MA014G
Algebra and Discrete Maths A
Assignment Block 1

To get the bonus point you must hand in your solutions by 10am on Thursday 20 September.

Exercise 1

Let $A = \{a, b, c, d\}$, $B = \{a, b, e\}$ and $C = \{a, b, e, f, g\}$ be three subsets of the same universal set $G = \{a, b, c, d, e, f, g\}$. Justifying your answers, decide for each of the following expressions whether it is true or false.

(a) $A \cap (B \cap C) = G$;

(b) $A \cup (B \cup C) = G$;

(c) $A - B \subset A - C$;

(d) $A - B \subseteq A - C$;

(e) $\{b, a, b, e\} \subset C$;

(f) $\{a, \{b\}, e\} = B$;

(g) $\{\{b\}\} \in B$;

(h) $\{\{b\}\} \subseteq B$;

(i) $\{\{b\}\} \subseteq \mathcal{P}(B)$.

Exercise 2

(a) Give the number of 10-bit binary strings.

(b) Give the number of 10-bit binary strings with precisely 3 zeroes.

(c) Let M be a set with 10 elements.

(i) State the cardinality of $\mathcal{P}(M)$.

(ii) How many subsets of M contain precisely 7 elements?

Exercise 3

(a) Let A,B and C be subsets of the universal set $\mathcal{U}$.

(i) Shade the area $X = (C \cup B) \cap \overline{A}$ in a Venn diagram.

(ii) Let $Y = C \cup (B \cap \overline{A})$. Decide whether $X = Y$ for *all* choices of A, B and C. Justify your answer!

(iii) Find three non-empty sets of positive integers A, B and C such that

$$(C \cup B) \cap \overline{A} = C \cup (B \cap \overline{A}).$$

(b) Give the set

$$M_1 = \{0, \tfrac{2}{3}, -\tfrac{4}{5}, \tfrac{6}{7}, -\tfrac{8}{9}, \tfrac{10}{11}, -\tfrac{12}{13}, \ldots\}$$

by the rules of inclusion method.

(c) Give the following sets by the listing method.

(i) $M_2 = \{x \in \mathbb{Z} \mid 3x^2 + 5x - 2 = 0\}$;

(ii) $\mathcal{P}(\{0, 1, b\})$.

Exercise 4

(a) In a dance academy there are 200 students, half of which are girls and half of which are boys. A pair to represent the academy in a competition is to be selected. In how many ways can the teacher select a pair to represent the academy if the pair must consist of one boy and one girl?

(b) The dance academy teaches the waltz, the rumba and the tango (among other dances), and 40 students are doing all of these three dances. 120 students do the tango, 120 do the rumba and 90 do the waltz. 60 of the tango students and 50 of the rumba students are also doing the waltz. 70 rumba students are also doing the tango.

(i) How many students are doing none of the three dances?

(ii) How many students are doing the tango, but not the waltz and not the rumba?

(iii) How many students are doing exactly two of the three dances?