Mid Sweden University NAT Björn Ivarsson, Abtin Daghighi

Testtenta in mathematics.

Fördjupningskurs i analys / Calculus II (MA059G/MA060G). May 2010 Time for tenta: 5 hours Allowed calculators and collections of formulas: Nonsymbolic calculators and approved "gymnasieformelsamling".

You must provide complete solutions. Motivations and calculations should be comprehensible and easy to follow. One problem per sheet and write only on one side of the paper.

The following guidelines will be used to grade: A 22 p, B 18 p, C 14p, D 10p, E 9p. The grade will be set by how satisfactory you meet the "lärandemål".

	(1)	(a) Give the formal definition of the statement: "The funct:	ion
		f(x) is continuous at the point $x = a$ ". (1)	1p)
ok!		$f(x) \text{ is continuous at the point } x = a".$ (b) Is the function defined as $f(x) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$	nu-
		ous at the point $x = 0$ ? (2)	2p)

Ok! (2) Is the integral

$$\int_{1}^{\infty} x e^{-x} dx$$

convergent or divergent? If it is convergent calculate the integrals value. (3p)

(3) An <u>orchard</u> has 60 trees and produce an average of 800 apples per tree per year. If more trees are planted the yield per tree will drop. For each tree planted the average yield per tree is reduced by 10 apples per year. How many more trees should be planted in order to maximize the yield from the orchard and what is the maximal yield? (3p)
(4) Find all solutions to

$$xy'' + y' + x = 0$$
, when  $x > 0$ .  
(Lestric: Lot  $u = y'$  on los for a first.)

orchard = fruktedling

(3p)

ok!

ok!

ok!

(5) Find the Taylor polynomial of degree 4 for  $\sin^2 x$  about x = 0and use it to calculate

$$\lim_{x \to 0} \frac{3\sin^2 x - 3x^2}{4x^4}$$

(1p)

(Ledhadi sin x = X Vx>0)

(6) (a) Does

(b) Does

$$\sum_{n=0}^{\infty} \frac{n}{n^2 + 1}$$
 converge or diverge?

ok!

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$$

converge or diverge? (2p)
(7) Let y(x) be the solution to the initial value problem y' = x - y<sup>2</sup> such that y(0) = 0. Use the Euler method with step size h = 0.2 to approximate y(1). (3p)
(8) (a) (MA059G) Use the definition of derivative to show that f(x) = sin x is differentiable at x = 0. (3p)
(b) (MA060G) Do 3 iterations, that is calculate x<sub>3</sub>, of the Newton-Raphson method to approximate the root of e<sup>x</sup> = 2 - x using x<sub>0</sub> = 0. (3p)

Good luck!

ok!

ok!

ok!

ч.

ok!

ok!

 $\mathbf{2}$ 

huntion 1.e utet m (s Car Solution 手很)= <u></u>手很和 defined as to the hunce 力 × éff O éff Ron 6 muons -761 the point af Solichion  $h = f(x) = +1 \neq 0$ 2) Is the integral of the CA Edenta e X l x - line te - te-- 100-10 Negral is convergent and fre de-2 NJO



4) Find all solutions of Xy +y + x = 0, when x >0  $\frac{531456}{\times 91} = \frac{1}{\times 91} = \frac{1}{\times 91}$ - u = y -X UE + UE = X Int, factor est dx = e hix (+C) = x  $\frac{d}{dx}(xu) = \chi^{e}u' + u = \chi$ Kn = X - tC y=n=x+ C  $y = \int \frac{x}{2} + \frac{c}{x} dx = \frac{x^2}{4} + C dnx + D$ 5) Find the Taytor polynomial of degree that sint about x=0 and use it to calculate din 3512× - 3×2-× -20 - 4/×9 4(x)=Sm2x  $f'(x) = 2 \sin x \cos x$  $f''(x) = 2(\cos^2 x - \sin^2 x) = 2\cos^2 x - 2\sin^2 x$ 1 (x) = 4 COSXSINX - 4 SINXCOJX = - 8 SINX COSX  $f^{(4)}(x) = -8(\cos^2 x - \sin^2 x)$ 

					And a second			And the second second second					-				and the second second second second			sector and an end of the sector							4 (My).44		CALLER CONTRACTOR										the second state independence						10 ( ) () () () () () () () () () () () ()	ang dag da ago to service and services		server was appropriate the server	1. AF 1. ANTONIO VE'NAMANANA ANTON				In the second second	
		J	p {							2					and a second second second				]	ß	1		à	)				, juni 			····		میں چاہد چاہد	Ł	1		0	)		2	? .													
				144		3	)	(	6						(	ð							and the second se	····	1000 - V. (000 -	The second s	4 J	ξ. ( 		)	/ {		)	یر. کند ا				8	a provincial and a second second									-						
			an			<		7)	2		X	,	2								 					SA-C	ζ 7/		X		1			(	8	7	X	25																
	<					- 2000											X	1999	2				Construction of the second	3		X	9	~	4	T	(	<b>D</b>	0	r. X			)		set a second							or or mandemannantic processor (come	:	+						
4	6	1 z			en de la composition en la composition de la en composition de la composition de la encomposition de la composition de			5	))		z>	2			(5								2	×		a dama adjawani jaman				X			*	X		3			Z	5/	ζ Χ	18	,				>				And a second		10° - 1 - 1 - 1 10000 - 11	 
					 						<	2			••• 		2			ţ	þ.			6	)/(		()			n (1754) 19 (1754)	ann Cana		X	5		e	, , ,				7		р. С.)	4		57.I								
	,						1	V	· · · · · · · · · · · · · · · · · · ·		······			5	5.)	~		3	X		geriores.		3.	X	2					 1				*	3	X	2			ť	4					X	5			^ 	32	2		
						X			6	and a second second second				94 <sup>4</sup>		~		1	7	ζ	4			2000 100 100 100 100 100 100 100 100 100			*****				>			A CONTRACTOR OF A CONTRACTOR O						1	~	× .	C						Pro					
											ı)		7			×				. ,	Y													¢					South and the second						8				>					and the second
				A	1	and the second se	X	D.	C	2						, de la composición d	,		×				1					6		>>>				Y				Y	Acres Processing in Processing	52		Î			3		-	*					10-12 (A)	- warmen and a second
							ł	ر هم				j J	4		None Asses			10	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	5	0	anna ann ann ann ann ann ann ann ann an		2	Z	Έλγ 7	er.		X	¥.4					1	f-	4.		10. J 1	< h :	P T T		12				<u> </u>	31	AY 			1		and and an an and a second designed as the second
						Į	1	r) h			A.		۱ 	 	No. 440		•••				2	Ŧ	`	4			×	A I	l	~			>	arrente frances and an arrente frances			· · · · · · · · · · · · · · · · · · ·	10 million and 10 mil				and analysis of the second sec												in a minimum provident of the
	L.						U	A.,		*****					<b>b</b>	2	···· ·			· · · · · · · · · · · · · · · · · · ·		a) (Containing Containing Contain		1					<u>(</u>	)										an and be			e :											······
	~	-3		\$		A second s	0	~							5		A			~?							J	and the second measured and the second se						and a second processing of the second s	annana (annana) a						**************************************	and and an an an and an an and an and and			v - 1		A DESCRIPTION OF A DESC		3					
							,			¢ 	and the second s	and the second se		N		2			- 1		ł	7				0 	k		~~~		5	Y-			The second se						1 2.00 A							1. The second se		and the second sec			1000 - 1000 - 1000 1000 - 1000 - 1000	
										· · ·	1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 -		 			· · ·														i una francesca l'accordentes pressa		••••	· · · · · · · · · · · · · · · · · · ·		4	,							And a second			and the second s								
							• .••					i minimus						2								ada - 17													• •••••	· · · ·														

b) Does Sm(2) converge of etalerge 0 = 5 m (2) < 12 and Z in 2 comages Z Sin (2) Course 1 Let y(x) be the solution to the Initial value problem y'= x - y 2 su that y (0)= 0. Use the Eular nithod with Step Size 1 = 0,2 to app 182 make y (I) SALL 50 1367 41 = 40 + f(x0, 40) h = 0 XFQZ  $n_{12} = 0 + 0.2 (0.2 - 0^2) = 0.54$ X2-OU  $y_3 = O_{104} \neq O_{1} (O_{14} = O_{10}, o_{14}) = O_{10} (0.8)$ X2=0,6 44 = 0,11968 + 0,2 (0,6 = 0,11968 2 Xy SOIR = 0,2:3681 45 = 0 23681 + 0,2 (0,8 - C) 23681-0,3855990-8 - (MAOS99)Use the definition of devitative to show that fait snix is differentiable at V=b 

Solution  $f(o) = Q_{4} + \frac{f(o)}{2} + \frac{f(o$ b) Do 3 ilevations of fue Deston -Raphson Milled to approximate the root of exe 2-x using 5=0 Solutions f(x)= ex+x=2(-0) - f'al=ex+1  $X_{min} = X_n - \frac{f(x_n)}{f(x_n)} = x_n - \frac{e^{x_n} + x_n - 2}{e^{x_n} + 1}$  $x_{0} = 0$   $x_{1} = 0$   $-\frac{e^{0} + 0}{e^{0} + 1} = \frac{1}{2}$  $X_2 = \frac{2}{2} = \frac{2}{2^2 + 2} = 0,443851 = \frac{2}{2^2 + 1}$  $X_3 = X_2 - e^{X_2} + X_2 - c = 6, 442854 = e^{X_2}$