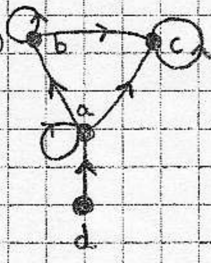
MAAA26 Assignment 12

(1) (a) (i) R is reflexive if aRa for all $a \in S$.

(ii) R is symmetric if whenever aRb then bRa also, for all $a,b \in S$.

(iii) R is transitive if whenever aRb and bRc then aRc also, for all $a,b,c \in S$.

(iv) R is antisymmetric if aRb and $bRa \Rightarrow a=b$, for all $a,b \in S$

(b) (i) 

(ii) To make R symmetric, we must make all "single-arrows" to "double-arrows", i.e. we must add the pairs

$$\{(b,a),(c,b),(c,a),(a,d)\}$$

(iii) To make R reflexive, we must make sure that all vertices in the graph have a loop, i.e. we must add the pair $\{(d,d)\}$

(iv) To make R transitive, we must assure that whenever there is a path $x \longrightarrow y \longrightarrow z$ in the graph, then there is an arc $x \longrightarrow z$ also. Hence we must add the pairs

$$\{(d,b),(d,c)\}.$$

(v) Yes, R is anti-symmetric, for there are no "double-arrows"  in the graph apart from when $x=y$.

(2) $A=\{0,1,2,\dots,9\}$ and $f: A \to \mathbb{Z}$ is the function given by the rule $f(x)=8x-5$

Thus f is given by the table

x	0	1	2	3	4	5	6	7	8	9
f(x)	-5	3	11	19	27	35	43	51	59	67

From the table we see that if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$ for all $x_1, x_2 \in A$

this shows that f is injective

From the table we see that the range (värdemängden) of f is

$$\{-5, 3, 11, 19, 27, 35, 43, 51, 59, 67\}$$

However, the codomain of f is $\mathbb{Z}$, so f is not surjective and thus not invertible.