

MA014G
Algebra och Diskret Matematik A
Inlämningsuppgifter till Block 1-2

För att få bonuspoängen måste du lämna in dina lösningar senast
tisdagen den 23 september kl. 8:15

Question 1

- (a) Let A, B and C be subsets of the universal set \mathcal{U} .
- (i) Shade the region $X = (A \cup B) \cap \overline{C}$ in a Venn diagram.
 - (ii) Let $Y = A \cup (B \cap \overline{C})$. Justifying your answer, say whether $X = Y$ for *all* possible choices of A , B and C .
 - (iii) Find three non-empty subsets A , B and C of the positive integers such that

$$(A \cup B) \cap \overline{C} = A \cup (B \cap \overline{C}).$$

- (b) Describe the set

$$M_1 = \{-2, \frac{4}{3}, -\frac{8}{9}, \frac{16}{27}, -\frac{32}{81}, \dots\}$$

by using the rules of inclusion method.

- (c) Describe the following two sets by the listing method.

- (i) $M_2 = \{x \in \mathbb{R} \mid 5x^2 + 2x - 3 = 0\}$;
- (ii) $\mathcal{P}(M_2 \cup \{0\})$.

Question 2

The sequence $\{s_n\}_{n=0}^{\infty}$ is defined by

$$s_n = 3s_{n-1} - 2n \quad \text{for } n \geq 1$$

and the initial term $s_0 = 2$.

Showing all our working, compute s_1, s_2, s_3, s_4, s_5 and s_6 .

Question 3

(a) Express the binary number $(110111011100001001010)_2$

(i) as a hexadecimal;

(ii) in base 10.

(b) Write down the following formula by using Σ -notation.

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

(c) Compute the following sum by using the formula from (b).

$$\sum_{n=20}^{100} (3n+1)^2.$$

Question 4

A small language school with 400 students is teaching English, French and German together with 7 other languages.

(a) The head teacher wants to pick 10 students to represent the school in an education fair. In how many ways can this be done?

(b) The 10 students picked in (a) have to be partitioned into 5 groups of 2 students each. In how many ways can this be done?

(c) Consider the set of students in the school doing at least one of the three languages English, French and German. Of these, 20 students study all three languages together. 150 students are doing English, 130 are doing French and 80 are studying German. 50 of the English students and 30 of the German students are also doing French. 60 German students are doing English.

(i) How many students in the school are not doing any of these three languages?

(ii) How many students are doing precisely one of these three languages?

(iii) How many students are doing French and German, but not English?