MA053G Assignment 2 VT08

(1) (a) $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

(b) $$\sum_{n=1}^{200} (2n-1)(2n+1) = \sum_{n=1}^{200}(4n^2-1) = 4\sum_{n=1}^{200} n^2 - \sum_{n=1}^{200} 1 = \frac{4\cdot 200\cdot 201\cdot 401}{6} - 200$$

using formula w. $n = 200$.

$$= \underline{10746600}$$

(c) $a_1 = 0$, $a_{n+1} = 2a_n + n$

$a_2 = 2a_1 + 1 = 2\cdot 0 + 1 = 1$

$a_3 = 2a_2 + 2 = 2\cdot 1 + 2 = 4$

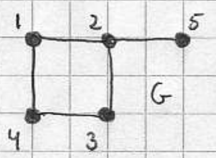
$a_4 = 2a_3 + 3 = 2\cdot 4 + 3 = 11$

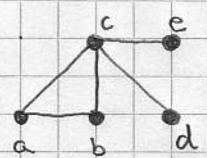
$a_5 = 2a_4 + 4 = 2\cdot 11 + 4 = 26$

$a_6 = 2a_5 + 5 = 2\cdot 26 + 5 = 57$

$a_7 = 2a_6 + 6 = 2\cdot 57 + 6 = 120$

(2) (a)





(b) A graph is bipartite if the vertex set can be partitioned into two non-empty parts X and Y such that all edges in the graph connect a vertex from X with a vertex from Y.

(c) Yes, G is bipartite. Let $X = \{1, 3, 5\}$ and $Y = \{2, 4\}$ then all edges are incident with one vertex from X and one vertex from Y.

(d) Two graphs G_1 and G_2 are isomorphic if there is a bijection $\varphi: V(G_1) \to V(G_2)$ between the vertices of G_1 and the vertices of G_2 such that if $v_i v_j$ is an edge of G_1, then $\varphi(v_i)\varphi(v_j)$ is an edge of G_2.

(e) G and H are not isomorphic. E.g. G has a vertex of degree 3, H has no vertex of degree 3.

③ (a) (i) $5 \cdot 4 \cdot 3 = \underline{\underline{60}}$

(ii) $3 \cdot 4 \cdot 3 = \underline{\underline{36}}$

(iii) $3 \cdot 4 \cdot 3 = \underline{\underline{36}}$

(iv) $3 \cdot 2 \cdot 3 = \underline{\underline{18}}$

(v) Using the Principle of inclusion-exclusion: $36 + 36 - 18 = \underline{\underline{54}}$

(b) $\{2n+3 \mid n \in \mathbb{Z},\ 0 \leq n \leq 50\} = \{2\cdot0+3, 2\cdot1+3, 2\cdot2+3, \ldots, 2\cdot50+3\} = \{3,5,7,9,11,\ldots,103\}$.

The set has 51 elements, so it has $\binom{51}{3}$ subsets of size 3.

$$\binom{51}{3} = \frac{51\cdot50\cdot49}{3\cdot2\cdot1} = \underline{\underline{20825}}$$

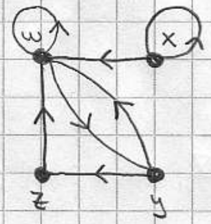
④ (a) R is reflexive if $(a,a) \in R$ for all $a \in S$.

R is symmetric if whenever $(a,b) \in R$ then $(b,a) \in R$ also, for all $a,b \in S$

R is transitive if whenever $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ also, for all $a,b,c \in S$.

R is antisymmetric if whenever $(a,b) \in R$ and $(b,a) \in R$ then $a=b$, for all $a,b \in S$.

(b) (i)



(ii) $\underline{\underline{\{(w,x), (w,z), (z,y)\}}}$

(iii) $\underline{\underline{\{(z,z), (y,y)\}}}$.

(iv) $\underline{\underline{\{(x,y), (x,z), (w,z), (y,y), (z,z), (z,y)\}}}$.

$(x,w), (w,y) \in R \Rightarrow (x,y) \in R$
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(v) No, for $(w,y), (y,w) \in R$ and $y \neq w$.

5 (a) (i) The domain for f is $\{a,b,c,d\}$

The codomain for f (målmgd) is $\{x,y,z,w\}$

The range of f (värdemgd) is $\{x,z,w\}$

(ii) f is not surjective because the codomain and range are not identical.

f is not injective because $f(a) = z = f(c)$, but $a \neq c$.

(b) (i) g is invertible because it is both injective and surjective, the graph of g is a straight line without 'gaps'.

(ii) $y = 5x - 3$

$\Downarrow$

$5x = y + 3$

$\Downarrow$

$x = \frac{y+3}{5}$

so $g^{-1}: \mathbb{R} \to \mathbb{R}$ is given by the rule $g^{-1}(x) = \frac{x+3}{5}$

6 (a)

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

(b) (i) $p \to q$ is true

(ii) $\neg q \to \neg p$ is the same as $p \to q$ and is thus true.

(iii) $q \to p$ is false e.g. $\gcd(62, 10) = 2$ but $62 \ngeq 10$.

(c)

p	q	$\neg q$	$\neg p$	$\neg q \to \neg p$	$q \vee \neg p$
0	0	1	1	1	1
0	1	0	1	1	1
1	0	1	0	0	0
1	1	0	0	1	1

so since the columns for $\neg q \to \neg p$ and $q \vee \neg p$ are the same, they are logically equivalent.