10^{-7}\ C}$$

4.4.5 The Haynes–Shockley Experiment

One of the classic semiconductor experiments is the demonstration of drift and diffusion of minority carriers, first performed by J. R. Haynes and W. Shockley in 1951 at the Bell Telephone Laboratories. The experiment allows independent measurement of the minority carrier mobility μ and diffusion coefficient D. The basic principles of the Haynes–Shockley experiment are as follows: A pulse of holes is created in an n-type bar (for example) that contains an electric field (Fig. 4–18); as the pulse drifts in the field and spreads out by diffusion, the excess hole concentration is monitored at some point down the bar; the time required for the holes to drift a given distance in the field gives a measure of the mobility; and the spreading of the pulse during a given time is used to calculate the diffusion coefficient.

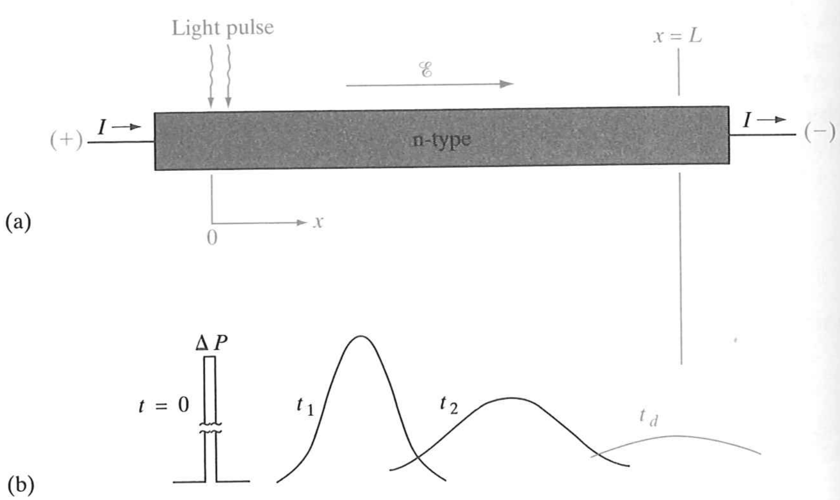

Figure 4–18 Drift and diffusion of a hole pulse in an n-type bar: (a) sample geometry; (b) position and shape of the pulse for several times during its drift down the bar.

In Fig. 4–18 a pulse of excess carriers is created by a light flash at some point $x = 0$ in an n-type semiconductor ($n_0 \gg p_0$). We assume that the excess carriers have a negligible effect on the electron concentration but change the hole concentration significantly. The excess holes drift in the direction of the electric field and eventually reach the point $x = L$, where they are monitored. By measuring the drift time t_d, we can calculate the drift velocity v_d and, therefore, the hole mobility:

$$v_d = \frac{L}{t_d} \tag{4-41}$$

$$\mu_p = \frac{v_d}{\mathscr{E}} \tag{4-42}$$

Thus the hole mobility can be calculated directly from a measurement of the drift time for the pulse as it moves down the bar. In contrast with the Hall effect (Section 3.4.5), which can be used with resistivity to obtain the *majority* carrier mobility, the Haynes–Shockley experiment is used to measure the *minority* carrier mobility.

As the pulse drifts in the $\mathscr{E}$ field it also spreads out by diffusion. By measuring the spread in the pulse, we can calculate D_p. To predict the distribution of holes in the pulse as a function of time, let us first reexamine the case of diffusion of a pulse *without drift, neglecting recombination* (Fig. 4–12). The equation which the hole distribution must satisfy is the time-dependent diffusion equation, Eq. (4–33b). For the case of negligible recombination (τ_p long compared with the times involved in the diffusion), we can write the diffusion equation as

$$\frac{\partial \delta p(x,t)}{\partial t} = D_p \frac{\partial^2 \delta p(x,t)}{\partial x^2} \tag{4-43}$$

The function which satisfies this equation is called a *gaussian distribution,*

$$\delta p(x,t) = \left[\frac{\Delta P}{2\sqrt{\pi D_p t}}\right]e^{-x^2/4D_p t} \tag{4-44}$$

where ΔP is the number of holes per unit area created over a negligibly small distance at $t = 0$. The factor in brackets indicates that the peak value of the pulse (at $x = 0$) decreases with time, and the exponential factor predicts the spread of the pulse in the positive and negative x-directions (Fig. 4–19). If we designate the peak value of the pulse as $\delta\hat{p}$ at any time (say t_d), we can use Eq. (4–44) to calculate D_p from the value of δp at some point x. The most convenient choice is the point $\Delta x/2$, at which δp is down by $1/e$ of its peak value $\delta\hat{p}$. At this point we can write

$$e^{-1}\delta\hat{p} = \delta\hat{p}e^{-(\Delta x/2)^2/4D_p t_d} \tag{4-45}$$

$$D_p = \frac{(\Delta x)^2}{16 t_d} \tag{4-46}$$

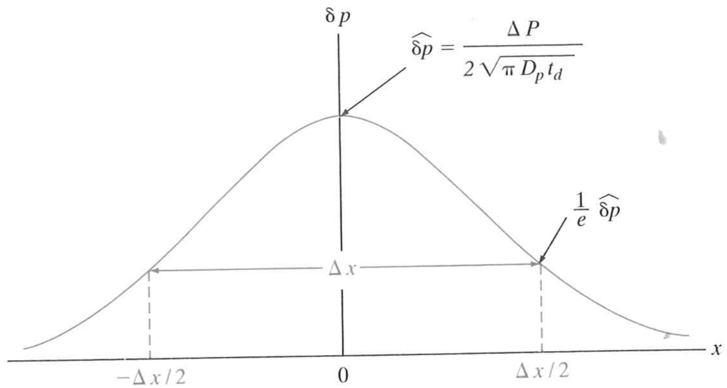

Figure 4–19 Calculation of D_p from the shape of the δp distribution after time t_d. No drift or recombination is included.

Since Δx cannot be measured directly, we use an experimental setup such as Fig. 4–20, which allows us to display the pulse on an oscilloscope as the carriers pass under a detector. As we shall see in Chapter 5, a forward-biased p-n junction serves as an excellent injector of minority carriers, and a reverse-biased junction serves as a detector. The measured quantity in Fig. 4–20 is the pulse width Δt displayed on the oscilloscope in time. It is related to Δx by the drift velocity, as the pulse drifts past the detector point (2)

$$\Delta x = \Delta t \mathrm{v}_d = \Delta t \frac{L}{t_d} \tag{4-47}$$

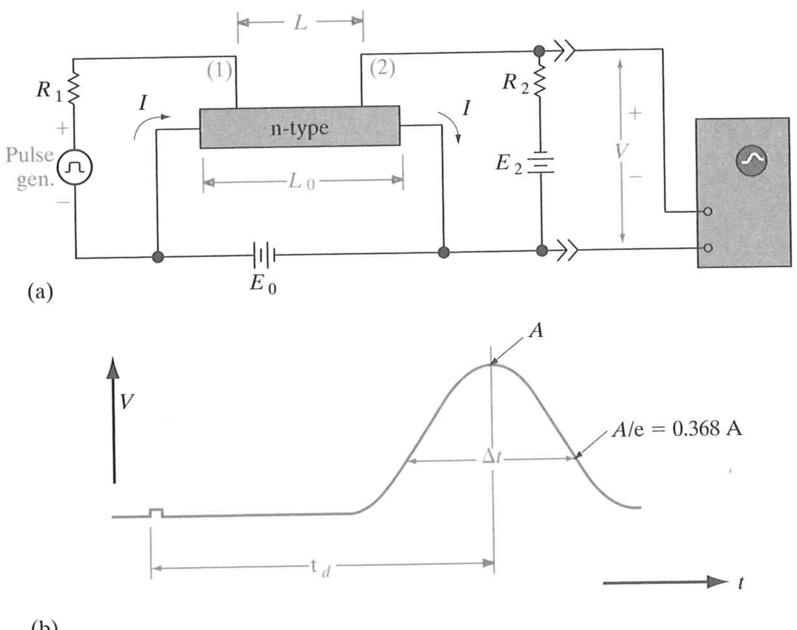

Figure 4–20 The Haynes–Shockley experiment: (a) circuit schematic; (b) typical trace on the oscilloscope screen.

EXAMPLE 4–6

An n-type Ge sample is used in the Haynes–Shockley experiment shown in Fig. 4–20. The length of the sample is 1 cm, and the probes (1) and (2) are separated by 0.95 cm. The battery voltage E_0 is 2 V. A pulse arrives at point (2) 0.25 ms after injection at (1); the width of the pulse Δt is 117 μs. Calculate the hole mobility and diffusion coefficient, and check the results against the Einstein relation.

SOLUTION

$$\mu_p = \frac{\mathrm{v}_d}{\mathscr{E}} = \frac{0.95/(0.25 \times 10^{-3})}{2/1} = 1900 \text{ cm}^2/(\text{V-s})$$

$$D_p = \frac{(\Delta x)^2}{16 t_d} = \frac{(\Delta t L)^2}{16 t_d^3}$$

$$= \frac{(117 \times 0.95)^2 \times 10^{-12}}{16(0.25)^3 \times 10^{-9}} = 49.4 \text{ cm}^2/\text{s}$$

$$\frac{D_p}{\mu_p} = \frac{49.4}{1900} = 0.026 = \frac{kT}{q}$$

4.4.6 Gradients in the Quasi-Fermi Levels

In Section 3.5 we saw that equilibrium implies no gradient in the Fermi level E_F. In contrast, any combination of drift and diffusion implies a gradient in the steady state quasi-Fermi level.

We can use the results of Eqs. (4–23), (4–26), and (4–29) to demonstrate the power of the concept of quasi-Fermi levels in semiconductors [see Eq. (4–15)]. If we take the general case of nonequilibrium electron concentration with drift and diffusion, we must write the total electron current as

$$J_n(x) = q\mu_n n(x)\mathscr{E}(x) + qD_n \frac{dn(x)}{dx} \tag{4-48}$$

where the gradient in electron concentration is

$$\frac{dn(x)}{dx} = \frac{d}{dx}\left[n_i e^{(F_n - E_i)/kT}\right] = \frac{n(x)}{kT}\left(\frac{dF_n}{dx} - \frac{dE_i}{dx}\right) \tag{4-49}$$

Using the Einstein relation, the total electron current becomes

$$J_n(x) = q\mu_n n(x)\mathscr{E}(x) + \mu_n n(x)\left[\frac{dF_n}{dx} - \frac{dE_i}{dx}\right] \tag{4-50}$$

But Eq. (4–26) indicates that the subtractive term in the brackets is just $q\mathscr{E}(x)$, giving a direct cancellation of $q\mu_n n(x)\mathscr{E}(x)$ and leaving

$$J_n(x) = \mu_n n(x) \frac{dF_n}{dx} \tag{4-51}$$

Thus, the processes of electron drift and diffusion are summed up by the spatial variation of the quasi-Fermi level. The same derivation can be made for holes, and we can write the current due to drift and diffusion in the form of a *modified Ohm's law*

$$J_n(x) = q\mu_n n(x)\frac{d(F_n/q)}{dx} = \sigma_n(x)\frac{d(F_n/q)}{dx} \tag{4-52a}$$

$$J_p(x) = q\mu_p p(x)\frac{d(F_p/q)}{dx} = \sigma_p(x)\frac{d(F_p/q)}{dx} \tag{4-52b}$$

Therefore, any drift, diffusion, or combination of the two in a semiconductor results in currents proportional to the gradients of the two quasi-Fermi levels. Conversely, a lack of current implies constant quasi-Fermi levels. One can use a hydrostatic analogy for quasi-Fermi levels and identify it as water pressure in a system. Just as water flows from a high-pressure region to a low-pressure region, until in equilibrium the water pressure is the same everywhere, similarly electrons flow from a high- to low-electron quasi-Fermi level region, until we get a flat Fermi level in equilibrium. Quasi-Fermi levels are sometimes also known as electrochemical potentials because, as we just saw, the driving force for carriers is governed partly by gradients of electrical potential (or electric field), which determines drift, and partly by gradients of carrier concentration (which is related to a thermodynamic concept called *chemical potential*), giving rise to diffusion.

SUMMARY

4.1 *Excess* carriers, above the equilibrium values contributed by doping, may be created *optically* (or by electrical *biasing* in devices). *Generation–recombination* (G–R) of electron–hole pairs (*EHPs*) can occur by absorption of the photons with energy greater than the band gap, balanced by direct or indirect recombination.

4.2 G–R processes can be mediated by *traps*, especially deep traps near midgap. Band-to-band or trap-assisted G–R processes lead to an average *lifetime* for the excess carriers. Carrier lifetime multiplied by the optical generation rate establishes a steady state excess population of carriers. The square root of carrier lifetime multiplied by the diffusion coefficient determines the diffusion length.

4.3 In *equilibrium*, we have a *constant Fermi level*. In nonequilibrium with excess carriers, Fermi levels are generalized to separate *quasi-Fermi levels* for electrons and holes. The quasi-Fermi level splitting is a measure of the departure from equilibrium. Minority carrier quasi-Fermi levels change more than majority carrier quasi-Fermi levels because the relative change of minority carriers is larger. Gradients in the quasi-Fermi level determine the net drift–diffusion current.

4.4 *Diffusion flux* measures the flow of carriers from *high- to low-concentration* regions and is given by the *diffusivity* times the concentration *gradient*. The direction of diffusion current is opposite to the flux for the negative electrons, but in the same direction for the positive holes. Carrier diffusivity is related to mobility by the thermal voltage kT/q (*Einstein relation*).

4.5 When carriers move in a semiconductor due to drift or diffusion, the time-dependent carrier concentrations at different points is given by the *carrier continuity* equation, which says that if more carriers flow into a point than flow out, the concentration will increase as a function of time and vice versa. G–R processes also affect carrier concentrations.

PROBLEMS

4.1 With E_F located 0.4 eV above the valence band in a Si sample, what charge state would you expect for most Ga atoms in the sample? What would be the predominant charge state of Zn? Au? *Note:* By charge state we mean neutral, singly positive, doubly negative, etc.

4.2 A Si sample with 10^{16}/cm^3 donors is optically excited such that 10^{19}/cm^3 electron–hole pairs are generated per second uniformly in the sample. The laser causes the sample to heat up to 450 K. Find the quasi-Fermi levels and the change in conductivity of the sample upon shining the light. Electron and hole lifetimes are both 10 μs. D_p = 12 cm^2/s; D_n = 36 cm^2/s; $n_i = 10^{14}$ cm^{-3} at 450 K. What is the change in conductivity upon shining light?

4.3 Construct a semilogarithmic plot such as Fig. 4–7 for Si doped with 2×10^{15} donors/cm^3 and having 4×10^{14} EHP/cm^3 created uniformly at $t = 0$. Assume that $\tau_n = \tau_p = 5$ μs.

4.4 Calculate the recombination coefficient α_r for the low-level excitation described in Prob. 4.3. Assume that this value of α_r applies when the GaAs sample is uniformly exposed to a steady state optical generation rate $g_{op} = 10^{19}$ EHP/cm^3-s. Find the steady state excess carrier concentration $\Delta n = \Delta p$.

4.5 An intrinsic Si sample is doped with donors from one side such that $N_d = N_0 \exp(-ax)$. (a) Find an expression for the built-in electric field at equilibrium over the range for which $N_d \gg n_i$. (b) Evaluate the field when $a = 1$ (μm)$^{-1}$. (c) Sketch a band diagram such as in Fig. 4–15 and indicate the direction of the field.

4.6 A Si sample with 10^{15}/cm^3 donors is uniformly optically excited at room temperature such that 10^{19}/cm^3 electron–hole pairs are generated per second. Find the separation of the quasi-Fermi levels and the change of conductivity upon shining the light. Electron and hole lifetimes are both 10 μs. D_p = 12 cm^2/s.

4.7 An n-type Si sample with $N_d = 10^{15}$ cm^{-3} is steadily illuminated such that $g_{op} = 10^{21}$ EHP/cm^3-s. If $\tau_n = \tau_p = 1$ μs for this excitation, calculate the separation in the quasi-Fermi levels, $(F_n - F_p)$. Draw a band diagram such as Fig. 4–11.

4.8 For a 2-cm-long doped Si bar ($N_d = 10^{16}$ cm^{-3}) with a cross-sectional area = 0.05 cm^2, what is the current if we apply 10V across it? If we generate 10^{20} electron-hole pairs per second per cm^3 uniformly in the bar and the lifetime $\tau_n = \tau_p = 10^{-4}$ s, what is the new current? Assume the low-level α_r doesn't change for high-level injection. If the voltage is then increased to 100,000 V, what is the new current? Assume $\mu_p = 500$ cm^2/V-s, but you must choose the appropriate value for electrons.

4.9 Design and sketch a photoconductor using a 5-μm-thick film of CdS, assuming that $\tau_n = \tau_p = 10^{-6}$ s and $N_d = 10^{14}$ cm^{-3}. The dark resistance (with $g_{op} = 0$) should be 10 MΩ, and the device must fit in a square 0.5 cm on a side; therefore, some sort of folded or zigzag pattern is in order. With an excitation of $g_{op} = 10^{21}$ EHP/cm^3-s, what is the resistance change?

4.10 A 100-mW laser beam with wavelength λ = 6328 Å is focused onto a GaAs sample 100 μm thick. The absorption coefficient at this wavelength is 3×10^4 cm^{-1}. Find the number of photons emitted per second by radiative recombination in the GaAs, assuming perfect quantum efficiency. What power is delivered to the sample as heat?

4.11 Assume that a photoconductor in the shape of a bar of length L and area A has a constant voltage V applied, and it is illuminated such that g_{op} EHP/cm^3-s are generated uniformly throughout. If $\mu_n \gg \mu_p$, we can assume the optically induced change in current ΔI is dominated by the mobility μ_n and lifetime τ_n for electrons. Show that $\Delta I = qALg_{op}\tau_n/\tau_t$ for this photoconductor, where τ_t is the transit time of electrons drifting down the length of the bar.

4.12 For the steady state minority hole distribution shown in Fig. 4–17, find the expression for the hole quasi-Fermi level position $E_i - F_p(x)$ while $p(x) \gg p_0$ (i.e., while F_p is below E_F). On a band diagram, draw the variation of $F_p(x)$. Be careful—when the minority carriers are few (e.g., when δp is n_i), F_p still has a long way to go to reach E_F.

4.13 In an n-type semiconductor bar, there is an increase in electron concentration from left to right and an electric field pointing to the left. With a suitable sketch, indicate the directions of the electron drift and diffusion current flow and explain why. If we double the electron concentration everywhere, what happens to the diffusion current and the drift current? If we add a constant concentration of electrons everywhere, what happens to the drift and diffusion currents? Explain your answers with appropriate equations.

4.14 The current required to feed the hole injection at $x = 0$ in Fig. 4–17 is obtained by evaluating Eq. (4–40) at $x = 0$. The result is $I_p(x = 0) = qAD_p\Delta p/L_p$. Show that this current can be calculated by integrating the charge stored in the steady state hole distribution $\delta p(x)$ and then dividing by the average hole lifetime τ_p. Explain why this approach gives $I_p(x = 0)$.

4.15 The direction of the built-in electric field can be deduced without math by sketching the result of a doping gradient on the band diagram. Starting with a flat Fermi level at equilibrium, place E_i near or far from E_F as the doping is varied for the two cases of a gradient in donor or acceptor doping as in Prob. 4.5. Show the electric field direction in each case, based on Eq. (4–26). If a *minority* carrier is injected into the impurity gradient region, in what direction is it accelerated in the two cases? This is an interesting effect that we will use later in discussing bipolar transistors.

4.16 In Prob. 4.5, the direction of the built-in electric field due to a gradient in doping was determined from Eqs. (4–23) and (4–26). In this problem, you are asked to explain qualitatively why the field must arise and find its direction. (a) Sketch a donor doping distribution as in Prob. 4.5, and explain the field required to keep the mobile electrons from diffusing down the gradient. Repeat for acceptors and holes. (b) Sketch a microscopic region of the doping distribution, showing ionized donors and the resulting mobile electrons. Explain the origin and direction of the field as the electrons attempt to diffuse toward lower concentrations. Repeat for acceptors and holes.

4.17 We wish to use the Haynes–Shockley experiment to calculate the hole lifetime τ_p in an n-type sample. Assume that the peak voltage of the pulse displayed on the oscilloscope screen is proportional to the hole concentration under the collector terminal at time t_d and that the displayed pulse can be approximated as a gaussian, as in Eq. (4–44), which decays due to recombination by e^{-t/τ_p}. The electric field is varied and the following data are taken: For t_d = 200 μs, the peak is 20 mV; for t_d = 50 μs, the peak is 80 mV. What is τ_p?

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SELF QUIZ

Question 1

Consider a p-type semiconductor that has a band gap of 1.0 eV and a minority electron lifetime of 0.1 μs, and is uniformly illuminated by light having photon energy of 2.0 eV.

(a) What rate of uniform excess carrier generation is required to generate a uniform electron concentration of 10^{10}/cm^3?

(b) How much optical power per cm^3 must be absorbed in order to create the excess carrier population of part (a)? (You may leave your answer in units of eV/s-cm^3.)

(c) If the carriers recombine via photon emission, approximately how much optical power per cm^3 will be generated? (You may leave your answer in units of eV/s-cm^3.)

Question 2

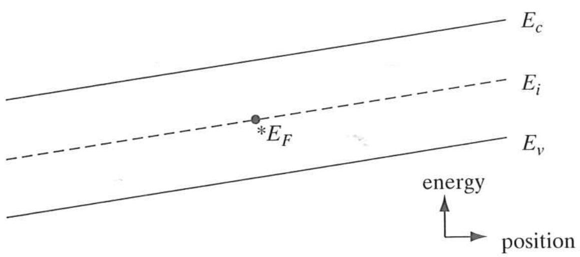
(a) What do we mean by "deep" versus "shallow" traps? Which are more harmful for semiconductor devices and why? What is an example of a deep trap in Si?

(b) Are absorption lengths of slightly above band gap photons longer in Si or GaAs? Why?

(c) Do absorption coefficients of photons increase or decrease with photon energy? Why?

Question 3

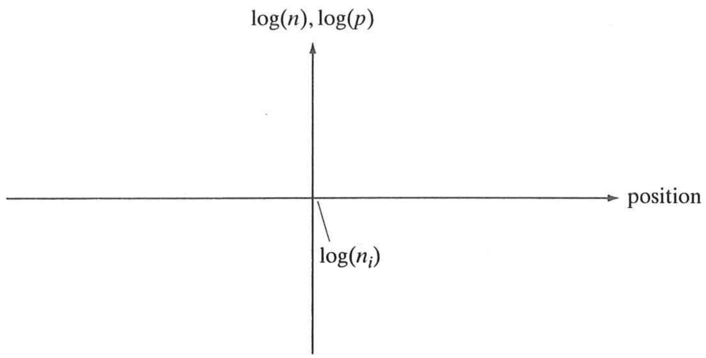
Consider the following equilibrium band diagram for a portion of a semiconductor sample with a built-in electric field $\mathscr{E}$:



(a) Sketch the Fermi level as a function of position *through* the indicated point, E_F, across the width of the band diagram above.

(b) On the band diagram, sketch the direction of the electric field. Is the field constant or position dependent?

(c) On the following graph, sketch and label both the electron and hole concentrations as a function of position across the full width of the sample. Note that the carrier concentration scale is logarithmic such that exponential variations in the carrier concentration with position appear as straight lines. Note also that the horizontal axis corresponds to the intrinsic carrier concentration of n_i.



Question 4

(a) Indicate the directions of the hole and electron flux densities ϕ due to diffusion and drift under these *equilibrium* conditions corresponding to the previous Question 3.

(b) Indicate the directions of the hole and electron current densities j due to diffusion and drift under these *equilibrium* conditions.

Question 5

(a) What are the relevant equations that must be solved in general for a semiconductor device problem?

(b) In general how many components of conduction current can you have in a semiconductor device? What are they?

Question 6

(a) Consider a region in a semiconductor with an electric field directed toward the right ($\rightarrow$) and carrier concentrations increasing toward the left ($\leftarrow$). Indicate the directions of particle fluxes ϕ (circle one for each) and charge currents j due to drift and diffusion within that region.

(b) Based on your answers to part (a), indicate the directions of the charge currents j due to drift and diffusion within that region.