MA014G
Algebra and Discrete Mathematics A
Assignment 4

To get the bonus points you must submit your solutions by 10am on Monday 8 October 2007.

Question 1

(a) Explain what it means for a relation R on a set S to be

(i) reflexive;

(ii) symmetric;

(iii) transitive;

(iv) anti-symmetric.

(b) Let R be the following relation on the set $S = \{a, b, c, d, e\}$.

$$R = \{(a, a),\ (b, b),\ (c, c),\ (e, e),\ (a, e),\ (e, c),\ (d, b),\ (c, d)\}$$

(i) Draw the relationship digraph for R.

(ii) The relation R is not symmetric. List the smallest set of pairs which must be added to R in order to make R symmetric.

(iii) The relation R is not reflexive. List the smallest set of pairs which must be added to R in order to make R reflexive.

(iv) The relation R is not transitive. List the smallest set of pairs which must be added to R in order to make R transitive.

(v) Justifying your answer, say whether the relation R is anti-symmetric.

Question 2

Let R be the relation on the set $X = \{-100, -10, -1, 0, 1, 10, 100, 1000\}$ defined by $(x, y) \in R$ if $xy > 0$.

(a) List the elements of R.

(b) Draw the relationship digraph for R.

(c) Justifying your answer, say whether R is an equivalence relation on X.

Question 3

(a) Compute the multiplication table for $\mathbb{Z}_{12}$.

(b) Solve the equation $[8] \odot [x] = [4]$ in $\mathbb{Z}_{12}$.

(c) Solve the equation $[8] \odot [x] = [2]$ in $\mathbb{Z}_{12}$.

Question 4
Find all solutions $[x] \in \mathbb{Z}_{3571}$ to the equation

$$[1753] \odot [x] = [3].$$

Hint: Use Question 3 from Assignment 3.

Question 5
Solve the equation $[14] \odot [x] = [4]$

(a) in $\mathbb{Z}_{150}$;

(b) in $\mathbb{Z}_{151}$.