MA014G

Mittuniversitetet
MID SWEDEN UNIVERSITY

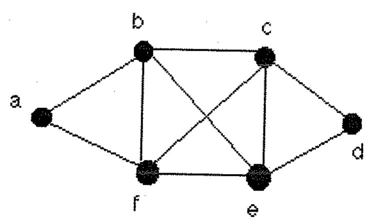
Algebra och Diskret Matematik A
Svar till Inlämningsuppgifter Block 6

Uppgift 1

[J] 6.2.23 (6.2.23) [8.2.23]
$\partial(v_1) = 2$,
$\partial(v_2) = 2$,
$\partial(v_3) = 3$,
$\partial(v_4) = 6$,
$\partial(v_5) = 2$,
$\partial(v_6) = 3$,
$\partial(v_7) = 4$,
$\partial(v_8) = 4$,
$\partial(v_9) = 4$,
$\partial(v_{10}) = 2$.

Uppgift 2

[J] 6.3.11 (6.3.11) [8.3.11]



A Hamiltonian cycle is (a,b,c,d,e,f, a) and since this cycle does not use every edge of the graph, it is not an Euler cycle. The graph is Eulerian though as all vertices have even degree, one possible Euler cycle is (a,b,c,d,e,c,f,e,b,f,a).
(Note that neither the Euler cycle nor the Hamiltonian cycle are unique.)

Uppgift 3

[J] 6.4.3 (6.4.3) [8.4.3]
(a)
Using Dijkstra's Algorithm as in the lecture notes p. 160, we find the shortest path from a to z to be (a,b,c,d,z) which has length 10. The vertices become marked in one of the following four orders:
a, b, h, c, e, i f, d, z
a, b, h, c, e, i f, d, j, z
a, b, h, e, c, i f, d, z
a, b, h, e, c, i f, d, j, z

(b)
Using Kruskal's algorithm we find the minimal spanning tree with edges (listed in the order chosen by the algorithm)
(b,c) of weight 2,
(h,i) of weight 2,
(f,c) of weight 2,

(d,z) of weight 2,
(b,a) of weight 3,
(f,j) of weight 3,
(d,c) of weight 3,
(a,h) of weight 4,
(e,f) of weight 4,
(g,j) of weight 4.

The total weight of the minimal spanning tree is 29.
(Note that the solution is not unique, but that all minimal spanning trees have the same total weight)

Uppgift 4

Yes, graphs G and H are isomorphic. An isomorphism Φ from V(G) to V(H) is defined by the following:

Φ(a) = 5,
Φ(b) = 4,
Φ(c) = 2,
Φ(d) = 1,
Φ(e) = 7,
Φ(f) = 6,
Φ(g) = 3,

because it is a 1-1 correspondence between the vertices of G and the vertices of H which preserves the edges:

edge (a,b) maps to (5,4)
edge (b,c) maps to (4,2)
edge (b,d) maps to (4,1)
edge (c,d) maps to (2,1)
edge (c,e) maps to (2,7)
edge (d,f) maps to (1,6)
edge (e,f) maps to (7,6)
edge (f,g) maps to (6,3).

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