SENSOR DEVICES
MECHANICAL SENSORS
OUTLINE

• 4 Mechanical Sensors
  • Introduction
  • General mechanical properties
  • Piezoresistivity
  • Piezoresistive sensors
  • Capacitive sensors
  • Applications
INTRODUCTION

• MECHANICAL SEMICONDUCTOR SENSORS
  • Combine electronic properties of semiconductors with its excellent mechanical properties

• TWO MAJOR CLASSES OF MECHANICAL SENSORS
  • Piezo resistive sensors (material property in silicon)
  • Capacitive sensors (relative motion of electrodes)

• APPLICATIONS
  • Pressure sensors
  • Accelerometer
  • Flow sensors

"Piezo" = "squeeze" or "press"
INTRODUCTION

Piezoresistive sensing applications

Load cell  Accelerometer  Pressure sensor
General mechanical properties

Hooke's law: \[ \sigma = E \varepsilon \]

Stress: \[ \sigma = \frac{F}{\text{Area}}, \quad [\text{N/m}^2] \]

Strain: \[ \varepsilon = \frac{\Delta L}{L} \]

→ Young's module: \( E, \quad [\text{N/m}^2=\text{Pa}] \),

Volumetric change:
\[
\frac{\Delta V}{V} \approx \frac{\Delta L}{L} - \frac{2\Delta L_l}{L} = (1 - 2\nu) \frac{\Delta L}{L}
\]

→ Poisson's ratio: \( \nu = \frac{\Delta L_l}{\Delta L} = -\frac{\varepsilon_l}{\varepsilon}, \quad l = \text{lateral} \)

Silicon: \( E_s \approx 190 \text{ GPa}, \quad \nu_s \approx 0.28 \)
General mechanical properties

- **Silicon**
  - Linear elasticity until fracture
  - No plastic deformation
  - Excellent for sensor applications

- **Metals**
  - Linear elastic behaviour for small strain only

- **Strength**
  - Yield strength = tensile stress where plastic deformation starts (non-reversible)
  - Tensile strength = maximum tensile stress before fracture

*Figure 19.2* Stress-strain curve for a typical metal as well as for a brittle material like silicon (high Young’s modulus and no plastic deformation region) (*reprinted with permission from Madou [7]. Copyright CRC Press*).
## General mechanical properties

<table>
<thead>
<tr>
<th></th>
<th>Yield Strength $(10^9 \text{ Pa})$</th>
<th>Young’s Modulus $(10^9 \text{ Pa})$</th>
<th>Density $(\text{g/cm}^3)$</th>
<th>Thermal Conductivity $(\text{W/cm} \degree \text{C})$</th>
<th>Thermal Expansion $(10^{-6}/\degree \text{C})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamond (single crystal)</td>
<td>53.0</td>
<td>1035.0</td>
<td>3.5</td>
<td>20.0</td>
<td>1.0</td>
</tr>
<tr>
<td>SiC (single crystal)</td>
<td>21.0</td>
<td>700.0</td>
<td>3.2</td>
<td>3.5</td>
<td>3.3</td>
</tr>
<tr>
<td>Si (single crystal)</td>
<td>7.0</td>
<td>190.0</td>
<td>2.3</td>
<td>1.6</td>
<td>2.3</td>
</tr>
<tr>
<td>$\text{Al}_2\text{O}_3$</td>
<td>15.4</td>
<td>530.0</td>
<td>4.0</td>
<td>0.5</td>
<td>5.4</td>
</tr>
<tr>
<td>$\text{Si}_3\text{N}_4$ (single crystal)</td>
<td>14.0</td>
<td>385.0</td>
<td>3.1</td>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>Gold</td>
<td>—</td>
<td>80.0</td>
<td>19.4</td>
<td>3.2</td>
<td>14.3</td>
</tr>
<tr>
<td>Nickel</td>
<td>—</td>
<td>210.0</td>
<td>9.0</td>
<td>0.9</td>
<td>12.8</td>
</tr>
<tr>
<td>Steel</td>
<td>4.2</td>
<td>210.0</td>
<td>7.9</td>
<td>1.0</td>
<td>12.0</td>
</tr>
<tr>
<td>Aluminum</td>
<td>0.2</td>
<td>70.0</td>
<td>2.7</td>
<td>2.4</td>
<td>25.0</td>
</tr>
</tbody>
</table>
Stress in thin film cause a curvature of the sample, which can be measured using a laser system.

\[
\frac{1}{R} = 6(1 - \nu) \frac{\Delta s}{ET^2}
\]

\[\Delta s = \sigma t\]

= surface stress [N/m]

\(R\) = radius
\(E\) = Young's module in substrate
\(T\) = substrate thickness
\(t\) = thin film thickness
\(\nu\) = Poisson’s ratio in substrate
General mechanical properties

– Cantilever beams
  • Max deflection
  • Max longitudinal stress
  • Resonant frequency

– Square membranes
  • Max deflection
  • Max longitudinal and transverse stress
  • Resonant frequency
General mechanical properties

Cantilever beam with uniform distributed load \((P=F/\Delta x)\)

Beam equation: \[ EI \frac{d^4w(x)}{dx^4} = P \]

Beam stiffness: \[ EI = Eat^3/12 \]

\[ I = at^3/12 \quad \text{(2nd moment of inertia)} \]

Deflection: \[ w(x) = \frac{P}{24EI} x^2 \left(6L^2 - 4Lx + x^2\right) \]

\[ w(L) = \frac{PL^4}{8EI} \]

Surface stress: \[ \sigma(x) = -\frac{tE}{2} \frac{d^2w(x)}{dx^2} \]

Max stress: \[ \sigma(0) = \frac{PL^2t}{4l} = \frac{3PL^2}{at^2} \]

SENSOR:

\[ P = at \rho \cdot \text{acceleration} \]

\[ \Delta P = at \rho_{\text{measurand}} \cdot g \]
General mechanical properties

Cantilever beam point load at the end

Deflection: \( w(x) = \frac{Qx^2}{6EI} (3L - x) \)

Max stress: \( \sigma(0) = \frac{QLt}{2I} = \frac{6QL}{at^2} \)

Resonant frequency:

\[
F_0 = 0.161 \frac{t}{L^2} \sqrt{\frac{E}{\rho}} = 0.161 \frac{t}{L} \sqrt{\frac{Eta}{ML}}
\]

Quasi-static sensing

\( f_{\text{measure}} < F_0 \)

SENSOR:

\( Q = \text{mass} \cdot \text{acceleration} \)
General mechanical properties

SQUARE MEMBRANES (UNIFORM LOAD)

Plate equation: \( D \nabla^4 w(x, y) = P \)

Membrane stiffness: \( D = \frac{Et^3}{12(1-\nu^2)} \)

Max deflection: \( w_{\text{max}} = 0.001265 \frac{Pa^4}{D} \)

Max longitud. stress: \( \sigma_l = 0.3081 \frac{Pa^2}{t^2} \)

Max transverse stress: \( \sigma_t \approx \nu \sigma_l \)

Resonant frequency: \( F_0 = 1.654 \sqrt{\frac{12D}{\rho ta^4}} \)
Piezoresistivity

Resistivity change in semiconductor:
- Resistivity change due to mechanical stress
  - piezoresistive effect
- Large resistivity change
- Dependence on
  - Doping (n- or p-type, doping concentration)
  - Temperature
  - Direction of force and direction of current flow
    (anisotropic effect)

\[
R = \rho \frac{L}{a^2}
\]

\[
\rho = \rho(\sigma)
\]

\[
R(\sigma) = R + \rho'\sigma \frac{L}{a^2}
\]

\[
\Delta R/R = \left(\frac{\rho'}{\rho}\right)\sigma = \pi\sigma = \pi E\varepsilon
\]

\[
\Delta R/R \geq 100\varepsilon
\]

Resistivity change in metal strain gauge – mainly due to geometric effect

\[
R = \rho \frac{L}{a^2}
\]

\[
\frac{\Delta R}{R} = \left(1 + 2\nu\right)\frac{\Delta L}{L} \sim 2\varepsilon
\]
Piezoresistivity

Longitudinal and transverse piezoresistance coefficients

- **Longitudinal stress:** $\Delta R/R = \pi_l \sigma_l$
  - $\pi_l$ = longitudinal piezoresistance coefficient

- **Transverse stress:** $\Delta R/R = \pi_t \sigma_t$
  - $\pi_t$ = transverse piezoresistance coefficient

**Stress:** $\sigma = \text{Force/Area}$

\[
E_i = \sum_j \rho_{ij} j_j
\]

\[
\rho_{ij} = \sum_k \sum_l \pi_{ijkl} \sigma_{kl}
\]

**In general** we have both longitudinal and transverse stresses:

\[
\Delta R/R = \pi_t \sigma_t + \pi_l \sigma_l
\]
Piezoresistance coefficients in p-type silicon [10^{-11} /Pa]

- 100 plane
- Upper half
  - Longitudinal coefficient $\pi_l$
  - Lower half
    - Transverse coefficient $\pi_t$

In sensor application the $\pi$ should be as large as possible, i.e. resistors along $<110>$ direction

\[
\pi_l = \frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44})
\]
\[
\pi_t = \frac{1}{2}(\pi_{11} + \pi_{12} - \pi_{44})
\]
PIEZORESISTIVITY

Resistors along \(<110>\) direction in (100) wafers (common for bulk micromachining)

\[
\pi_l = \frac{1}{2} (\pi_{11} + \pi_{12} + \pi_{44})
\]

\[
\pi_t = \frac{1}{2} (\pi_{11} + \pi_{12} - \pi_{44})
\]

\[
\frac{\Delta R}{R} = \pi_l \sigma_l + \pi_t \sigma_t
\]

\[
\approx \frac{\pi_{44}}{2} (\sigma_l - \sigma_t)
\]

\(\Delta R \approx \frac{\pi_{44}}{2} \sigma \approx \frac{\pi_{44}}{2} E \varepsilon \approx 100 \varepsilon\)

Table 18.1. Typical room-temperature piezoresistance coefficients for n- and p-type silicon [98].

<table>
<thead>
<tr>
<th>Type</th>
<th>Resistivity</th>
<th>(\pi_{11})</th>
<th>(\pi_{12})</th>
<th>(\pi_{44})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>(\Omega\text{-cm})</td>
<td>(10^{-11} \text{ Pa}^{-1})</td>
<td>(10^{-11} \text{ Pa}^{-1})</td>
<td>(10^{-11} \text{ Pa}^{-1})</td>
</tr>
<tr>
<td>n-type</td>
<td>11.7</td>
<td>-102.2</td>
<td>53.4</td>
<td>-13.6</td>
</tr>
<tr>
<td>p-type</td>
<td>7.8</td>
<td>6.6</td>
<td>-1.1</td>
<td>138.1</td>
</tr>
</tbody>
</table>
Piezoresistive Sensors

Piezoresitive pressure sensor

- Membrane fabrication
  - Anisotropic etch
- Piezoresistor fabrication
  - doped area
  - or deposited polysilicon resistor on an insulator (SiO$_2$ or Si$_3$N$_4$)
- Piezoresistor position
  - at the edges of the membrane where the stress is maximal

Fig. 13 Schematic cross-section of a piezoresistive pressure sensor.
Piezoresistive Sensors

Piezoresistive accelerometer

- The piezoresistor must be placed where the stress is maximal
- To increase the sensitivity an inertial mass is included
Piezoresistivity

\[ R = \rho \frac{L}{w \cdot t} \]

Fig. 10 Piezoresistance factor \( P(N, T) \) as a function of impurity concentration and temperature for \( p \)-type silicon. (After Ref. 49)
Piezoresistive Sensor

Wheatstone Bridge Configuration

- R1 and R3 under lateral stress and decrease
  \[ \frac{\Delta R}{R} \approx -\frac{\pi_{44}}{2} \sigma \]
- R2 and R4 under longitudinal stress and increase
  \[ \frac{\Delta R}{R} \approx \frac{\pi_{44}}{2} \sigma \]

\[ R_1 = R_3 = R - \Delta R \]
\[ R_2 = R_4 = R + \Delta R \]

\[ \frac{\Delta R}{R} = \frac{\pi_{44}}{2} \left( \sigma_l - \sigma_t \right) \]

\[ V_m = V_b \left( \frac{\Delta R}{R} \right) = V_b \frac{\pi_{44}}{2} \left( \sigma_l - \sigma_t \right) \]

Figure 19.7 (A) Schematic drawing of the position and orientation of four piezoresistive elements on a silicon membrane with arrows delineating anisotropic silicon etching of a (100) silicon wafer; (B) Wheatstone bridge configuration of the four piezoresistive elements, with arrow directions indicating resistance increases or decreases for membrane deflection downward (after Sue [15], © 1994, reprinted with permission of John Wiley & Sons, Inc.).
**Piezoresistive Sensors**

**Figure 10.7** (A) Schematic drawing of the position and orientation of four piezoresistance elements on a silicon membrane with sides defined by anisotropic silicon etching of a (100) silicon wafer; (B) Wheatstone bridge configuration of the four piezoresistive elements, with arrow directions indicating resistance increases or decreases for membrane deflection downward (after Sze [15], © 1994, reprinted with permission of John Wiley & Sons, Inc.).

Pressure sensitivity for constant $V_b$:

$$S_v = \frac{\Delta V / V_b}{\Delta P} \quad [\text{mV/V-bar}]$$

$$= \frac{\Delta R / R}{\Delta P} = \frac{1}{2\Delta P} \pi_{44} (\sigma_i - \sigma_t)$$

Pressure sensitivity for constant $I_b$:

$$S_i = \frac{\Delta V / I_b}{\Delta P} \quad [\text{mV/ma-bar}]$$

$$= \frac{\Delta R}{\Delta P} = \frac{1}{2\Delta P} R \pi_{44} (\sigma_i - \sigma_t)$$
Piezoresistive Sensors
Offset voltage

Symmetrical mismatch of the resistors, caused by difference in layout (parallel and perpendicular to the edges of the membrane)

\[ R_1 = R_3 = R \]
\[ R_2 = R_4 = R + r \]
\[ V_o = V_b \left( \frac{r}{2R+r} \right) \]
\[ O = \frac{V_o}{V_b} = \frac{r}{2R+r} = \frac{1}{1+2R/r} \]

Offset also caused by residual stress on resistors
- pre-stress due to passivation layers and packaging
Piezoresistive Sensors

Temperature coefficient of offset

\[
\frac{\partial O}{\partial T} = \frac{2Rr}{(2R\pi + r)^2} \left( \frac{r}{r} - \frac{R}{R} \right)
\]

If resistors have equal temperature coefficients

\[
\frac{r}{r} = \frac{R}{R}, \quad \frac{\partial O}{\partial T} = 0
\]

However, temperature dependence of pre-stress might be significant.
Piezoresistive Sensors

Temperature coefficient of sensitivity

Constant bridge voltage

\[
S \propto \pi_{44} (\sigma_i - \sigma_t)
\]

\[
\text{TCS}_v = \frac{1}{S} \frac{\partial S}{\partial T} = \frac{1}{\pi_{44}} \frac{\partial \pi_{44}}{\partial T} + \frac{1}{\sigma_i - \sigma_t} \frac{\partial (\sigma_i - \sigma_t)}{\partial T}.
\]

Temperature coefficient for \(\pi_{44}\) can be high for \(p\)-type silicon

Constant bridge current

\[
S \propto \pi_{44} R (\sigma_i - \sigma_t)
\]

\[
\text{TCS}_i = \frac{1}{S} \frac{\partial S}{\partial T} = \frac{1}{\pi_{44}} \frac{\partial \pi_{44}}{\partial T} + \frac{1}{R} \frac{\partial R}{\partial T} + \frac{1}{\sigma_i - \sigma_t} \frac{\partial (\sigma_i - \sigma_t)}{\partial T}.
\]
Piezoresistance Sensors

Place your resistors this way:

\[ \sigma_t = 10 \text{ MPa}, \sigma_i = 50 \text{ MPa}; \]
\[ \Delta R/R = 3\% \]

Up to about 10 mV/kPa sensitivity of a final pressure sensor

FIGURE 4.18 Measuring on a membrane with piezoresistors. (A) Wheatstone-bridge configuration of four in-diffused piezoresistors. The arrows indicate resistance changes when the membrane is bent downward. (B) Maximizing the piezoresistive effect with p-type resistors. The A resistors are stressed longitudinally and the B resistors are stressed transversally. (From Peeters, E., Ph.D. Thesis, KUL, Belgium, 1994. With permission.)
Piezoresistive Sensors

- Convex corners bounded by \{111\} planes are attacked
Capacitive Sensors

Capacitor two electrodes separated by a dielectric

Electronic should be close to the sensor, minimising the stray capacitance

\[ C_0 = \varepsilon \frac{S}{d} \]

\[ \frac{\Delta C}{\Delta d} = -\varepsilon \frac{S}{d^2} \]

High sensitivity means large area \( S \) and a small distance \( d \)

\[ \Delta d \ll d \] results in

Advantage with capacitive sensors no direct sensitivity to temperature
Capacitive Sensors

a) Pressure sensor
b) Accelerometer

\[ C(\theta) = \frac{\varepsilon S}{l \theta} \ln \left( \frac{2d + l\theta}{2d + l\theta} \right) \]
## Comparison of different technologies

### TABLE 3 Comparison of Some Characteristics of Three Common Sensing Technologies for Accelerometers (Adapted from Ref. 67)

<table>
<thead>
<tr>
<th></th>
<th>Capacitive</th>
<th>Piezoelectric</th>
<th>Piezoresistive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impedance</td>
<td>High</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Size</td>
<td>Medium</td>
<td>Small</td>
<td>Medium</td>
</tr>
<tr>
<td>Temperature range</td>
<td>Very wide</td>
<td>Wide</td>
<td>Medium</td>
</tr>
<tr>
<td>Linearity error (sensor only)</td>
<td>High</td>
<td>Medium</td>
<td>Low</td>
</tr>
<tr>
<td>DC response</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>AC response</td>
<td>Wide</td>
<td>Wide</td>
<td>Medium</td>
</tr>
<tr>
<td>Damping available</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>High</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>Zero shifts due to shock</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Electronics required</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Cost</td>
<td>Medium</td>
<td>High</td>
<td>Low</td>
</tr>
</tbody>
</table>
Applications

Symmetric capacitive accelerometer with low thermal sensitivity

In some cases the movable electrode must be damped to avoid serious oscillations. A small cavity with a viscous liquid or gas can fulfil the requirements.
Flow sensors (gas)

With no flow the two sensors display the same temperature.

With gas flow the first sensor is cooled, while the second is heated up by the gas.
Some examples, Bulk micro-machined piezoresistive sensor

Laboratory for Electron Devices
Ljubljana SLOVENIA
Surface Micromachined Motor

Rotary Electrostatic Micromotor

Fan Long-Shen, Tai Yu-Chong and Muller R S 1989 IC-processed electrostatic micromotors Sensors Actuators 20 41–7
World's Smallest Guitar

A 10 μm long Si guitar (same size as a single cell) with six strings, each ~50 nm (100 atoms) wide. (Cornell University)
Automotive Airbag Accelerometer

Bulk micromachined proof mass suspended between mass deflection electrodes attached by wafer bonding
Automotive Airbag Accelerometer

A 3-Axis Force Balanced Accelerometer Using a Single Proof-Mass

Mark A. Lemkin, Bernhard E. Boser, David Auslander*, Jim H. Smith**
Fluidic MEMS

Electrostatic micropump with two one-way check valves

Zengerle R, Ulrich J, Kluge S, Richter M and Richter A
1995, A bi-directional silicon micropump Sensors
Actuators A50 81–6
EXERCISES

1) Calculate the maximum deflection and maximum stresses for a square silicon membrane of thickness 10 μm and side length 2 mm for an applied pressure of 1000 Pa. For silicon, $E = 190$ GPa and $\nu = 0.28$.

2) A silicon cantilever beam with a piezoresistor located at the point of maximum stress is subjected to a point load $Q$ at the end of the beam. $Q$ is 10 μN, the length of the beam is 1000 μm, and the beam thickness is 3 μm. Calculate the beam width that results in a 3% resistance change for the piezoresistor due to the load $Q$. Assume the beam lies perpendicular to the silicon $<110>$ lattice direction.

3) What is the resonant frequency $F_o$ for a silicon cantilever beam 1000 μm long, 100 μm wide, and 3 μm thick? The density of silicon is 2.3 g/cm$^3$. 

4) A polysilicon cantilever fabricated using surface micromachining is 500 μm long, 75 μm wide, and 3 μm thick. The sacrificial layer thickness for the process was 2.0 μm. The electrically grounded beam is electrostatically actuated using a positive voltage V applied to a conducting bottom electrode (length 20 μm, width 75 μm) under the end of the beam. Neglecting fringing effects, estimate the voltage V required to deflect the beam by 0.2 μm.

\[ Q = 0.5 \varepsilon_0 \varepsilon_r \frac{AV^2}{d^2} \]
EXERCISES

ANSWER:
1) $W_{\text{max}} = 1.17 \mu m$, $\sigma_l = 12.3 \text{ MPa}$, $\sigma_t = 3.45 \text{ MPa}$
2) $\sigma_{\text{max}} = 4.3 \times 10^7 \text{ Pa}$, $a = 150 \mu m$
3) $f_0 = 4.39 \text{ kHz}$
4) $V = 9.6 \text{ V}$