

DHY ASSIGNMENT 3 - SPRING 2007

Question 1

$$A = \{a, b, c\} \quad B = \{a, b, g\} \quad C = \{c, d, e, f\} \quad G = \{a, b, c, d, e, f, g\}$$

(a) (i) TRUE, as $B \cup C = \{a, b, g\} \cup \{c, d, e, f\} = \{a, b, c, d, e, f, g\} = G$

(ii) FALSE, as $A \cap B = \{a, b, c\} \cap \{a, b, g\} = \{a, b\} \neq G$.

(iii) FALSE as $G \setminus A = \{a, b, c, d, e, f, g\} \setminus \{a, b, c\} = \{d, e, f, g\} \neq C$

(iv) FALSE, as $G \setminus B = \{a, b, c, d, e, f, g\} \setminus \{a, b, g\} = \{c, d, e, f\} = C$

(v) TRUE, see (iv)

(vi) TRUE, as $\{a, b, b\} = \{a, b\} \in A$.

(vii) FALSE as $g \in B$ but $g \notin \{a, b, \{g\}\}$.

(viii) FALSE, $b \in B$, so $\{b\} \subseteq B$, $\{b\} \in \mathcal{P}(B)$, $\{\{b\}\} \subseteq \mathcal{P}(B)$

(ix) FALSE, see (viii)

(x) TRUE, see (viii).

(b) (i) $S_1 = \{2\}$ as $-2 \notin \mathbb{N}$.

(ii) $S_2 = \{\dots, -9, -6, -3, 0, 3, 6, 9, \dots\}$

(iii) $S_3 = \emptyset = \{\}$ as $x^2 + 4 = 0$ has no solutions $x \in \mathbb{R}$.

(iv) $S_4 = \{-2, 2\}$

(c) (i) $\{-7, 7, -8, 8, -9, 9, -10, 10\} = \underline{\{n \in \mathbb{Z} \mid 7 \leq |n| \leq 10\}}$

(ii) $\{0, -2, 2, -4, 4, \dots\} = \underline{\{2n \mid n \in \mathbb{Z}\}}$

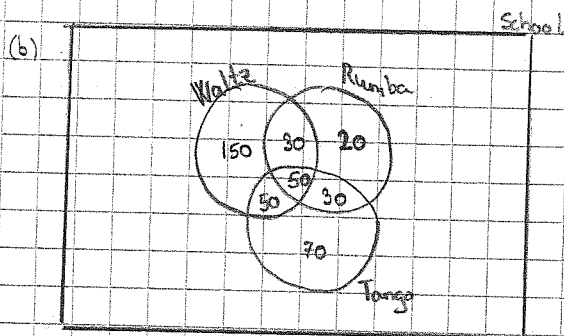
(iii) $\{1, 2, 4, 8, 16, 32, \dots\} = \underline{\{2^n \mid n \in \mathbb{N}\}}$.

Question 2

$X = (B \cup C) \setminus (A \cap C)$ (note: the brackets are necessary here!)

Question 3

(a) $250 \text{ girls} \times 250 \text{ boys} = \underline{62500 \text{ couples}}$



By the 'Venn diagram' the number of students doing at least one of the three dances is

$$150 + 30 + 20 + 50 + 50 + 30 + 70 = 400$$

and we know the school has 500 students

(i) So 100 students are not doing any of the three dances.

(ii) 200 students are doing the tango, but of these $50 + 50 + 30$ are also doing either waltz or rumba, so $200 - (50 + 50 + 30) = \underline{70}$ are doing tango only.

(iii) The number of students doing precisely two dances is

$$50 + 30 + 30 = \underline{110}$$

as 50 learn just waltz and tango,

30 learn just waltz and rumba, and

30 learn just rumba and tango.