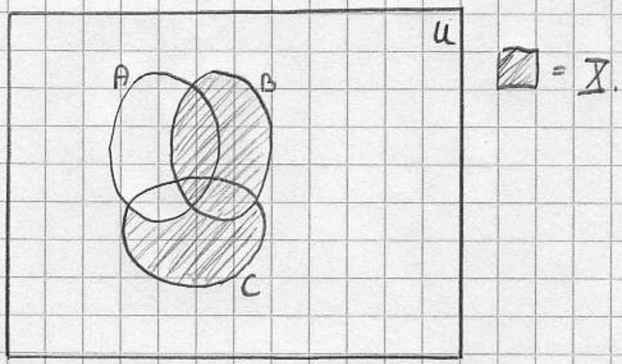
MA053G Diskret matematik för yrkeshögskoleutbildning 5 juni 2008 - svar.

1 (a)



(b) $\{3m-2 : m \in \mathbb{Z}, -2 \le m \le 3\} = \{-8, -5, -2, 1, 4, 7\}$

$\{2n^3 : n \in \mathbb{Z}, -2 \le n \le 3\} = \{-16, -2, 0, 2, 16, 54\}$.

(c) $\{\frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \dots\} = \{(-1)^{n+1}/n+1 : n \in \mathbb{Z}_+\}$. Note that the answer here is not unique.

2 (a) $t = (\underbrace{1\,0\,1\,0}_{10_{10}}\ \underbrace{1\,1\,0\,1}_{13_{10}}\ \underbrace{0\,1\,0\,0}_{4_{10}})_2$

$= (A\ D\ 4)_{16}$

(b) $x = \dfrac{-17 \pm \sqrt{17^2 - 4\cdot 2\cdot 21}}{42} = \begin{cases} -1/7 \\ -2/3 \end{cases}$

so solutions are $x = -\frac{1}{7}$, $x = -\frac{2}{3}$

(c) $3\cdot 3^{2x+y} / 9^{x-1} = 3^{2x+y+1} / 3^{2x-2} = 3^{2x+y+1-2x+2} = 3^{y+3}$

③ (a) $\sum_{n=5}^{8} \frac{1}{n} = \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

$$= \frac{6\cdot7\cdot8 + 5\cdot7\cdot8 + 5\cdot6\cdot8 + 5\cdot6\cdot7}{5\cdot6\cdot7\cdot8}$$

$$= 533/840$$

(b) $S = \sum_{i=17}^{1000}(2i+1) = \sum_{i=1}^{1000}(2i+1) - \sum_{i=1}^{16}(2i+1)$

$$= 2\sum_{i=1}^{1000} i + 1000 - 2\sum_{i=1}^{16} i - 16$$

$$= \frac{\cancel{2}\cdot1000\cdot1001}{\cancel{2}} + 1000 - \frac{2\cdot16\cdot17}{2} - 16$$

$$= 1001000 + 1000 - 16\cdot17 - 16$$

$$= 1002000 - 16\cdot18 = 1002000 - 288 = \underline{\underline{1001712}}$$

(c) $a_0 = 0$

$$a_1 = a_0 - \frac{1}{(1-0)(3-0)} = 0 - \frac{1}{1\cdot3} = -\frac{1}{3}$$

$$a_2 = a_1 - \frac{1}{(1-2)(3-2)} = -\frac{1}{3} - \frac{1}{(-1)(1)} = -\frac{1}{3} + 1 = \frac{2}{3}$$

$$a_3 = a_2 - \frac{1}{(1-4)(3-4)} = \frac{2}{3} - \frac{1}{(-3)(-1)} = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$$a_4 = a_3 - \frac{1}{(1-6)(3-6)} = \frac{1}{3} - \frac{1}{(-5)(-3)} = \frac{1}{3} - \frac{1}{15} = \frac{5-1}{15} = \frac{4}{15}$$

$$a_5 = a_4 - \frac{1}{(1-8)(3-8)} = \frac{4}{15} - \frac{1}{(-7)(-5)} = \frac{4}{15} - \frac{1}{35} = \frac{4\cdot7-3}{3\cdot5\cdot7} = \frac{25}{3\cdot5\cdot7} = \frac{5}{21}$$

$$a_6 = a_5 - \frac{1}{(1-10)(3-10)} = \frac{5}{21} - \frac{1}{(-9)(-7)} = \frac{5}{21} - \frac{1}{63} = \frac{15-1}{63} = \frac{14}{63} = \frac{2}{9}$$

4 (a) Kruskal's algorithm for finding an MST in a weighted graph w. n vertices:

1. choose an edge of lowest weight and add it to the tree.

2. repeat 1. until you have chosen $n-1$ edges,

subject to the condition that you never choose an edge which creates a cycle.

(b) The graph has 11 vertices, a possible MST by Kruskal would consist of the 10 edges

ab, xg, eh, ec, ck, eb, ik, ed, xa, hf

listed in the order chosen by the algorithm. (Note, answer is not unique!)

(c) $5 \cdot 1 + 4 \cdot 2 + 3 = 16$ is the weight of the MST

5 (a) R is reflexive if $(a,a) \in R$ for all $a \in S$.

R is symmetric if $(a,b) \in R \Rightarrow (b,a) \in R$ for all $a,b \in S$.

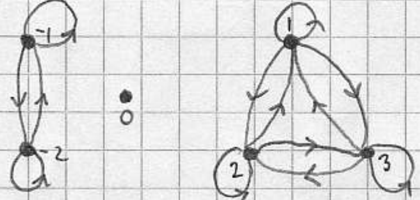
R is transitive if whenever $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$, for all $a,b,c \in S$.

R is antisymmetric if $(a,b) \in R$, $(b,a) \in R \Rightarrow a=b$, for all $a,b \in S$.

R is an equivalence relation if it is reflexive, symmetric and transitive.

R is a partial order if it is reflexive, antisymmetric and transitive.

(b) (i)



(ii) $(0,0) \notin R$ so R is not reflexive, hence it is not a partial order

(iii) as R is not reflexive, it is not an equivalence relation either.

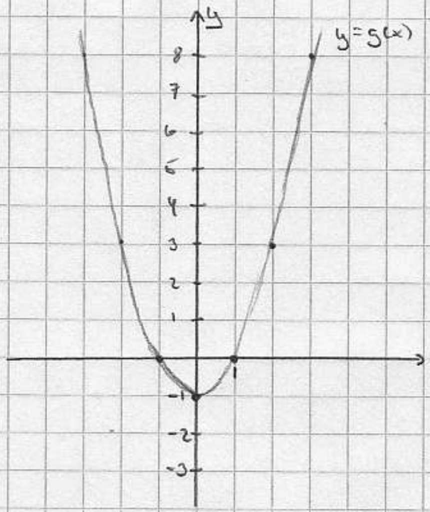
(6) (a) (i) $D_f = A$, The codomain $= \mathbb{Z}$, the range is $\{2, 7, 12, 17, 22\}$.

(ii) f is not surjective as $\mathbb{Z} \neq \{2, 7, 12, 17, 22\}$ so the codomain and the range of f are not equal.

(iii) f is injective as $x \neq y \Rightarrow f(x) \neq f(y)$. This can be seen from the following table:

x	1	2	3	4	5
$f(x)$	2	7	12	17	22

(b)



g is not injective as e.g. $g(1) = g(-1)$ while $1 \neq -1$.

(7) (a)

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$(\neg p) \vee (\neg q)$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

as the two columns for $\neg(p \wedge q)$ and $(\neg p) \vee (\neg q)$ are identical, we have $\neg(p \wedge q) \Leftrightarrow (\neg p) \vee (\neg q)$.

(b) $p \Rightarrow q$ is true for all values of n

$q \Rightarrow p$ is false, as $n = 10$ makes q true and p false.

(c) (i) $x = 10$, $y = 10$, $z = 10$

$r: x < 10$ is false so $\neg r$ is true

$s: y \leq 5$ is false

$t: 2z > 10$ is true

$\neg r$ is true, so $(\neg r) \vee (s \wedge t)$ is true

(ii) $x = 5$, $y = 5$, $z = 5$

$r: x < 10$ is true so $\neg r$ is false

$s: y \leq 5$ is true

$t: 2z > 10$ is false

so $s \wedge t$ is false

hence $(\neg r) \vee (s \wedge t)$ is false

(8) (a) $a \equiv b \pmod{13}$ means that $13 \mid (a-b)$

$[2]_{13} = \{13x+2 \mid x \in \mathbb{Z}\}$.

$[5]_{13} \odot [7]_{13} = [35]_{13} = [9]_{13}$

(b) $114 = 72 + 42$

$72 = 42 + 30$

$42 = 30 + 12$

$30 = 12 \cdot 2 + 6$ so $\gcd(114, 72) = 6$

$12 = 6 \cdot 2 + 0$

(c) Using Euclid's algorithm backwards

$6 = 30 - 12 \cdot 2 = 30 - [42 - 30] \cdot 2 = 30 \cdot 3 + 42(-2) = [72 - 42] \cdot 3 + 42(-2) = 72 \cdot 3 + 42(-5)$

$= 72 \cdot 3 + (114 - 72)(-5) = 72 \cdot 8 + 114(-5)$

so e.g. $s = 8$, $t = -5$

(d) As $6 \nmid 2$ this congruence has no solutions

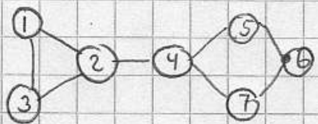
(e) As $6 \mid 6$ and $\gcd(72, 114) = 6$

this congruence has the same solution as $12x \equiv 1 \pmod{19}$ which has a unique congruence class of solutions in $\mathbb{Z}_{19}$. To find this class:

By (c) $6 = 72 \cdot 8 + 114 \cdot (-5)$, so $1 = 12 \cdot 8 + 19 \cdot (-5)$ and thus when $x \equiv 8 \pmod{19}$ solves

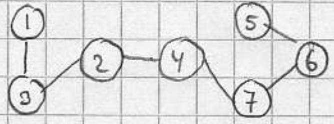
So $x = 19n + 8 \quad n \in \mathbb{Z}$

9

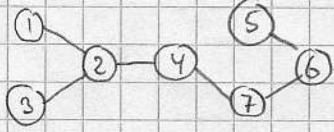
(a) 

(b) A graph is a tree if it is connected and has no cycles.

(c) Two graphs H_1 and H_2 are isomorphic if there exists a bijective function f from the set of vertices of H_1 to the set of vertices of H_2 such that whenever vertices v and w are adjacent in H_1 then $f(v)$ and $f(w)$ are adjacent in H_2

(d) 

is not isomorphic to the other two as it has no vertex of degree 3.



is not isomorphic to the tree below as the vertex of degree 3 has neighbours of degree 1, 1, 2 while in the graph below the vertex of degree 3 has neighbours of degree 1, 2, 2.