Revision

- Read through the whole course once
- Make summary sheets of important definitions and results,
  you can use the following pages as a start and fill in more yourself
- Do the assignments again
- Do the model examination paper
- If you have more time, do some more exercises:
  A number of old exam questions, some with solutions, will available in pdf-format via the course website, and you can also do more exercises from the book.

Section A - Sets and numbers

Main Topics in Section A

- Sets
  - Sets and subsets. The powerset of a set.
  - Using rules of inclusion (set builder notation) and Venn diagrams to represent sets.
  - Complement, union, intersection and difference of sets. Representation of these operations in Venn diagrams.
  - Laws of set operations. Associative laws, distributive laws, De Morgan's laws. Verifying these using Venn diagrams.
  - The sets $\mathbb{Z}_+$, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$
  - Intervals
- Absolute Values
- Inequalities

- Equations
  - Polynomial equations
    - Linear equations
      - Drawing the graph of a linear polynomial function
      - Finding the linear polynomial function corresponding to a given line
    - Quadratic equations, the method of completing the square
    - Drawing the graph of a quadratic
    - Higher order polynomial equations
      - The Rational Root Test
      - The Factor Theorem

  - Equations with roots, exponential functions and logarithms
    - Checking for fake roots

  - Trigonometric equations
    - Trigonometric identities

Section B - Complex Numbers

Main Topics in Section B

- The set of complex numbers $\mathbb{C}$
  - Definition, arithmetic (sums, products, quotients) with complex numbers. Geometric interpretation of multiplication in $\mathbb{C}$
  - Argument, absolute value, complex conjugate of a complex number
  - The Cartesian (sv:rektangulär) form $a + ib$
  - The polar form $r\cos(\theta) + ir\sin(\theta)$
  - The exponential form $re^{i\theta}$

- The Fundamental Theorem of Algebra (without proof) and how to find the two complex roots of any complex quadratic equation.

- The Factor Theorem.
- In any polynomial equation with real coefficients, if a complex number z_0 is a root, so is its conjugate $\overline{z_0}$.

- Powers and roots of complex numbers, De Moivre's Theorem
  - In particular the n complex nth roots of unity

Section C - Functions

Main Topics in Section C

- Functions and their domains, range and graphs.
- Some elementary functions, their domains and ranges and their graphs:
  - Polynomial functions
  - Rational functions
    - Partial fractions (sv:partialbråksuppdelning)
  - Power functions
  - Exponential and logarithmic functions
    - In particular $\exp(x)$, $y = 2^x$, $\ln(x)$ and $\log_2(x)$
    - The laws of powers and logarithms
  - Trigonometric functions
    - The unit circle and how to find $\sin x$ and $\cos x$ on it. Degrees and radians.
- Algebra of functions
- Composing functions
- Invertible functions, how to show that a function is invertible by checking that it is one-to-one (sv:injektiv) and onto (sv:surektiv)
- Important Theorem: Strictly increasing and strictly decreasing functions are invertible

Section D - Sequences, Sums and Σ-notation

Main Topics in Section D

- Sequences
  - Definition by general term
  - Definition by initial term and recurrence relation
- Arithmetic and geometric sequences and sums
- Expressing and manipulating sums in Sigma notation
- Three useful sums:

$$\sum_{r=1}^{n} 1 = n, \qquad \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

- The binomial theorem (without proof):

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r, \text{ where } \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

- Some logic and proof
  - Why fake roots occur
  - Proof by induction (not on the tenta HT08)

Section E - Limits and Continuity

Main Topics in Section E

- The definition of limit, limit from the left and limit from the right
- An important theorem:
  The double-sided limit $\lim_{x \to a} f(x)$ exists
  if and only if both limits
  $\lim_{x \to a+} f(x)$ **and** $\lim_{x \to a-} f(x)$ exist **and** they are equal.
- The definition of continuity
- An important theorem: f is continuous at $x = a$ if and only if $\lim_{x \to a+} f(x) = \lim_{x \to a-} f(x) = f(a)$
- Rules for calculating finite and infinite limits to a, limits to $\pm\infty$
  - Remember NEVER to do arithmetic with $\pm\infty$. Writing any expression like "$\frac{\infty}{\infty}$" or "$\frac{0}{\infty}$" results in a zero mark for the exercise in which you wrote it
  - Sandwich Theorem
  - Limits of sums, products and quotients
  - Limits of rational functions:

$$\lim_{x \to \infty} \frac{p(x)}{q(x)} = \begin{cases} 0 & \text{if deg } p < \text{deg } q \\ \pm\infty & \text{if deg } p > \text{deg } q \\ \frac{\text{(lead coef p)}}{\text{(lead coef q)}} & \text{if deg } p = \text{deg } q \end{cases}$$

- Some standard limits without proof, e.g.
  - $\lim_{x\to 0} \frac{\exp x - 1}{x} = 1,$
  - $\lim_{x\to 0} \frac{\sin x}{x} = 1,$
  - $\lim_{x\to 0} \frac{1-\cos x}{x} = 0,$
  - $\lim_{x\to 0} \frac{1-\cos x}{x^2} = 1/2.$

- Hint for limit calculations: any quotient containing a denominator like $(\sqrt{x} + a)$ may become easier if prolonged by $(\sqrt{x} - a)$ to obtain $(x - a^2)$.

- Theorems concerning continuous functions, e.g. the Intermediate Value Theorem, Bolzano's Theorem (Sats 8.2, 8.3 in [RS])

- Limits of composite functions: Substitutionssatsen (Sats 8.1 p. 243 in [RS])

- Functions covered on this course are contiunous nearly everywhere in their domains, there are only few points at which they are not defined or have 'jumps'.

Section F - Differentiation

Main Topics in Section F

- The definition (and geometric interpretation) of the derivative $f'(x)$ in terms of limits.

- The definition of differentiability.

- An important theorem: Differentiable functions are continuous.

- The various notations for the derivative $f'(x)$

- Differentiation from first principles

- Differentiation of sums, products and quotients of functions

- Differentiation of composite functions: the Chain Rule

- An important derivative without proof:

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$