MA055G & MA056G
Introduktionskurs i matematik

Lecture Notes 6

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VECTORS IN 2-SPACE

Definition

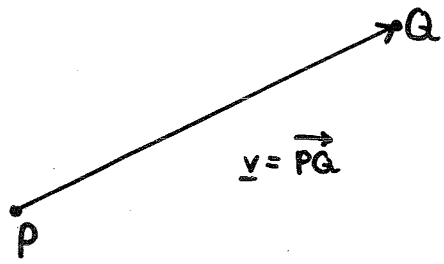
We call a 2-dimensional space such as a plane 2-space

Definition

A vector is a directed line segment.

Each vector has a length and a direction.

Given two points P and Q in 2-space, we denote the vector which starts at P and ends at Q by $\overrightarrow{PQ}$.

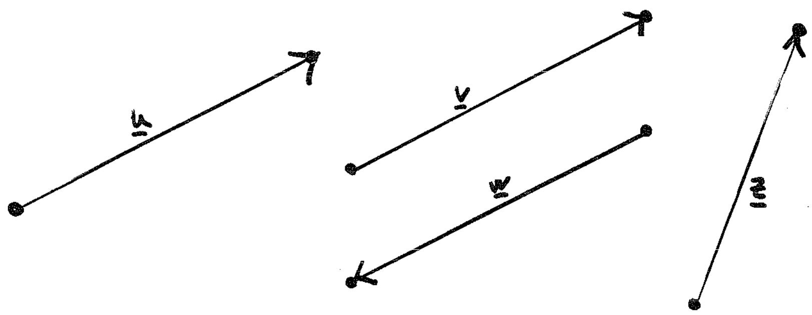


NOTE

We consider two vectors with the same length and the same direction as being equal, even if they lie in different positions in 2-space or 3-space.

EXAMPLE

Consider the following vectors $\underline{u}, \underline{v}, \underline{w}$ and $\underline{z}$ in 2-space.



Here $\underline{u} = \underline{v}$ as $\underline{u}$ and $\underline{v}$ have the same length and point in the same direction.

While $\underline{u} \neq \underline{w}$ and $\underline{u} \neq \underline{z}$.

COORDINATES FOR VECTORS

We usually have a coordinate system for 2-space with origin O and two perpendicular lines which we call the x-axis and the y-axis.

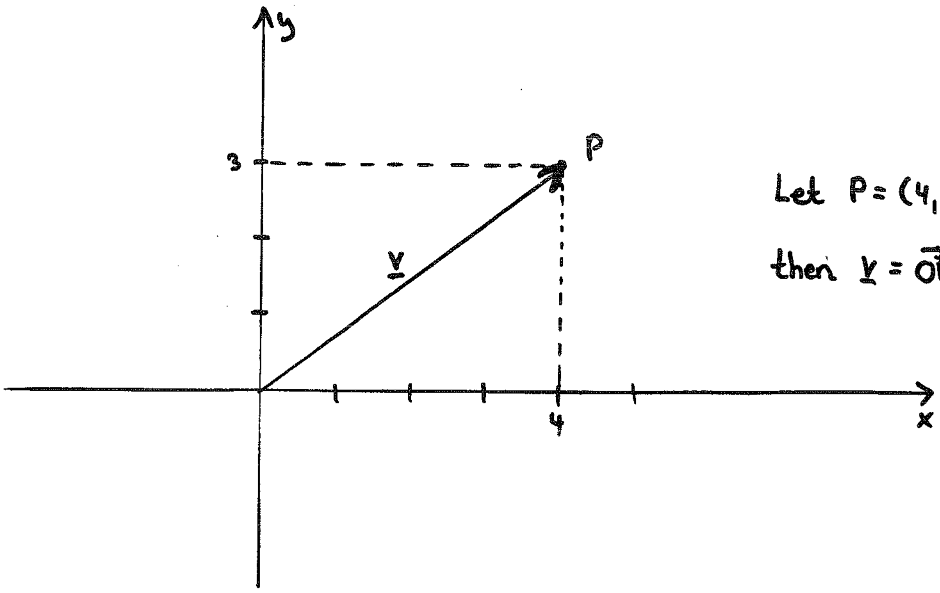
Any point P is represented by an ordered pair (p_x, p_y) where p_x is the x-coordinate of P and p_y is the y-coordinate of P.

Given any vector $\underline{v}$ in 2-space, we can draw $\underline{v}$ with starting point at the origin O. If $\underline{v} = \overrightarrow{OP}$ and P has coordinates (v_x, v_y), we shall also use the ordered pair (v_x, v_y) to represent $\underline{v}$.

(v_x, v_y) is called the <u>coordinates (or components) of vector $\underline{v}$</u>.

We shall write $\underline{v} = (v_x, v_y) = \overrightarrow{OP}$

We shall also refer to $\overrightarrow{OP}$ as the position vector for the point P.



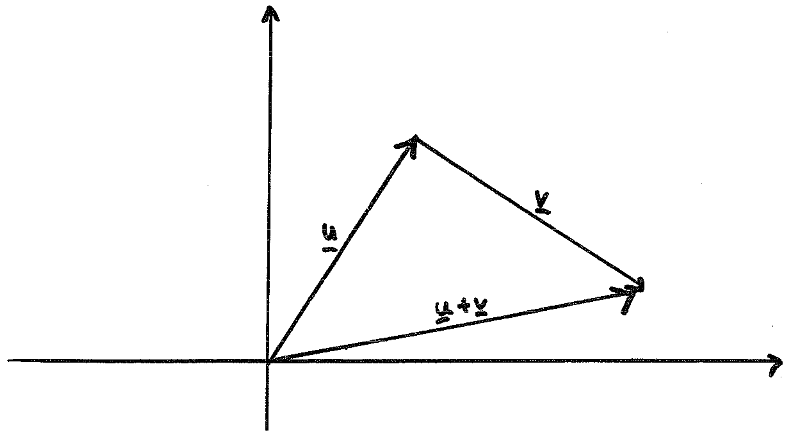
Let $P = (4,3)$,

then $\underline{v} = \overrightarrow{OP} = (4,3)$.

SUMS, DIFFERENCES AND SCALAR MULTIPLICATION

Let $\underline{u}$ and $\underline{v}$ be vectors. Then the sum $\underline{u}+\underline{v}$ is the vector obtained by the following construction:

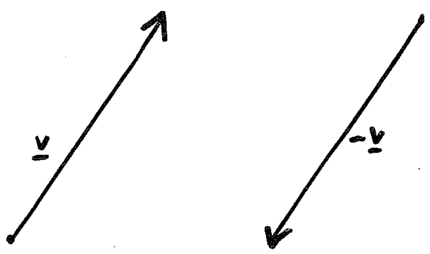
- Draw $\underline{u}$ and $\underline{v}$ with $\underline{v}$ starting at $\underline{u}$'s end point.
- Then $\underline{u}+\underline{v}$ is the vector given by the line segment from the starting point of $\underline{u}$ to the end point of $\underline{v}$.



If $\underline{u}$ has coordinates (u_x, u_y) and $\underline{v}$ has coordinates (v_x, v_y) then $\underline{u}+\underline{v}$ has coordinates (u_x+v_x, u_y+v_y).

Definition

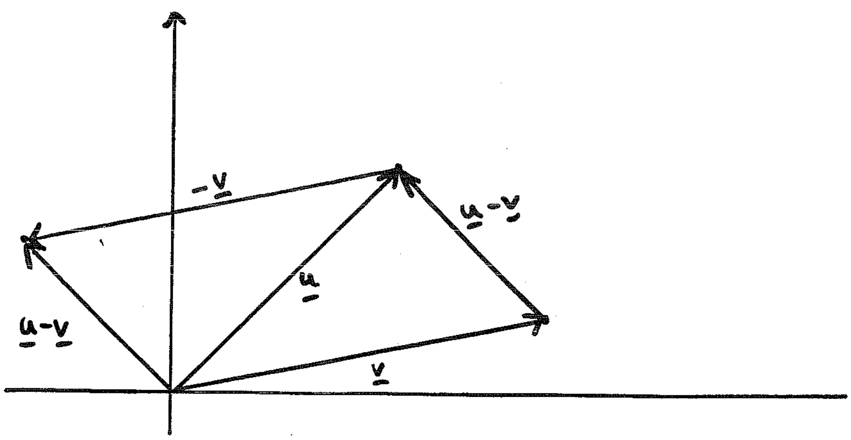
Let $\underline{v}$ be a vector. Then the inverse of $\underline{v}$ (also called the negative of $\underline{v}$) is the vector with the same length as $\underline{v}$, but with opposite direction. We denote it by $-\underline{v}$



If $\underline{v}$ has coordinates (v_x, v_y) then $-\underline{v}$ has coordinates $(-v_x, -v_y)$.

The difference $\underline{u}-\underline{v}$ is defined as $\underline{u}+(-\underline{v})$

Geometrically we can construct $\underline{u}-\underline{v}$ as follows:



If $\underline{u}$ has coordinates (u_x, u_y) and $\underline{v}$ has coordinates (v_x, v_y) then $\underline{u}-\underline{v}$ has coordinates $(u_x - v_x, u_y - v_y)$.

Definition

Let $\underline{v}$ be a vector and $k \in \mathbb{R}$ be a number (called a scalar). Then $k\underline{v}$ is the vector defined as follows:

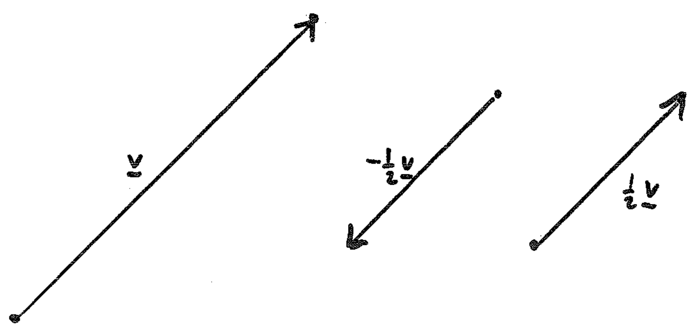
- If $k \geq 0$ then $k\underline{v}$ has the same direction as $\underline{v}$ and length k times the length of $\underline{v}$.

- If $k \leq 0$ then $k\underline{v}$ has the opposite direction of $\underline{v}$ and length $|k|$ times the length of $\underline{v}$

This operation is known as scalar multiplication of vectors.

If $\underline{v}$ has coordinates (v_x, v_y) then $k\underline{v}$ has coordinates (kv_x, kv_y).

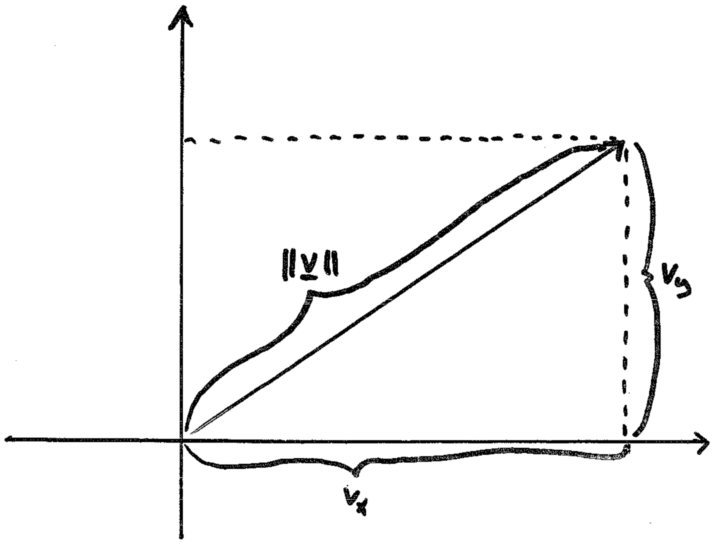
Example



LENGTH OF VECTORS

We shall use $\|\underline{v}\|$ to denote the length (also called norm) of a vector $\underline{v}$

If $\underline{v} = (v_x, v_y)$ then $\underline{\underline{\|\underline{v}\| = \sqrt{v_x^2 + v_y^2}}}$

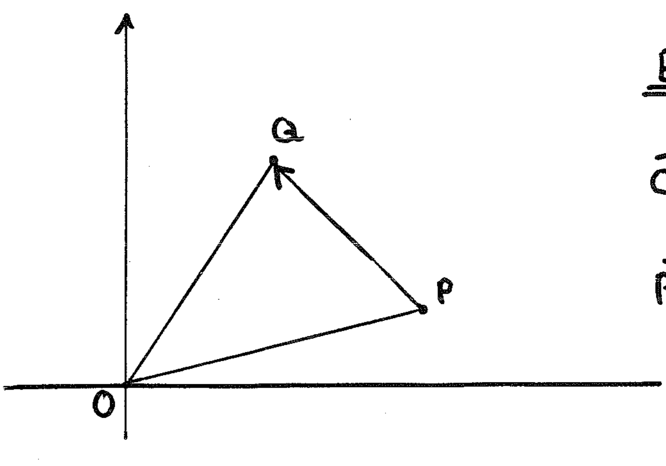


Distances between points in 2-space (sv: avståndsformeln)

Suppose that $P(p_1, p_2)$ and $Q(q_1, q_2)$ are two points.

Then $\overrightarrow{PQ} = (q_1 - p_1, q_2 - p_2)$, and the distance between P and Q is

$$\|\overrightarrow{PQ}\| = \sqrt{(q_1-p_1)^2 + (q_2-p_2)^2}$$



Example

$\overrightarrow{OP} = (4,1) \qquad \overrightarrow{OQ} = (2,3)$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (2,3) - (4,1) = (-2,2)$

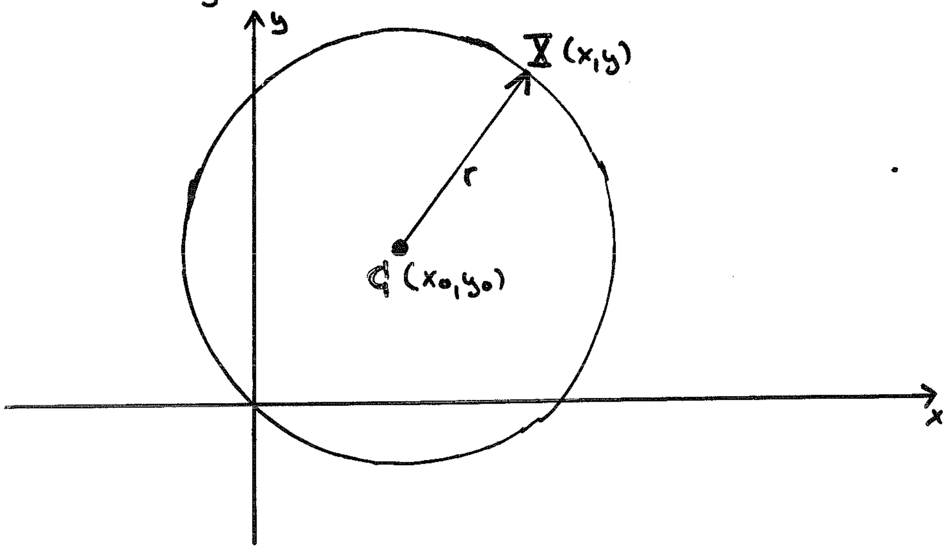
$\|\overrightarrow{PQ}\| = \sqrt{(-2)^2 + (2)^2} = \sqrt{8} = \underline{\underline{2\sqrt{2}}}$

CIRCLES

The equation of a circle with centre at $C = (x_0, y_0)$ and radius r is

$$(x-x_0)^2 + (y-y_0)^2 = r^2$$

Such an equation describes a curve in the coordinate system which is NOT the graph of a function (as to some x-values there are two y-values)



We can describe a point $X = (x, y)$ on the circle as a point having $\|\vec{CX}\| = r$, i.e. a point such that $\|\vec{OX} - \vec{OC}\| = r$

Using coordinates, this becomes

$$\sqrt{(x-x_0)^2 + (y-y_0)^2} = r,$$

hence the equation for the circle is

$$(x-x_0)^2 + (y-y_0)^2 = r^2$$