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HOMEWORK PROBLEM – SIMULATION OF COMMUNICATION SYSTEMS:

# *Old exam problems from basic courses in computer networks and telecommunications*

In the end of this document you will find some common formulas within the area of telecommunications.

Reply by handwritten answers. Present all calculations, and use correct units. You should have at least 50% correct answers for approval. Be prepared to present your answers on the whiteboard. Requirement for approval: 50% correct answers. You may to some extent cooperate and discuss with your class mates, but everyone must write down and understand their own answers.

**1.** A modulated signal has the power S = 10 mW (milliwatt). It is interfered by the noise power N = 1 μW (microwatt). The channel bandwidth is 10MHz. (8 credit points.)

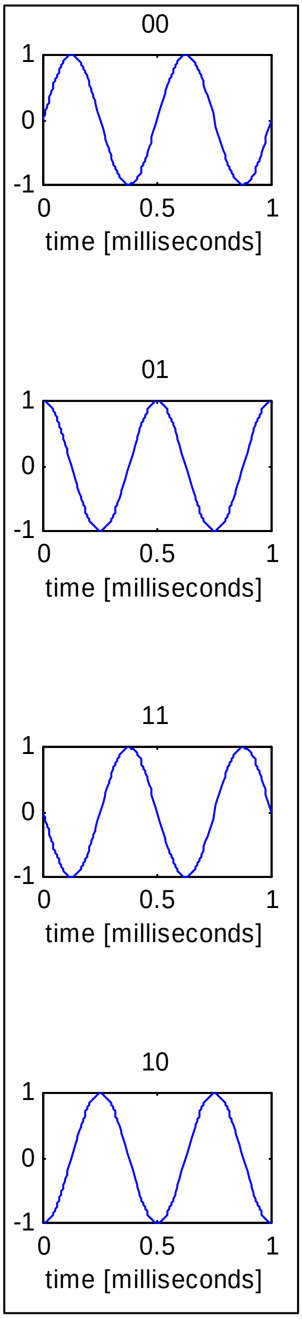
a) What is the signal-to-noise ratio S/N in times?

b) What is the signal-to-noise ratio SNR in decibel? (See the formulas in the end of the document.)

c) How many bit per second net bit rate (i.e. of useful information) can in theory be transferred by means of this signal without errors, if you add an ideal forward error correction code (FEC), and use ideal modulation. (Hint: Use the Shannon-Hartley theorem.)

d) If we want to transfer twice as many bits/second, what SNR should we use in decibel?

**2.** To the right you see four symbols that are used by a so called 4PSK-modem (PSK=Phase Shift Keying, with 4 symbols). The four symbols are representing the bit sequences 00, 01, 11 and 10 respectively. (6 credit points)



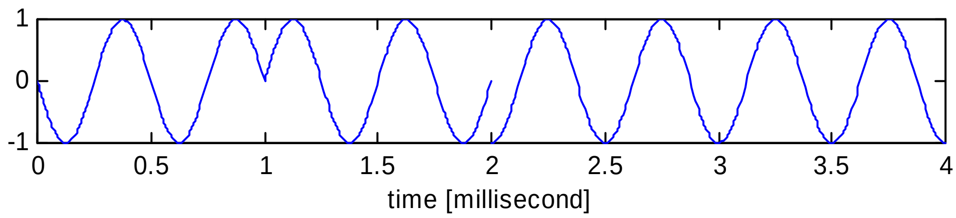

a) Below you find the output signal from the transmitting modem. What message, i.e. what bit sequence, is transferred?

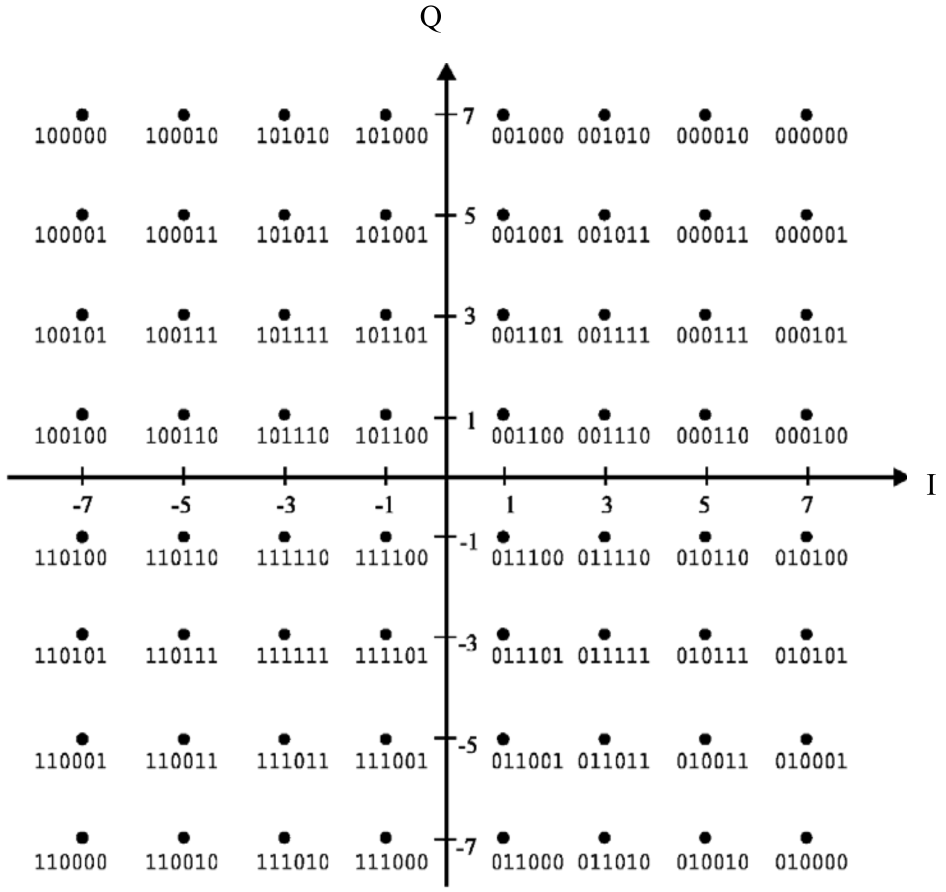
b) What is the symbol rate in baud or symbols/second?

c) What is the bit rate in bit per second (bps)?

d) What is the carrier frequency $f_c$ in Hertz? (1 divided by the sine wave period time.)

e) The bandwidth $B$ in Hertz is approximately equal to the symbol rate. The bandwidth $B = f_{max} - f_{min}$, where $f_{max}$ and $f_{min}$ are the maximum and minimum frequencies of the signal spectrum, or the cutoff frequencies of the band pass filter in the receiver. The carrier frequency $f_c$ is the center frequency of the spectrum, i.e. the average of $f_{max}$ and $f_{min}$. Thus $f_{max} = f_c$ + B/2, and $f_{min} = f_c$ – B/2. **Question:** What are the cut-off frequencies $f_{max}$ and $f_{min}$?






**3.** A 64QAM modem is using symbols according to the above constellation diagram. The amplitudes are in Volts. The bitrate is 100 kbit/s. The carrier frequency is $f_c$ = 300 kHz. (20 credit points)

a) What is $M$, i.e. how many symbols are there?

b) How many bits are transferred per symbol? (You can see it directly in the diagram, but please show how this can be calculated based on that you know the value of $M$.)

c) What is the symbol rate in baud or symbols/second?

d) We want to use an ideal bandpass filter in the transmitter and the same filter in the receiver in view to reduce the noise, and increase the signal-to-noise ratio. What bandwidth $B$ in Hertz, and what upper and lower cut-off frequencies $f_{max}$ and $f_{min}$ should the filter have? (Hint: See the above problem. $B$ is approximately equal to the symbol rate. $f_{max}$ and $f_{min}$ are equally spaced around the carrier frequency $f_c$, which is the center frequency of the spectrum.)

e) **Repeat the calculations in the following example for the bit sequence 001100. Replace all underlined expressions.**

**Example:** The constellation diagram shows that for the bit sequence **010011**, the modulator first generates the I-signal (inphase signal) = 5 Volt, and the Q signal (quadrature phase) = -5 Volt during this symbol. The I and Q signals are multiplied with sine and cosine waves respectively, if a sine wave is used as reference phase. The transmitted physical signal during this symbol is consequently:

$$u(t) = I\sin(2\pi f_c t) + Q\cos(2\pi f_c t) = \underline{\underline{5\sin(2\pi\cdot 300000t) - 5\cdot\cos(2\pi\cdot 300000t)}}.$$

Sometimes, for example in Matlab simulations, a cosine wave is used as reference phase. In that case the modulated signal is defined as:

$$u(t) = I\cos(2\pi f_c t) - Q\sin(2\pi f_c t) = \underline{\underline{5\cos(2\pi 300000t) + 5\cdot\sin(2\pi 300000t)}}.$$

Note the negative sign.

Instead of simulating or analyzing this physical high-frequency signal (or so called bandpass signal), we often represent the modulated signal by a complex so called baseband signal C = $I+jQ$= $\underline{5 - j5}$ Volt, where $j = \sqrt{-1}$ is the imaginary unit. Sometimes the *C* signal is also called the equivalent low-pass signal. A vector representation of the complex number is the position of the symbol in the constellation diagram, where the *I* axis is the real part, and the C axis is the imaginary part of the complex value. This representation, with a sine reference phase, resembles to what in electric circuit theory is called the $j\omega$ method, or the complex method, which some of you may have met.

The amplitude of the physical signal is the absolute value of the baseband signal:

$$|C| = \sqrt{I^2 + Q^2} = \underline{\underline{\sqrt{5^2 + (-5)^2} = 5\sqrt{2} \approx 7.07}} \text{ Volts}$$

according to Pythagoras' theorem. It is the distance from origo and the symbol 010011 in the constellation diagram. The phase φ of the physical single is the complex argument of C, i.e. the angle of the C complex vector relative to the I axis in the constellation diagram, which you easily can find graphically as $\underline{\underline{\varphi = -45° = -\frac{\pi}{4}}}$, or calculate according to the following:

$$\varphi = \arg(I + jQ) = \begin{cases} \arctan\frac{Q}{I}, & \text{if } I \geq 0 \\ \arctan\frac{Q}{I} + \pi, & \text{if } I \leq 0 \end{cases}.$$

The complex baseband signal may be described as above in rectangular form, i.e. $C = I + jQ$, but also in polar form as

$$C = |C|e^{j\varphi} = \underline{\underline{5\sqrt{2}e^{-j\pi/2}}}$$

If we assume a cosine wave as reference phase, the physical high-frequency signal (the bandpass signal) can be calculated by means of the C representation, as:

$$u(t) = |C|\cos(2\pi f_c t + \varphi) = \underline{\underline{5\sqrt{2}\cos(2\pi 300000t - \frac{\pi}{2})}}.$$

f) The I(t), Q(t) and C(t) signals may be constant during one symbol, but are varying in time if you consider a sequence of several symbols. They can be described as pulse amplitude modulated signals (PAM) that change voltage for every new symbol. Draw the I and Q signals, if we transmit the message **010011 001100,** i.e. a sequence of the two symbols in the above problem. Use correctly graded time and voltage axes.

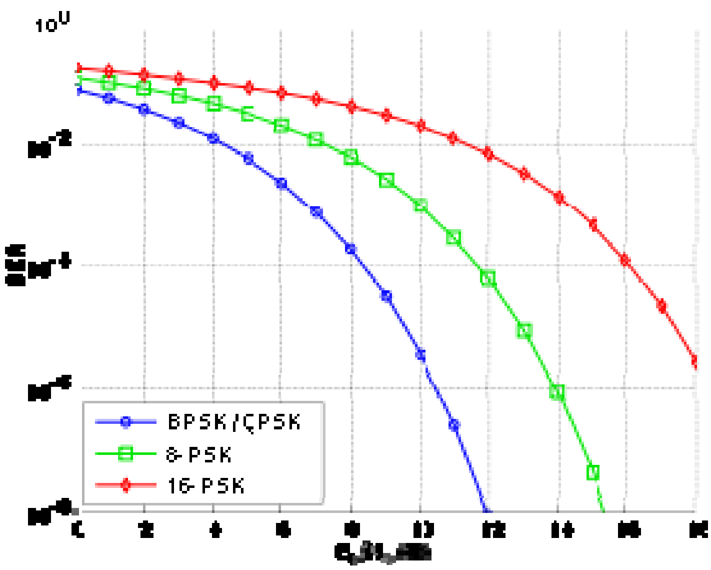
g) If the Signal-to-noise ratio is 16 dB, what is the normalized signal-to-noise ratio in times (the energy-per-bit to noise ratio $E_b/N_0$?) Use the following formula:

$$\frac{E_b}{N_0} = \frac{S}{N} \cdot \frac{B}{R}$$

where the Bandwidth $B$ is approximately equal to the symbol rate, R is the bit rate and $S/N$ is the signal-to-noise ratio in times. Convert the $E_b/N_0$ ratio to decibel!

h) Find the the bit error probability Pe (the bit error rate BER) from the following plot: (Note that that the $E_b/N_0$ ratio is converted to decibels).

[MY MISTAKE – THIS PLOT IS NOT FOR 64QAM. THEN WE ACCEPT ANY OF THE THREE CURVES IN THE MARKING.]



i) Assume a packet size of 1000 bits. What is the packet error rate PER or block/packet error probability? (See list of equations in the end of the document.)

## Common Formulas in the area of Telecommunications

**Bit rate of "M-ary" digital modulation:**

$f_b = f_S \log_2 M$

where $f_s$ is the symbol rate in symbols/s or baud.

**Shannon-Hartley formula:**

$R \le B \log_2\left(1 + \frac{S}{N}\right)$

where $R$ is the net bit rate in bit/s (exclusive of forward error correction coding), $B$ is the bandwidth in Hertz, $S$ is the averge signal power, and N is the noise power.

**Decibel:**

Signal-to-noise ratio $SNR_{dB} = 10 \log \frac{S}{N}$

where S is the Signal power and N the noise power in Watt or Volt$^2$

Power amplification (gain) $G_{dB} = 10 \log \frac{P_{ut}}{P_{in}}$

Voltage amplification (gain) $G_{dB} = 20 \log \frac{U_{ut}}{U_{in}}$

Attenuation: $A_{dB} = -G_{dB}$

**Mean voltage of a periodic signal:**

$$U_0 = \frac{1}{T}\int_0^T u(t)dt$$

**RMS voltage (root-mean-square) of a periodic signal:** $\mathrm{U}_{\mathrm{RMS}} = \sqrt{\frac{1}{T}\int_0^T (u(t))^2\, dt}$

**RMS voltage of a sine wave:**

$$\mathrm{U}_{\mathrm{RMS}} = \frac{\hat{U}}{\sqrt{2}}$$

**Fourier series development:**

$$f(t) = A_0 + A_1 \cos(wt) + A_2 \cos(2wt) + \ldots + A_n \cos(nwt) + B_1 \sin(wt) + B_2 \sin(2wt) + \ldots + B_n \sin(nwt)$$

$$A_0 = \frac{1}{T}\int_0^T f(t)\, dt$$

$$A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)dt$$

$$B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)dt$$

**Noise power of white noise:** $N = N_0 B$ [W], where $B$ is the bandwidth in Hertz.

Spectral density of thermal noise: $N_0 = kT$ [W/Hz], where $k$ is the Boltzmann constant and $T$ is the absolute temperature.