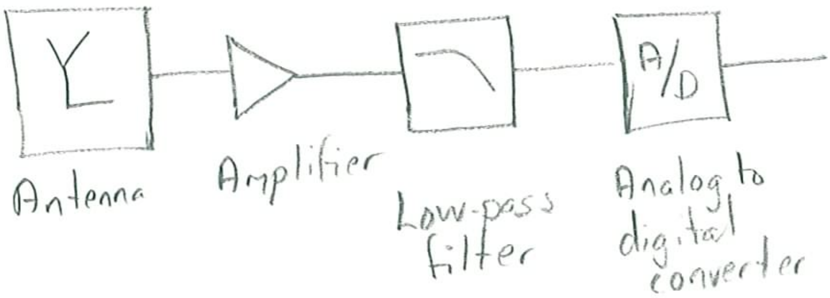
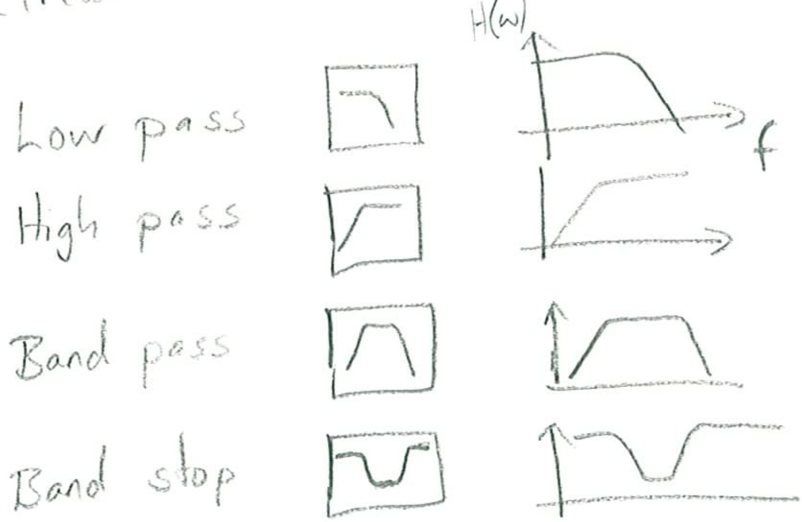
Analog electronics

Electronic systems can be described by a block diagram



Filter

Circuit that let som frequencies pass and stops other



jω-method

Impedance

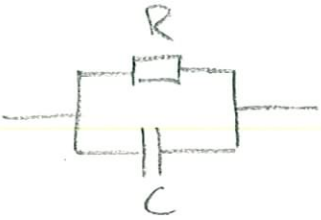
$Z = R$

$Z = \frac{1}{j\omega C}$

$Z = j\omega L$

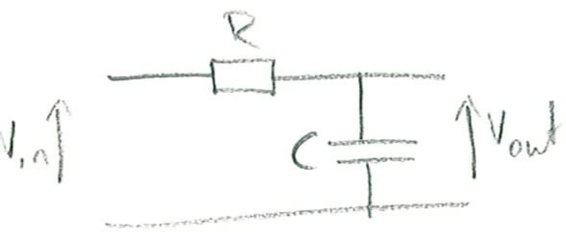
The same calculations can be used for impedances as resistances

$Z = Z_R + Z_L = R + j\omega L$



$$Z = Z_R // Z_C = \frac{Z_R \cdot Z_C}{Z_R + Z_C} = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC}$$

RC-filter



Voltage division

$$\Rightarrow V_{out} = V_{in} \frac{Z_C}{Z_R + Z_C} = V_{in} \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$V_{out} = V_{in} \frac{1}{1 + j\omega RC}$$

Transfer function $H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\frac{\omega}{\omega_0}} = \frac{1}{1 + j\frac{f}{f_0}}$

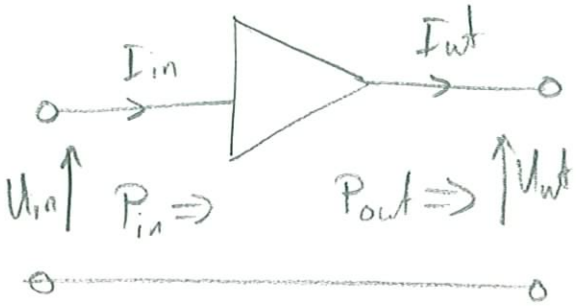
Where ω_0 or f_0 is the cut of frequency.

$$|H(\omega_0)| = \frac{1}{\sqrt{1 + \left(\frac{\omega_0}{\omega_0}\right)^2}} = \frac{1}{\sqrt{2}}$$

$\therefore$ At the cut-off frequency the power is halfed

Power $\propto H(\omega)^2 \Rightarrow P(\omega_0) = \frac{P_{in}}{2}$

Amplifier



Voltage amplification

$$A_V = \frac{V_{out}}{V_{in}}$$

Current amplification

$$A_i = \frac{I_{out}}{I_{in}}$$

Power amplification

$$A_P = \frac{P_{out}}{P_{in}}$$

Decibel

It is very useful to express amplification and attenuation in dB

$$A_{P_{dB}} = 10 \log A_P = 10 \log \frac{P_{out}}{P_{in}}$$

as $P = U \cdot I = \frac{U^2}{R}$

$$A_{P_{dB}} = 10 \log \frac{P_{out}}{P_{in}} = 10 \log \frac{\left(\frac{V_{out}}{\cancel{R}}\right)^2}{\left(\frac{V_{in}}{\cancel{R}}\right)^2} = 10 \log \left(\frac{V_{out}}{V_{in}}\right)^2 = \underline{20 \log \frac{V_{out}}{V_{in}} = A_{V_{dB}}}$$

$$A_{V_{dB}} = 20 \log \frac{V_{out}}{V_{in}}$$

dB	A_V	A_P
-20 dB	$1/10$	$1/100$
-3 dB	$1/\sqrt{2}$	$1/2$
0 dB	1	1
+3 dB	$\sqrt{2}$	2
+20 dB	10	100

Bode diagram

Absolut value $|H(\omega)|_{dB}$ and phase $\phi = \text{angle}(H(\omega))$ is plotted in two separate graphs with logarithmic frequency axis.

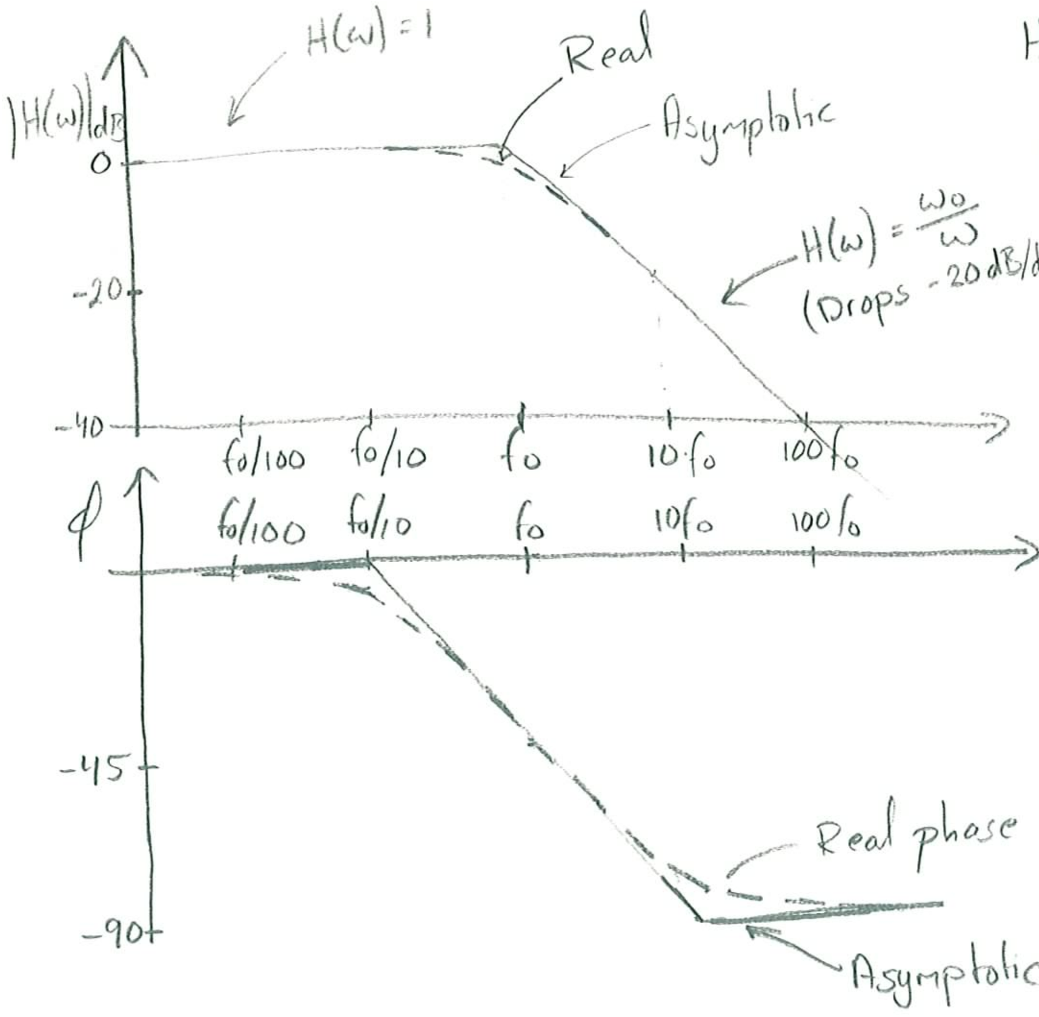
For the RC-filter

$$|H(\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}$$ where $\omega_0 = \frac{1}{RC}$ cut-off frequency

$$\phi = \text{angle}(H(\omega)) = \arctan\left(\frac{Im(H(\omega))}{Re(H(\omega))}\right)$$

$$H(\omega) = \frac{1}{1+j\frac{\omega}{\omega_0}} = \frac{1-j\frac{\omega}{\omega_0}}{\underbrace{\left(1+j\frac{\omega}{\omega_0}\right)\left(1-j\frac{\omega}{\omega_0}\right)}_{\text{Real value}}} \Rightarrow \phi = \arctan\left(\frac{-\frac{\omega}{\omega_0}}{1}\right) = \arctan\left(-\frac{\omega}{\omega_0}\right)$$

$$\phi = -\arctan\frac{\omega}{\omega_0}$$



$$H(\omega) = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}$$

Asymptotic
$\omega \ll \omega_0 \Rightarrow H(\omega) = 1$
$\omega \gg \omega_0 \Rightarrow H(\omega) = \frac{\omega_0}{\omega}$

$\arctan(0,1) = 5,7°$
$\arctan(1) = 45°$
$\arctan(10) = 84,3°$

Complicated expression can be factorized into supexpressions such as

C (const), $j\frac{\omega}{\omega_0}$, $\frac{1}{j\frac{\omega}{\omega_0}}$, $1+j\frac{\omega}{\omega_0}$, $1-j\frac{\omega}{\omega_0}$

The total amplification A_{tot} is then

$$A_{tot} = A_1 \cdot A_2 \cdot A_3$$

or

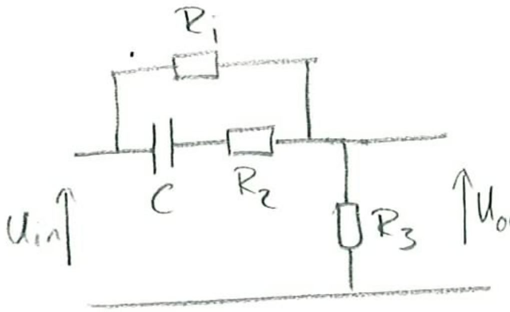
$$A_{tot_{dB}} = A_{1dB} + A_{2dB} + A_{3dB}$$ (According to logarithmic laws)

and the phase

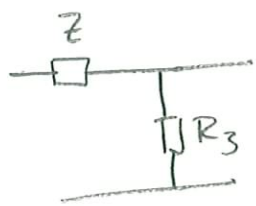
$$\text{angle}(A_{tot}) = \text{angle}(A_1) + \text{angle}(A_2) + \text{angle}(A_3)$$

Lect 1

Ex



$\Longleftrightarrow$



$$Z = R_1 // \left(R_2 + \frac{1}{j\omega C}\right) = \frac{R_1\left(R_2 + \frac{1}{j\omega C}\right)}{R_1 + R_2 + \frac{1}{j\omega C}} = \frac{R_1(1 + j\omega R_2 C)}{1 + j\omega(R_1+R_2)C}$$

$$\frac{U_{out}}{U_{in}} = \frac{R_3}{R_3 + Z} = \frac{R_3}{R_3 + \frac{R_1(1+j\omega R_2 C)}{1+j\omega(R_1+R_2)C}} = \frac{R_3(1+j\omega(R_1+R_2)C)}{R_3(1+j\omega(R_1+R_2)C) + R_1(1+j\omega R_2 C)}$$

$$= \frac{R_3(1+j\omega(R_1+R_2)C)}{R_1 + R_3 + j\omega(R_1R_2 + R_1R_3 + R_2R_3)C} = $$ / Break out real part to leave 1 instead /

$$= \frac{R_3}{R_1+R_3}\,\frac{1+j\omega(R_1+R_2)C}{1+j\omega\frac{R_1R_2+R_1R_3+R_2R_3}{R_1+R_3}C} = K\,\frac{1+j\frac{\omega}{\omega_1}}{1+j\frac{\omega}{\omega_2}} = K\,\frac{1+j\frac{f}{f_1}}{1+j\frac{f}{f_2}}$$

where $K = \frac{R_3}{R_1+R_3}$

$f_1 = \frac{1}{2\pi(R_1+R_2)C}$

$f_2 = \frac{R_1+R_3}{2\pi(R_1R_2+R_1R_3+R_2R_3)C}$

Using

$R_1 = 99k$

$R_2 = 9{,}9k$

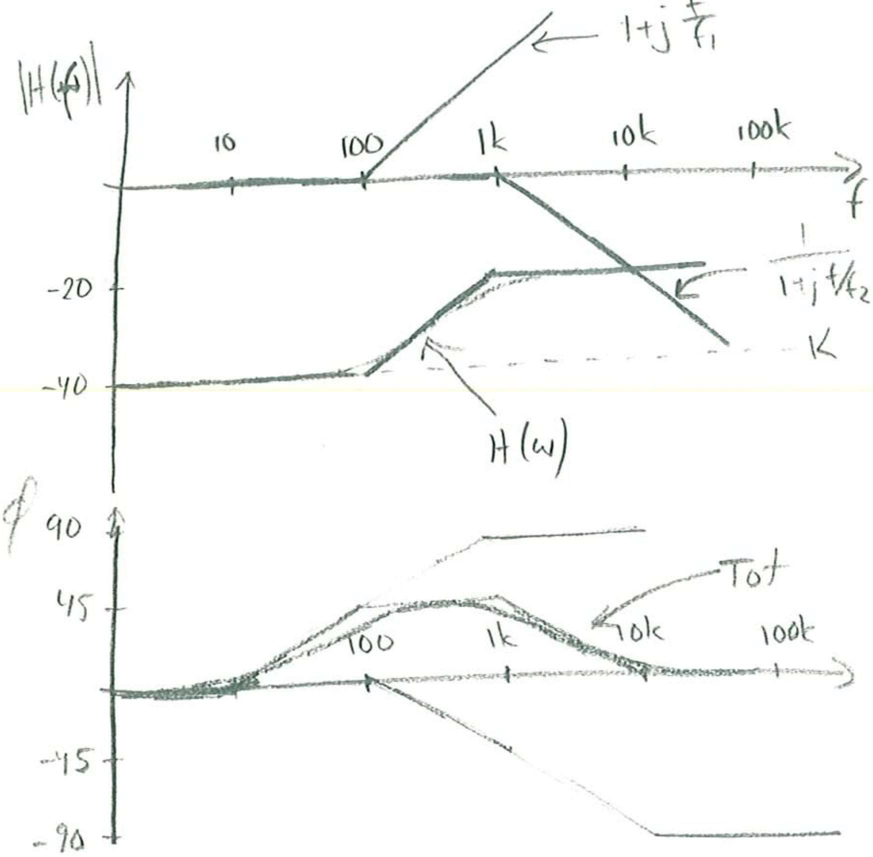
$R_3 = 1{,}0k$

$C = 14{,}6nF$

$K = 0{,}01$ or $-40dB$

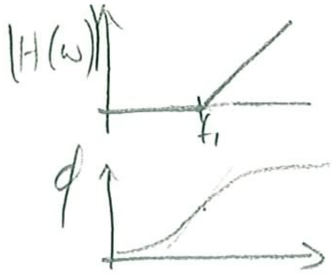
$f_1 = 100$ Hz

$f_2 = 1000$ Hz



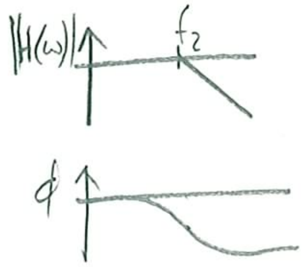
Pole

$1+j\frac{f}{f_1}$

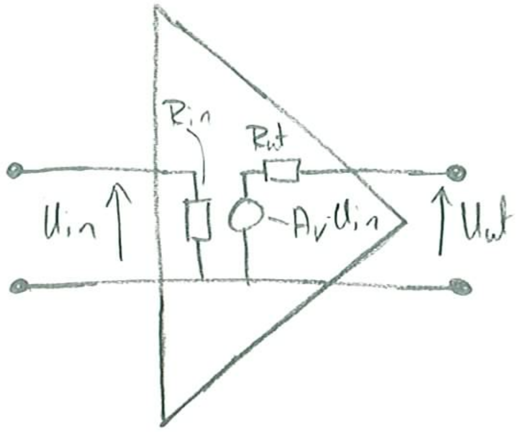


Zero

$\frac{1}{1+j\frac{f}{f_2}}$



Amplifier model

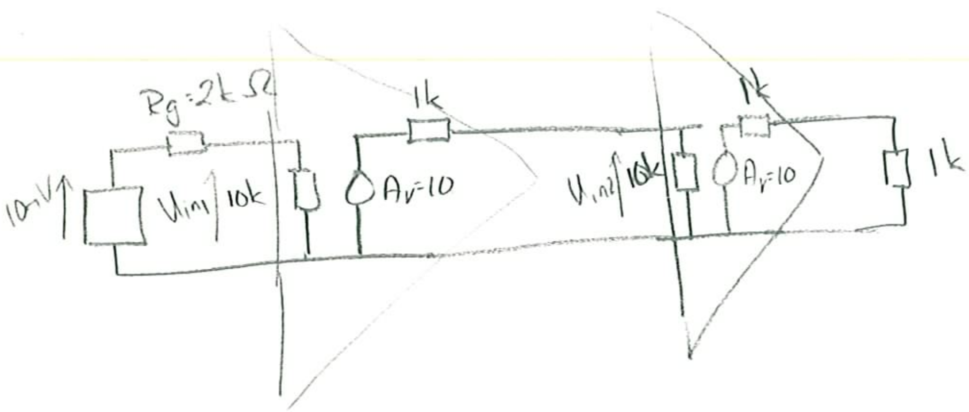


R_{in} – Input resistance

R_{out} – Output resistance

A_V – Open loop gain

Ex 2 Amplifiers with $A_V = 10$ $R_{in} = 10k$ and $R_{out} = 1k$ is cascaded between a generator with $U_{in} = 10mV$ and $R_g = 2k\Omega$ and a load of $1k\Omega$.
What is the Voltage at the load



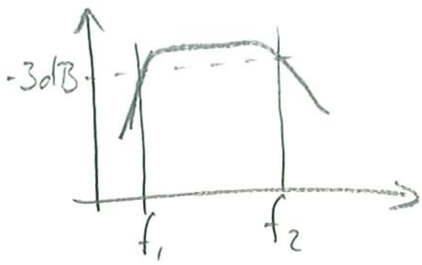
$$V_{in1} = V_{in} \frac{10}{10+2} = \frac{V_{in}}{1,2}$$

$$V_{out_1} = V_{in2} = 10 \cdot V_{in1} \frac{10}{10+1} = 10 \cdot V_{in1} \cdot \frac{1}{1,1} = \frac{10 \cdot V_{in}}{1,2 \cdot 1,1}$$

$$V_{out2} = V_{in2} \cdot 10 \cdot \frac{1}{1+1} = \frac{10 \cdot V_{in}}{1,1 \cdot 1,2} \cdot 10 \frac{1}{2} = 37,9\, V_{in} = 379\,mV$$

Bandwidth

The frequency range for flat gain

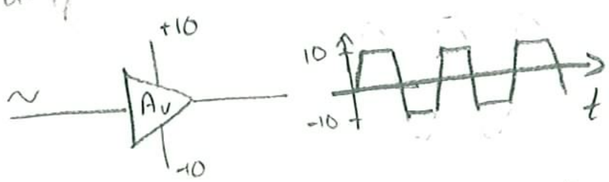


$BW = f_2 - f_1$

Distortion

A nonlinear transfer funktion between U_{in} and U_{out} distorts the signal

- An amplifier that saturates at high voltages



- A diode that conducts only in forward direction