On Agrawal's conjecture for triple arrays

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A *triple array* is an $r \times c$ array on v symbols arranged so that no symbol occurs more than once in any row or column, and satisfies the following four conditions:

TA1. Each symbol occurs k times (equireplicate).
TA2. Any two distinct rows contain λ_{rr} common symbols.
TA3. Any two distinct columns contain λ_{cc} common symbols.
TA4. Any row and column contain λ_{rc} common symbols.

For a general triple array we use the notation $TA(v, k, \lambda_{rr}, \lambda_{cc}, \lambda_{rc} : r \times c)$.

Agrawal [1] suggested a method for constructing triple arrays from symmetric 2-designs. However, there is one step in the construction that is not proved, giving rise to what is called *Agrawal's conjecture*.

Conjecture 1. *If there is a symmetric* 2-$(v+1, r, \lambda_{cc})$ *design with* $r - \lambda_{cc} > 2$, *then there is a* $TA(v, k, \lambda_{rr}, \lambda_{cc}, \lambda_{rc} : r \times c)$ *with* $v = r + c - 1$.

The converse of Agrawal's conjecture was proven in [3] and many examples have been constructed. However, until now only one infinite family, called *Paley triple arrays*, has been proved to exist. This has been done to different degrees in [6], [2] and [5], and can be summarized as follows.

Theorem 1. *Let* $q \geq 5$ *be an odd prime power. Then there exists a* $q \times (q+1)$ *triple array.*

In this talk we point out some problems around, and approaches to Agrawal's conjecture. We also introduce a new infinite family of triple arrays from our present work [4].

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[2] S. Bagchi: On two-way designs, *Graphs Combin.* **14** (1998), no. 4, 313–319.

[3] J. P. McSorley, N. C. K. Philips, W. D. Wallis, J. L. Yucas: Double arrays, triple arrays and balanced grids, *Des. Codes Cryptogr.* **35** (2005), 21–45.

[4] T. Nilson, P. J. Cameron: Triple arrays from difference sets, *To appear.*

[5] D. A. Preece, W. D. Wallis, J. L. Yucas: Paley triple arrays, *Australas. J. Comb.* **33** (2005), 237–246.

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