MA014G Assignment 2 (autumn 2007)

Question 1

(a) $\sum_{i=3}^{10}(2i-5) = 1+3+5+7+9+11+13+15 = 64$

(b) $\sum_{k=-3}^{4}(-k)^3 = \cancel{3^3}+\cancel{2^3}+\cancel{1^3}+0^3-\cancel{1^3}-\cancel{2^3}-\cancel{3^3}-4^3 = -64$

(c) $\sum_{k=-2}^{50} 5 = 53\cdot 5 = 265$

Question 2

(a) $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

(b) $$\sum_{n=10}^{100}(2n+1)^2 = \sum_{n=10}^{100}(4n^2+4n+1) = 4\sum_{n=10}^{100} n^2 + 4\sum_{n=10}^{100} n + \sum_{n=10}^{100} 1$$

$$= 4\left(\sum_{n=1}^{100} n^2 - \sum_{n=1}^{9} n^2\right) + 4\left(\sum_{n=1}^{100} n - \sum_{n=1}^{9} n\right) + 91$$

$$= \frac{4\cdot 100\cdot 101\cdot 201}{6} - \frac{4\cdot 9\cdot 10\cdot 19}{6} + \frac{4\cdot 100\cdot 101}{2} - \frac{4\cdot 9\cdot 10}{2} + 91$$

$$= 1353400 - 1140 + 20200 - 180 + 91$$

$$= \underline{1372371}$$

(c) Guest 1 shakes hands with 49 guests = 49 handshakes
Guest 2 shakes hands with 48 apart from g.1 = 48 handshakes
Guest 3 shakes hands with 47 apart from g.1, 2 = 47 handshakes
⋮
Guest 49 —"— 1 other = 1 handshakes
Guest 50 has already shaken hands with all = 0 handshakes,

So number of handshakes $= \sum_{i=1}^{50}(50-i) = \sum_{i=0}^{49} i$

Question 3

(a) $1\,\underbrace{1\,0\,1\,1}_{B}\,\underbrace{1\,0\,1\,1}_{B}\,\underbrace{0\,1\,0\,0}_{4}\,\underbrace{0\,0\,1\,1}_{3}\,\underbrace{1\,0\,1\,0}_{A}$

$1 \quad B \quad B \quad 4 \quad 3 \quad A$

(i) $(1BB43A)_{16}$

(ii) $10+3\cdot 16+4\cdot 16^2+11\cdot 16^3+11\cdot 16^4+1\cdot 16^5 = 1815610$

(b) $\overset{D}{1\,1\,0\,1}\;\overset{B}{1\,0\,1\,1}\;\overset{9}{1\,0\,0\,1}$

$(1\,1\,0\,1\,1\,0\,1\,1\,1\,0\,0\,1)_2$

(c) $(2102)_3 = 2+0\cdot 3+1\cdot 3^2+2\cdot 3^3 = 2+9+54 = 65 = (1\,0\,0\,0\,0\,0\,1)_2$

Question 4

$s_k = s_{k-3} - 3s_{k-2} + 2s_{k-1}$ for $k \geq 4$

$s_1 = 1$
$s_2 = -1$
$s_3 = 4$

$s_4 = s_1 - 3s_2 + 2s_3 = 1-3(-1)+2\cdot 4 = 12$

$s_5 = s_2 - 3s_3 + 2s_4 = -1-3(4)+2(12) = 11$

$s_6 = s_3 - 3s_4 + 2s_5 = 4-3\cdot 12+2\cdot 11 = -10$

$s_7 = s_4 - 3s_5 + 2s_6 = 12-3\cdot 11+2(-10) = -41$

Question 5

Let a_n = the number of ways of using n coins.

For $n=1$: $a_1 = 1$ as he can buy coffee only

For $n=2$: $a_2 = 2$ as he can either buy 2 coffees or 1 coffee + 1 snack.

For $n \geq 3$: $a_n = a_{n-1} + a_{n-2}$, for there are two disjoint possibilities:

either he buys 1 coffee first and then has $n-1$ coins left to spend, yielding $1 \cdot a_{n-1}$ ways of spending the coins

or he buys a coffee + a snack first and then has $n-2$ coins left to spend, yielding $1 \cdot a_{n-2}$ ways of spending the n coins this way.

So the required recurrence relation is $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$

with initial conditions $a_1 = 1$, $a_2 = 2$.