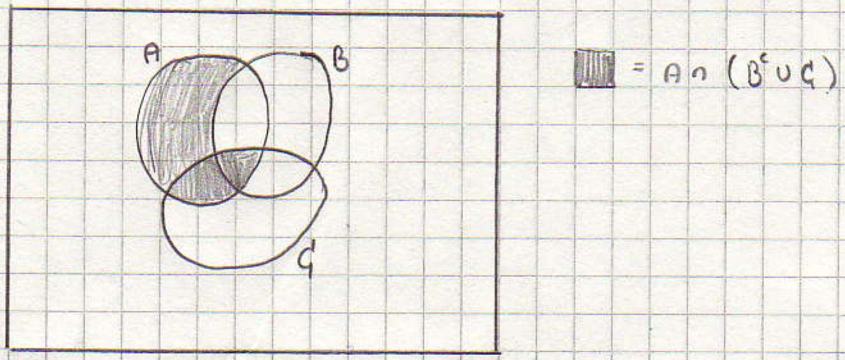
MA053G Assignment 1 VT08

(1) (a) (i) - (ii)



(a) (iii) $B^c = \{b, c, e, f\}$, $B^c \cup C = \{a, b, c, d, e, f\}$.

$A \cap (B^c \cup C) = \{a, c, e, f, h\} \cap \{a, b, c, d, e, f\} = \underline{\underline{\{a, c, e, f\}}}$

(b) (i) $\{7r+2 : r \in \mathbb{Z}, -2 \le r \le 3\} = \underline{\underline{\{-12, -5, 2, 9, 16, 23\}}}$

(ii) $\{5^s \mid s \in \mathbb{Z}, -2 \le s \le 3\} = \underline{\underline{\{\frac{1}{25}, \frac{1}{5}, 1, 5, 25, 125\}}}$

(c) This can be done in more than one way, the most compact solution is

$$\underline{\underline{\left\{\frac{2n-1}{2} : n \in \mathbb{Z}_+\right\}}}$$

(2) (a) (ii) $(\underbrace{1\ 0\ 1\ 1}_{B}\ \underbrace{1\ 0\ 0\ 0}_{8}\ \underbrace{1\ 1\ 0\ 0}_{C}\ \underbrace{1\ 0\ 1\ 0}_{A})_2 = \underline{\underline{(B8CA)_{16}}}$

(i) $B8CA_{16} = 10 + 12\cdot 16 + 8\cdot 16^2 + 11\cdot 16^3 = \underline{47306}$

(b) $x = \left\{\frac{-2 \pm \sqrt{4 + 4\cdot 5\cdot 3}}{10}\right. = \left\{\frac{-2 \pm 8}{10}\right. = \begin{cases} -1 \\ 3/5 \end{cases}$

so $\underline{\underline{x = -1 \text{ or } x = 3/5}}$

(c) $\frac{x\sqrt{y} - y\sqrt{x}}{\sqrt{y} - \sqrt{x}} = \frac{-\sqrt{x}\sqrt{y}\,(-\sqrt{x} + \sqrt{y})}{-\sqrt{x} + \sqrt{y}} = \underline{\underline{-\sqrt{xy}}}$

③ (a) False $6 \nmid 2$.

(b) True $6 = 2 \cdot 3$, so $2 \mid 6$.

(c) False e.g. let $n = 6$, $a = 3$, $b = 8$ then $6 \mid 3 \cdot 8$ but $6 \nmid 3$ and $6 \nmid 8$.

(If n were a prime, then the statement would be true).

(d) False E.g. if $\{a, b\} = A$, $\{a, c, d\} = B$ then $|A \cup B| = |\{a, b, c, d\}| = 4$ while

$|A| + |B| = 2 + 3 = 5$.

(e) True $A \times B$ is the set of ordered pairs with first coordinate from A and second coordinate from B. If $|A| = 10$, $|B| = 3$ there are indeed $|A||B| = 30$ such pairs.

(f) False $\emptyset \subseteq \emptyset$ but not $\emptyset \subset \emptyset$.

(g) True

(h) True e.g. $\{\emptyset\}$

④ (a)(i) $7 \mid (a-b)$

(ii) $[0]_7 = \{7x \mid x \in \mathbb{Z}\}$ $[4]_7 = \{7x + 4 \mid x \in \mathbb{Z}\}$

$[1]_7 = \{7x+1 \mid x \in \mathbb{Z}\}$ $[5]_7 = \{7x+5 \mid x \in \mathbb{Z}\}$

$[2]_7 = \{7x+2 \mid x \in \mathbb{Z}\}$ $[6]_7 = \{7x+6 \mid x \in \mathbb{Z}\}$

$[3]_7 = \{7x+3 \mid x \in \mathbb{Z}\}$

(iii)

$\odot$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

omitting $[\cdot]_7$ - brackets

(b) $\underline{13} = \underline{8} + \underline{5}$
$\underline{8} = \underline{5} + \underline{3}$
$\underline{5} = \underline{3} + \underline{2}$
$\underline{3} = \underline{2} + \underline{1}$
so $\gcd(13, 8) = 1$

Using Euclid's algorithm backwards we get

$\underline{1} = \underline{3} - \underline{2} = \underline{3} - (\underline{5} - \underline{3}) = 2 \cdot \underline{3} - \underline{5} = 2 \cdot (\underline{8} - \underline{5}) - \underline{5} = 2 \cdot \underline{8} - 3 \cdot \underline{5} = 2 \cdot 8 - 3(\underline{13} - \underline{8})$

$= 5 \cdot \underline{8} + (-3) \cdot \underline{13}$

So $\underline{1} = 5 \cdot \underline{8} + (-3) \cdot \underline{13} \Rightarrow \underline{2} = 10 \cdot \underline{8} + (-6) \cdot \underline{13}$. Hence let $s = 10$, $t = -6$

(c) By (b) $8 \cdot 10 \equiv 2 \pmod{13}$, so all solutions are in the class $[10]_{13}$:

$[10]_{13} = \{\ldots, -29, -16, -3, 10, 23, 36, \ldots\}$