OMY ASSIGNMENT 2 — SPRING 2007

1. (a) $x^2-9 = 0 \iff x^2 = 9 \iff x = \pm\sqrt{9} \iff \underline{x = 3 \text{ or } x = -3}$

(b) $x^2+9 = 0 \iff x^2 = -9 \iff \underline{\text{no solutions } x \in \mathbb{R}}$,

(c) $x^2-9 = 3 \iff x^2 = 12 \iff x = \pm\sqrt{12} = \pm 2\sqrt{3} \iff \underline{x = 2\sqrt{3} \text{ or } x = -2\sqrt{3}}$

(d) $2x^2+3x+1 = 0 \iff x = \dfrac{-3 \pm \sqrt{9-4\cdot 2\cdot 1}}{4} = \dfrac{-3 \pm 1}{4} \iff \underline{x = -1 \text{ or } x = -\tfrac{1}{2}}$

2. (a) $$\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = \frac{\left(\sqrt{1+x}+\sqrt{1-x}\right)^2}{\left(\sqrt{1+x}-\sqrt{1-x}\right)\left(\sqrt{1+x}+\sqrt{1-x}\right)} = \frac{(1+x)+(1-x)+2\sqrt{(1+x)(1-x)}}{(1+x)-(1-x)}$$

$$= \frac{2+2\sqrt{1-x^2}}{2x} = \frac{1+\sqrt{1-x^2}}{x}$$

So for $x = 3/5$ we get $$\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = \frac{1+\sqrt{1-x^2}}{x} = \frac{1+\sqrt{1-\left(\frac{3}{5}\right)^2}}{3/5}$$

$$= \frac{1+\sqrt{\frac{25}{25}-\frac{9}{25}}}{3/5}$$

$$= \frac{1+4/5}{3/5} = \frac{9/5}{3/5} = \frac{9}{5}\cdot\frac{5}{3} = \underline{3}$$

(b) $f(x) = x - \frac{1}{x}$ and $g(x) = 3x^2+1$

So $f(x)+g(x) = x - \frac{1}{x} + 3x^2 + 1 = \underline{3x^2 + x - \frac{1}{x} + 1} = (3x^3+x^2+x-1)/x$

$f(x)\cdot g(x) = \left(x-\frac{1}{x}\right)(3x^2+1) = 3x^3 - 3x + x - \frac{1}{x} = \underline{3x^3 - 2x - \frac{1}{x}} = (3x^4-2x^2-1)/x$

$f(x)/g(x) = \left(x-\frac{1}{x}\right)/3x^2+1 = (x^2-1)/x(3x^2+1)$

$$= \frac{x^2-1}{3x^2+x}$$

(c) $$\frac{(6a+b)^2-9a^2}{(6a+b)^2-b^2} = \frac{\left((6a+b)+3a\right)\left((6a+b)-3a\right)}{\left((6a+b)+b\right)\left((6a+b)-b\right)} = \frac{(9a+b)(3a+b)}{(6a+2b)(6a)}$$

$$= \frac{9a+b}{12a}$$

(d) $$\frac{a\sqrt{b}+b\sqrt{a}}{\sqrt{a}+\sqrt{b}} = \frac{\sqrt{ab}\left(\sqrt{a}+\sqrt{b}\right)}{\sqrt{a}+\sqrt{b}} = \underline{\sqrt{ab}}$$