Nonlinear evolution equations: Bäcklund transformations and recursion operators.

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Abstract

The important role played by Bäcklund transformations in investigating nonlinear evolution equations is well known (see, for instance, [16, 17]) and twofold. Indeed, on one hand, via Bäcklund transformations solutions to initial boundary value problems can be obtained, (as an example, see [12, 13]) and, on other hand, properties they enjoy, such as the existence of a recursion operator and Hamiltonian/biHamiltonian structure can be revealed. Here, the attention is focussed on nonlinear evolution equations which admit a (hereditary) recursion operator. Bäcklund Charts connecting hierarchies of nonlinear evolution equations such as the Korteweg deVries, modified Korteweg deVries and Dym equations are well known [11]. Links among the related recursion operators are expressed via Bäcklund transformations. Here recent results, obtained in the framework of a joint research project with C. Schiebold, are presented. Specifically, referring to the approach proposed by Aden and Carl [1] and further developed by Carl and Schiebold [7, 8, 18], nonlinear evolution equations in which the unknown function is an element of a Banach space are considered and termed operator nonlinear evolution equations. Note that, in this noncommutative setting, a special case are matrix nonlinear evolution equations. Recursion operator are obtained in the case of Burgers [3] and of Korteweg deVries, modified Korteweg deVries equations [4] and, hence the corresponding hierarchies are generated. In addition, the related recursion operators are explicitly proved to satisfy the algebraic properties which characterize a recursion operator. Hence, in turn, the Korteweg deVries, modified Korteweg deVries [5], and Burgers [6] noncommutative hierarchies are generated. Notably, the operators obtained in this framework, coincide with the corresponding ones obtained via different approaches by Gürses, Karasu and Sokolov[14] and by Gürses, Karasu and Turhan[15].

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