MA014G ASSIGNMENT 1 (Autumn 2007)

Question 1

$A = \{a,b,c,d\}$ $B = \{a,b,e\}$ $C = \{a,b,e,f,g\}$

$G = \{a,b,c,d,e,f,g\}$

(a) FALSE as $A \cap (B \cap C) = A \cap \{a,b,e\} = \{a,b\} \neq G$

(b) TRUE as $A \cup (B \cup C) = A \cup C = \{a,b,c,d,e,f,g\} = G$

(c) FALSE as $A-B = \{c,d\}$ and $A-C = \{c,d\}$ $A-B = A-C$ and $A-B$ is thus not a proper subset of $A-C$

(d) TRUE but $A-B = A-C$ so $A-B \subseteq A-C$

(e) TRUE as $\{b,a,b,e\} = \{a,b,e\} \subset G$

(f) FALSE as $\{b\} \notin B$.

(g) FALSE as $\{\{b\}\} \notin B$

(h) FALSE as $\{b\} \notin B$ the set $\{\{b\}\}$ is not a subset of B.

(i) TRUE as $\{b\} \subseteq B$ we have $\{b\} \in \mathcal{P}(B)$ and so $\{\{b\}\} \subseteq \mathcal{P}(B)$

Question 2

(a) $2^{10} = \underline{1024}$

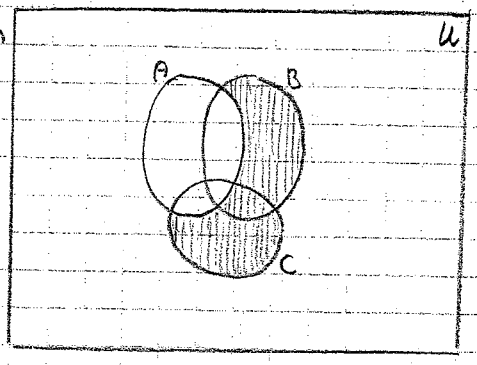
(b) $\binom{10}{3} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = \underline{120}$

(c) (i) $\mathcal{P}(M)$ is in 1-1 correspondence with the set of binary strings of length 10, hence $\mathcal{P}(M)$ has cardinality $2^{10} = \underline{1024}$

(ii) $\binom{10}{7} = \binom{10}{3} = \underline{120}$

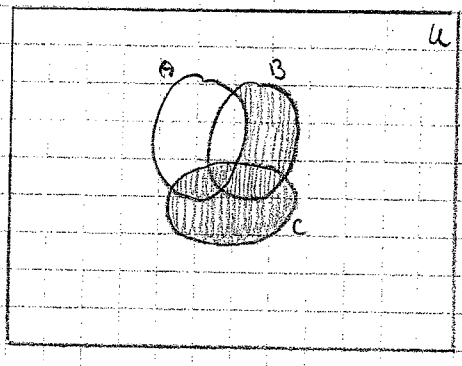
Question 3

(a) (i)



$\blacksquare = (C \cup B) \cap \bar{A} = X$

(ii)



$\blacksquare = C \cup (B \cap \bar{A}) = Y$

as the two Venn diagrams for X and Y are not identical, X and Y are not the same for all choices of A, B and C.

(iii) Let e.g. $A = \{1,2,3\}$, $B = \{1,2,4,5\}$ and $C = \{4,5,6\}$ (any three sets with $A \cap C = \varnothing$.)

then $(C \cup B) \cap \bar{A} = \{1,2,4,5,6\} \cap \bar{A} = \{4,5,6\}$

and $C \cup (B \cap \bar{A}) = C \cup \{4,5\} = \{4,5,6\}$

So here $(C \cup B) \cap \bar{A} = C \cup (B \cap \bar{A})$.

(b) $M_1 = \left\{ (-1)^{n+1} \frac{2n}{2n+1} \;\middle|\; n \in \mathbb{N} \right\}$

(c) (i) $3x^2 + 5x - 2 = 0 \Rightarrow x = \frac{-5 \pm \sqrt{25 + 4 \cdot 3 \cdot 2}}{6} = \frac{-5 \pm 7}{6} = \begin{cases} 1/3 \\ -2 \end{cases}$

so $M_2 = \{ x \in \mathbb{Z} \mid 3x^2 + 5x - 2 = 0 \} = \{-2\}$ as $\frac{1}{3} \notin \mathbb{Z}$.

(ii) $\mathcal{P}(\{a,1,b\}) = \{ \varnothing, \{a\}, \{1\}, \{b\}, \{1,b\}, \{a,b\}, \{a,1\}, \{a,1,b\} \}$

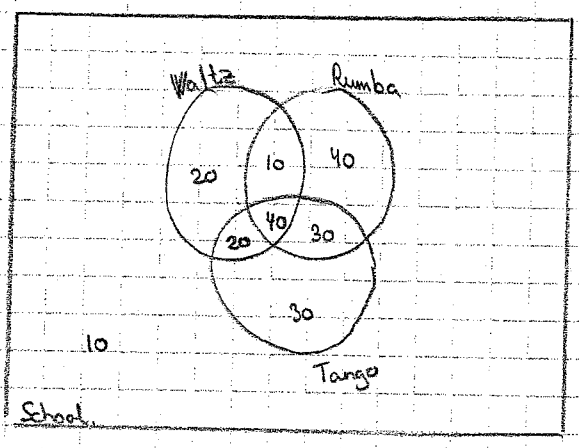
Question 4

(a) The school has 100 boys and 100 girls. A pair for the contest consists of 1 boy and 1 girl.

There are 100 ways to choose a boy and 100 ways to choose a girl.

Hence, by the Multiplication Principle, we get that the number of possible pairs = $100 \cdot 100 = \underline{\underline{10000}}$

(b)



By the 'Venn diagram' the number of students doing at least one of the three dances is

$20+10+40+20+40+30+30 = 190.$

We know the school has 200 students

(i) So 10 students are not doing any of the three dances

(ii) 120 students are doing the Tango, but of these $20+40+30$ are also doing either Waltz or Rumba, so $120 - (20+40+30) = 30$ are doing Tango only

(iii) the number of students doing precisely two dances is $10+20+30 = 60$

as 10 learn just Waltz and Rumba,

20 learn just Waltz and Tango and

30 learn just Rumba and Tango.