Mittuniversitetet
MID SWEDEN UNIVERSITY

MA014G
Algebra och Diskret Matematik A
Svar på uppgifter till Block 6

Referenser utan parenteser är till [J] edition 5, referenser i ()-parenteser är till [J] edition 4, och referenser i []-parenteser är till [J] edition 6.

Uppgifter i läsanvisningen

Uppgift B6.1 - B6.4

[J] Section 6.1 (6.1) [8.1]:

6 (2) [6].
The edge ab must be included, so we start with it. Now we have a choice: bc or bd. If we take bc we are left with the edge bd, which we have to take at some point. When we do take it, we will find ourselves stuck at b with no edge remaining by which to leave b. So we cannot take bc. If instead we take bd, we find we have the same problem, this time with bc remaining.

7 (3) [7].
We will have exactly the same problem as in exercise 6 (2) [6]. When we go through b we use up two edges, so we can go through it twice, but when we visit it the third time, we use the last edge and cannot leave again.

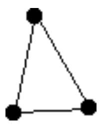
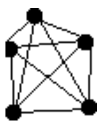
9 (5) [9]. abdefcebca

10 (6) [10]. abcfihfehgecgdebda

11 (7) [11]. Not simple as it contains both a loop (e_5) and parallel edges (e_1 and e_6). e_1 is incident on v_1 and v_2.

12 (8) [12]. Simple graph. e_1 is incident on v_2 and v_4.

13 (9) [13]. Simple graph. There is no e_1 .

14 (10) [14].  K_3  K_5

15 (11) [15]. n(n-1)/2.

16 (12) [16]. $K_{3,3}$ - see solution to exercise 20 (16) [24] in [J].

18 (14) [18]. Bipartite. $V_1=\{v_1, v_6, v_3, v_4, v_8, v_{10}, v_9\}$ $V_2=\{v_5, v_2, v_7\}$

19 (15) [19]. Bipartite. V_1={Gre, Buf, Dou, Sho, Mud} V_2={She, Wor, Cas, Lan, Gil}

21 (17) [21]. Not bipartite as it contains a triangle, $v_1v_2v_3v_1$.

22 (18) [22]. See 21 (17) [21] .

25 (21) [25]. mn

Uppgift B6.5

1,2,5,6,2,1 är en stig från 1 till 1, men den är inte enkel.

1,2,5,7 är inte en stig, ty det finns ingen kant (5,7) i grafen.

Uppgift B6.6 - B6.7

[J] Section 6.2 (6.2) [8.2]:

2 (2) [2]. simple path

3 (3) [3]. none

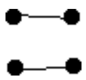
5 (5) [5]. none

6 (6) [6]. cycle

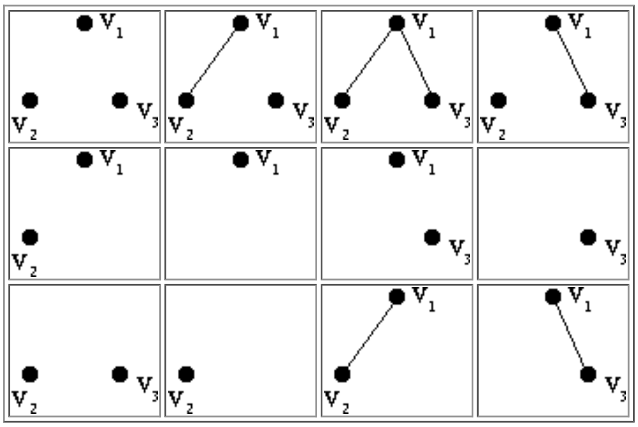
8 (8) [8]. simple path

9 (9) [9]. simple path

12 (12) [12].



26 (26) [26].



29 (29) [29]. Has an Eulerian cycle as all degrees are even.

30 (30) [30]. No Eulerian cycle as v_3 has odd degree.

32 (32) [32]. abdcbfgjfejhcideiha

33 (33) [33]. abcdehfijkihgfegdca

35 (35) [35]. when n is odd

36 (36) [36]. both m and n must be even

Uppgift B6.8

[J] Section 6.3 (6.3) [8.3]:

2 (2) [2]. abcdefnpmlkjoihga

4 (4) [4]. This graph is bipartite, with an uneven partition of the vertices so it is not Hamiltonian.

5 (5) [5]. The edges gi, ij, fg, gh and jg would all have to be included, which would lead to g being used too many times.

7 (7) [7]. abcglmrqpkjfeinotshda

8 (8) [8].
We can only use 2 edges at c, 2 at e and 2 at f so we don't use 2 edges at c, 1 edge at d and 3 edges at e leaving a total of 6 edges which is too few for a hamilton cycle.

10 (10) [10]. K_3

11 (11) [11]. K_5

Uppgift B6.9

[J] Section 6.4 (6.4) [8.4]:

2 (2) [2]. 11 abcg

3 (3) [3]. 10 abcdz

5 (5) [5]. 10 hfcd

Uppgift B6.10

[J] Section 6.5 (6.5) [8.5]:

2 (2) [2].

	a	b	c	d	e	f	g
a	0	2	0	0	0	0	0
b	2	0	1	0	1	0	0
c	0	1	2	0	1	0	0
d	0	0	0	2	1	0	1
e	0	1	1	1	0	1	0
f	0	0	0	0	1	0	1
g	0	0	0	1	0	1	0

8 (8) [8].

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
a	1	0	0	0	0	0	0	0	0	0
b	1	1	0	1	0	0	0	0	0	0
c	0	1	1	0	0	0	0	1	0	0
d	0	0	0	0	1	1	1	0	0	0
e	0	0	0	1	0	0	1	1	0	1
f	0	0	0	0	0	0	0	0	1	1
g	0	0	0	0	0	1	0	0	1	0

Uppgift B6.11

[J] Section 6.6 (6.6) [8.6]:

2 (2) [2].
Isomorphic, as there is a bijection f from the vertices of G_1 to the vertices of G_2 which preserves the edges. f is defined by f(a)=a', f(b)=c', f(c)=e', f(d)=g', f(e)=b', f(f)=d', f(g)=f'.

3 (3) [3].
These are not isomorphic. Choose ONE of the following to show this:

- G_1 contains triangles (such as abca) whereas G_2 contains no triangles.
- The vertices of G_2 cannot be coloured with 2 colours so that no two adjacent vertices are coloured with the same colour. G_2 can be coloured with 2 colours.
- G_2 is bipartite whereas G_1 is not.
- The complement of G_1 consists of one component whereas the complement of G_2 consists of two.

This list is probably not complete, try to think of some other structural differences yourself.
note that giving a failed attempt to pair up the vertices is **never enough** to show that two graphs are non-isomorphic unless you have tried **every** possible way to pair them up - that is, in this case all 6! = 720 ways.

5 (5) [5].
Isomorphic, as there is a bijection f from the vertices of G_1 to the vertices of G_2 which preserves the edges. f is defined by f(a)=b', f(b)=d', f(c)=e', f(d)=a', f(e)=c'.

6 (6) [6].
Here is a description of how I solved this problem. Note that such a description would not usually be included in an answer given to a question of this kind, one would just give the 'answer' below. First, I wrote the degree of each vertex next to the vertex in the graph, and then compared the vertices of each degree. There are two vertices of degree 5 in each graph, and as G_2 is symmetrical, it doesn't matter which way we pair them up. Take f(l)=l' and f(k)=k'. Now there are two vertices of degree 4 adjacent to both l and l'. Try pairing those (might be wrong - we'll see): f(a)=a', f(h)=d'. Now k is adjacent to two vertices of degree 4, namely, e and d, where d is adjacent to a. Similarly k' is adjacent to c' and b' where b' is adjacent to f(a)=a'. To preserve the edges, we pair d and b' so f(d)=b', leaving f(e)=c'. Now I look for vertices which are adjacent to those I have already paired up. g is adjacent to both l and h, and h' is adjacent to both f(l)=l' and f(h)=d' so f(g)=h'. f is adjacent to g,k,e and g' is adjacent to f(g)=h', f(k)=k' and f(e)=c' so f(f)=g'. Only one of the neighbours of a is not paired and that is b. Similarly, of the neighbours of f(a)=a', only e' is unpaired so f(b)=e'. c is adjacent to both b and d and f' is adjacent to both f(b)=e' and f(d)=b' so f(c)=f'. We are left we just i and j and i' and j'. By comparing neighbours, we see f(i)=i' and f(j)=j'. Now for the 'answer' to the question:

G_1 and G_2 are isomorphic because there is a bijection f mapping the vertices of G_1 to G_2 so that there is an edge uv in G_1 if and only if the corresponding edge f(a)f(b) is present in G_2. The function f is defined as follows: f(a)=a', f(b)=e', f(c)=f', f(d)=b', f(e)=c', f(f)=g', f(g)=h', f(h)=d', f(i)=i', f(j)=j', f(k)=k', f(l)=l'.

8 (8) [8].
Hint: look at the vertices of degree 3.

9 (9) [9].
First I looked at the degrees. It didn't help. Next I looked at the complements. Did not look too nice, so I decided to come back to that later if I could not find anything else. Then I noticed that in both graphs the only vertex of degree 4 is contained in two triangles in G_1 and just one triangle in G_2.

G_1 and G_2 are not isomorphic because (chose any ONE of the following)

- the only vertex e of degree 4 in G_1 is contained in two triangles abe and efg, whereas the only vertex f' of degree 4 in G_2 is contained in just one triangle f'g'e'.
- if we remove the vertices of degree 4 from G_1 and from G_2, one of the remaining graphs can be coloured with 2 colours whereas the other can not.
- G_1 contains two triangles whereas G_2 contains only one.

Uppgift B6.12

[J] Section 7.1 (7.1) [9.1]:

9 (2) [9].
Answer 4

10 (4) [10].
Answer 5

Uppgift B6.13

[J] Section 7.2 (7.2) [9.2]:

2 (2) [2]. Aphrodite, Uranus.

3 (3) [3]. Aphrodite, Kronos, Atlas, Prometheus.

5 (5) [5]. Zeus, Poseidon, Hades.

6 (6) [6]. One edge drawn vertically with Aphrodite as root and Eros as leaf.

24 (24) [24]. No such graph exists. A terminal node (leaf) has degree one.

26 (26) [26]. The root has degree three and each child of the root is adjacent to two leaves.

Uppgift B6.14

[J] Uppgift 7.3.9 (7.3.9) [9.3.9]:

The edges in the spanning tree are (a,c), (b,c), (c,d), (d,f), (f,e), (f,g), (f,h), (f,i).

Uppgift B6.15

[J] Section 7.4 (7.4) [9.4]:

2 (2) [2].
The edges in the minimal spanning tree are (1,2), (2,3), (3,6), (6,9), (2,5), (5,8), (8,7), (7,4).

3 (3) [3].
The edges in the minimal spanning tree are (4,1), (1,2), (2,5), (5,6), (6,3).

5 (5) [5].
The edges in the minimal spanning tree are
(1,2), (2,5), (2,7), (3,7), (4,8), (5,6), (5,9), (6,15), (7,11), (8,12), (9,10), (10,13), (11,14), (12,16), (15,16).

The authors would like to thank the following for their contribution to various updates of the original manuscript:
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Updated 070811