Revision

- Read through the whole course once
- Make summary sheets of important definitions and results, you can use the following pages as a start and fill in more yourself
- Do all assignments again
- Do the specimen examination paper
- If you have more time, do some more exercises:
  A number of old exam papers with solutions are available in pdf-format via the course website, and
  you can also find more exercises from the book.

Block 1 - Sets and counting

Main Topics in Block 1

- Sets
  - Sets and subsets. The powerset of a set. Use of binary strings to count and list subsets.
  - Using rules of inclusion and Venn diagrams to represent sets.
  - Complement, union, intersection and difference of sets. Representation of these operations in Venn diagrams.
  - Laws of set operations. Associative laws, distributive laws, De Morgan's laws. Verifying these using Venn diagrams.
  - Partitions of sets.
  - Cartesian product of sets.
  - The sets $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{Q}$.

- Counting.
  - Cardinality of sets.
  - The addition principle.
  - The multiplication principle.
  - The principle of inclusion-exclusion for 2 and 3 sets.
  - Permutations
  - Combinations. Binomial coefficients.

Block 2 - Sequences, strings and summation notation

Main Topics in Block 2

- Sequences
  - Definition by initial term and recurrence relation
  - Definition by general term
  - Proof by induction that a sequence defined by initial term and recurrence relation has a given general term (cf. Block 5)
  - Solution of linear homogeneous recurrence relations (cf. Block 5)
- Strings
  - In particular binary strings and their connection with subsets (cf. Block 1).
  - The null string λ.
  - Concatenation of strings.
- Sums and products
  - Expressing sums in $\sum$-notation.
  - Rules of arithmetic for $\sum$-notation.
  - Changing the variable in $\sum$-notation
  - Expressing products in $\prod$-notation.
  - The sum $\displaystyle\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$.

Main Topics in Block 2 (cont.)

- Number bases
  - The Division Algorithm.
  - Principle of place value.
  - Conversion between bases, in particular hexadecimal, octal, binary and decimal systems.

Block 3 - Integers and Divisibility

Main Topics in Block 3

- Integers
  - Multiples and factors, what it means for numbers to be multiples and factors of each other. The notation $b|a$.
  - Definition of composites, units and primes.
  - Some divisibility rules
  - A simple primality test
  - Factorisation. Statement of the Fundamental Theorem of Arithmetic.
- Greatest common divisor of two integers a and b.
  - Euclid's Algorithm.
  - Using Euclids Algorithm to express $\gcd(a, b)$ as a linear combination of a and b.
  - The set of linear combinations of a and b is the same as the set of all multiples of $\gcd(a, b)$.

Block 4 - Modular Arithmetic, Relations and functions

Main Topics in Block 4

- How to use a directed graph (digraph) to represent a relation on a finite set.
- Properties of relations
  - Symmetric relations
  - Antisymmetric relations
  - Transitive relations
  - Reflexive relations
  - Equivalence relations and equivalence classes
  - Partial orders
- The congruence (mod n) relation
  - Ways of defining congruence (mod n).
  - Rules of arithmetic for congruences.
  - Congruence classes.
  - Finding all solutions to congruences $ax \equiv b(\text{mod} n)$.

Main Topics in Block 4 (cont.)

- $\mathbb{Z}_n$ – the set of congruence classes modulo n
  - Defining addition $\oplus$ and multiplication $\odot$ for $\mathbb{Z}_n$.
  - Computing multiplication and addition tables for $\mathbb{Z}_n$.
  - Rules of arithmetic for $\mathbb{Z}_n$.
  - Definition of zero-divisors and multiplicative inverses.
  - $[a]_n$ has a multiplicative inverse in $\mathbb{Z}_n$ if and only if $\gcd(a, n) = 1$.
  - Using Euclid's Algorithm to find the multiplicative inverse of $[a]_n$ in $\mathbb{Z}_n$ if $\gcd(a, n) = 1$.
  - Finding all solutions to equations $[a] \odot [x] = [b]$ in $\mathbb{Z}_n$.

- Functions
  - Definition of a function
  - Domain and range
  - Injections (functions that are 1-1)
  - Surjections (functions that are onto)
  - Bijections (1-1 correspondences).
  - Composition of functions
  - Inverse function.
  - A function has an inverse if and only if it is 1-1 and onto.

Block 5 - Logic and Proof

Main Topics in Block 5

- Logic
  - Propositions.
  - Combining statements using NOT, AND and OR.
  - Conditional and Biconditional Statements. Necessary and Sufficient conditions.
  - Truth tables for p AND q, p OR q and $p \Rightarrow q$.
  - The contrapositive of a proposition.
  - The quantifiers $\forall$ and $\exists$
- Methods of Proof
  - Direct proof
  - Proof by contradiction
  - Proving $\iff$ by proving $\Rightarrow$ and $\Leftarrow$.
  - Proof by counterexample
  - Proof by induction
- More about sequences
  - Proof by induction that a sequence defined by initial term and recurrence relation has a given general term (cf. Block 2)
  - Solution of linear homogeneous recurrence relations (cf. Block 2)

Block 6 - Graph Theory

Main Topics in Block 6

- Graphs, simple graphs, loops, multiple edges
- Digraphs, arcs
- Subgraphs
- The degree of a vertex.
- The Handshaking Lemma.
- Paths, simple paths, cycles and simple cycles
- Connected graphs. Components.
- Complement of a graph
- Special types of graphs
  - Regular graphs
  - Complete graphs K_n
  - Bipartite graphs
  - Complete bipartite graphs $K_{m,n}$
  - Trees
- Adjacency matrices
- Incidence matrices
- Graph Isomorphisms

Main Topics in Block 6 (cont.)

- Trees
  - Definition: A tree is a connected graph without cycles
  - In a tree there is a unique (simple) path between any pair of vertices
  - A tree on n vertices has $n-1$ edges
  - Rooted trees and the terminology surrounding them
  - Spanning trees
  - Breadth-First-Search (BFS) spanning trees
  - Depth-First-Search (DFS) spanning trees
  - Minimal/Maximum weight spanning trees (MSTs) for weighted graphs

- Eulerian and Hamiltonian Graphs
  - A graph has an Euler cycle if and only if all vertices have even degree
  - Know how to find an Euler cycle in a small graph.
  - Know a necessary condition for a graph to have a Hamiltonian cycle.

Main Topics in Block 6 (cont.)

- Weighted Graphs
  - Dijkstra's Algorithm for finding the shortest path between two given vertices.
  - Kruskal's or Prim's algorithm for finding an MST

- Traveling Salesman Problem
  - Shortest route is not necessarily a Hamiltonian cycle unless the graph satisfies the triangle inequality.
  - Often an exact solution is too expensive to find, so an approximated solution may be preferable.
  - Nearest neighbour approximations to the solution.