MA014G
Algebra and Discrete Mathematics A
Assignment Block 3

To get the bonus points you must submit you solutions by 10am on Monday 1 October 2007.

Question 1
Carefully study Theorem B3.3 from the study guide for Block 3 and its proof which is given below. Then answer questions (a)-(e).

Theorem B3.3 *Let m, n and d be integers such that $d|m$ and $d|n$. Then $d|(sm+tn)$ for all integers s and t.*

Proof.
As m and n both are divisible by d, there exists two integer quotients q_1 and q_2 such that

$$m = dq_1 \text{ and } n = dq_2. \quad (*)$$

Thus

$$sm + tn \;=\; s(dq_1) + t(dq_2) \;=\; d(sq_1 + tq_2), \quad (**)$$

and so $d|(sm+tn)$. □

(a) What is meant by $d|m$, i.e. what is meant by saying that d divides m?

(b) Which theorem or definition guarantees the existence of q_1 and q_2 satisfying $(*)$?

(c) Explain carefully why the two =-signs hold in the computation $(**)$.

(d) Consider the computation $(**)$. Explain why the term $sq_1 + tq_2$ is an integer.

(e) Explain why the computation $(**)$ proves that $d|(sm+tn)$.

Question 2
Assume that $3|(2^{n-1} - 1)$ for some integer $n \geq 1$.
Prove that then $3|(2^{n+1} - 1)$ also.

Question 3

(a) Use *Euclid's Algorithm* to show that $\gcd(3571, 1753) = 1$.

(b) Find integers s and t with $s > 0$ and $t < 0$ such that $3571s + 1753t = 1$.

(c) Find integers k and ℓ with $\ell > 0$ and $k < 0$ such that $3571k + 1753\ell = 1$.

Question 4
The Fundamental Theorem of Arithmetic says that, apart from the order of the factors, there is a unique factorisation of 22374 into one unit and a finite number of positive primes. Showing all your working, find this factorisation of 22374 without using a calculator.