MA014G
Algebra and Discrete Mathematics A
Assignment 5

To get the bonus points you must hand in your solution by 10am on Friday 19 October 2007.

Question 1
Let n be an integer. State the contrapositive of the proposition

'If n^2 is odd then n is odd'

and prove the proposition.

Question 2
Give a proof by contradiction to show that if we put 201 balls into 50 boxes, then there is at least one box which contains five or more balls.

Question 3
Solve the recurrence relation

$$a_{n+2} = -6a_{n+1} - 9a_n \quad \text{for } n \geq 0,$$

with initial terms $a_0 = -1$ and $a_1 = 1$, that is
express a_n as a function of n for all $n \geq 0$.

Question 4
The sequence $\{s_n\}_{n=1}^{\infty}$ is defined by

$$s_n = s_{n-1} + n^2 \quad \text{for } n \geq 2,$$

and the initial term $s_1 = 1$.

(a) Use the recurrence relation to compute s_2, s_3, s_4 and s_5.

(b) Prove by induction that $s_n = \sum_{r=1}^{n} r^2$ for all $n \geq 1$.

(c) Prove (by induction) that

$$s_n = \frac{n(n+1)(2n+1)}{6}$$

for all $n \geq 1$.