# MA014G
# Algebra and Discrete Mathematics
# Assignment Block 2

**In order to get the bonus points you must hand in your solutions by 10am on Monday 24 September 2007**

**Question** 1
Compute the following sums.

(a) $\sum_{i=3}^{10}(2i-5)$.

(b) $\sum_{k=-3}^{4}(-k)^3$.

(c) $\sum_{k=-2}^{50}5$.

**Question 2**

(a) Express the following formula by using $\Sigma$-notation.

$$1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}.$$

(b) Compute the following sum by using the formula in (a):

$$\sum_{n=10}^{100}(2n+1)^2.$$

(c) One evening 50 people meet at a party. Everybody shakes hands with every other person precisely once. Using summation notation (Sigma notation), write down an expression for the total number of handshakes taking place at the party. (Actually there is also a combinatorial solution to this problem, you are welcome to write this down too).

**Question 3**

(a) Convert the binary number $(110111011010000111010)_2$

(i) to hexadecimal;

(ii) to base 10.

(b) Express the hexadecimal number $DB9_{16}$ in binary.

(c) Express the number $2102_3$ in base 2.

**Question 4**

A sequence $\{s_k\}_{k=1}^{\infty}$ is defined by the recurrence relation

$$s_k = s_{k-3} - 3s_{k-2} + 2s_{k-1} \quad \text{for } k \geq 4,$$

and the initial terms $s_1 = 1, s_2 = -1$ and $s_3 = 4$.
Compute $s_4, s_5, s_6$ and $s_7$.

**Question 5**
Sam has been collecting a large number of 5kr-coins which he intends to use for coffee and snacks during breaks. Each time there is a break in his Maths lectures he hurries down to the coffee shop. The coffee shop sells coffee and sweets. At each break he either buys coffee only, which costs him 5 kr, or he buys coffee and a sweet, which costs him 10 kr. He does this until he has used up all his coins.

Suppose that he has $n$ 5kr-coins initially, where $n \geq 1$. Write down a recurrence relation describing the number of possible purchasing sequences.

*Hint 1:*
If he has 3 coins, then there are 3 possible purchasing sequences, namely

(coffee+sweet, coffee), (coffee, coffee+sweet) and (coffee, coffee, coffee).

*Hint 2:*
Let $a_n$ be the number of ways of spending $n$ coins. Calculate $a_1$ and $a_2$. For $n \geq 3$, think about the case where you start the sequence by Sam purchasing coffee only. In this case, in how many ways can he spend the remaining coins? Next consider the case where his first purchase is coffee+sweet.