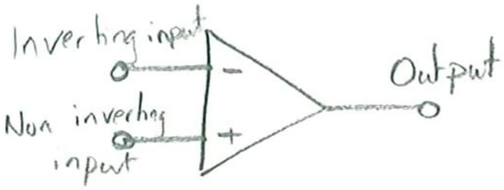
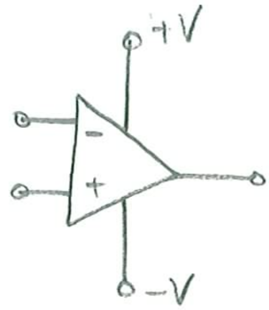
Operational Amplifier   OP Amp

Amplifier with differential input and high gain

- Differential input - Amplifies the Voltage difference

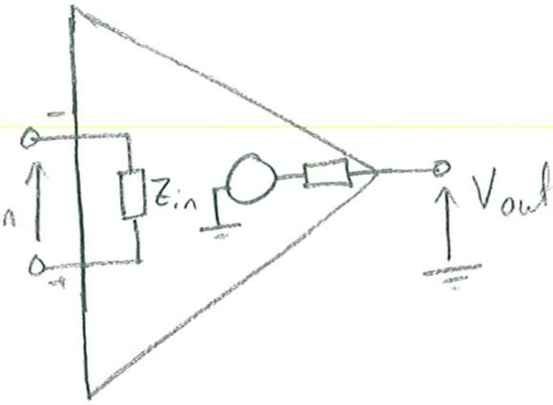




Symbol with supply connection

An operational amplifier is nearly ideal

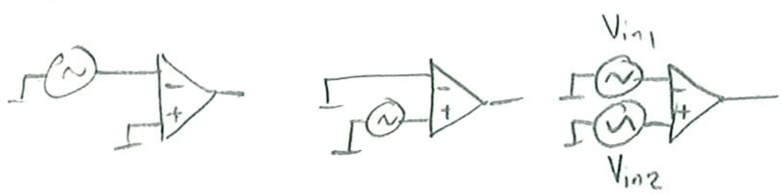
	Ideal	Practical
A_v	∞	Large
R_{in}	∞	Large
R_{out}	0	Small



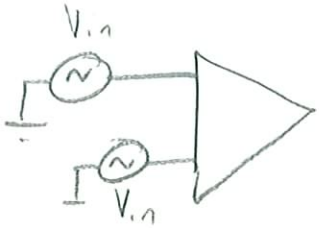
The OP-Amp is used to amplify the difference in voltage between the input

$$V_{out} = (V^+ - V^-) \cdot A_v$$

$\Rightarrow$ Called differential mode. Either between one input and ground or different signals on both iputs.



Applying the same signal on both input is called "Common mode" and should be as small as possible



Common mode rejection ratio – "CMRR" is the ratio

A_{ol} – "Open-loop gain"

$$CMRR = \frac{A_{ol}}{A_{cm}} \quad \text{or} \quad \underbrace{= 20 \log \frac{A_{ol}}{A_{cm}}}_{\text{in dB}}$$

Ex $A_{ol} = 100\ dB$
$A_{cm} = -14\ dB$

$100\ dB \Rightarrow 10^{\frac{100}{20}} = 100\ 000$
$-14\ dB \Rightarrow 10^{\frac{-14}{20}} = 0{,}2$

$$\underline{CMRR} = \frac{100\ 000}{0{,}2} = \underline{500\ 000}$$

$$20 \log 500\ 000 = \underline{114\ dB}$$

Any other way ??

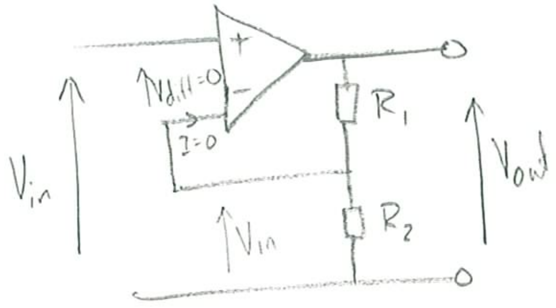
$$CMRR = 100 - (-14) = 114\ dB$$

Lect 2
↓↓

Basic OP-Amp circuit analysis

- In a circuit the OP-amp drives the output so that V_{diff} is minimized, read $V_{diff}=0$

Non inverting amplifier

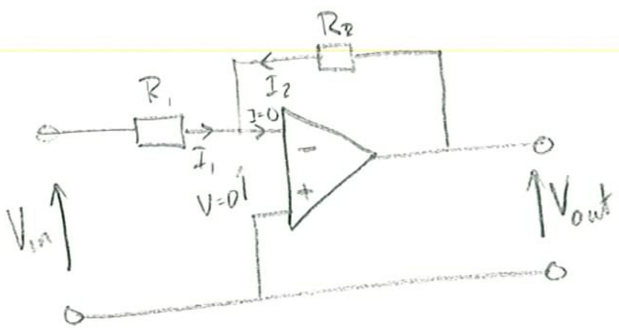


Voltage division

$$V_{in} = V_{out}\frac{R_2}{R_1+R_2}$$

$$\frac{V_{out}}{V_{in}} = A_V = \frac{R_1+R_2}{R_2}$$

Inverting amplifier

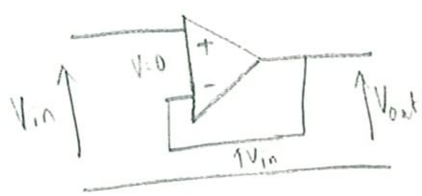


$$I_1 = \frac{V_{in}}{R_1} \qquad I_2 = \frac{V_{out}}{R_2} \qquad \text{KCL} \quad I_1 = -I_2$$

$$\frac{V_{out}}{R_2} = -\frac{V_{in}}{R_1} \qquad \frac{V_{out}}{V_{in}} = A_V = -\frac{R_2}{R_1}$$

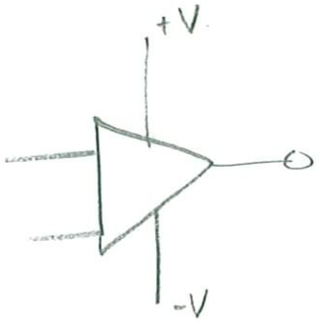
Inverted or $\phi = 180°$

Voltage follower



$$V_{out} = V_{in} \qquad \frac{V_{out}}{V_{in}} = 1$$

Maximal output swing, V_{OP-P}



⇒ the OP-amp is normally fed with a supply voltage that limits the output swing.

⇒ In practice the output is even less than this.

In a KA741 fed with +/-15 V the output is limited to $V_{OP-P} = \pm 13V$

Some OP-Amps labeled Rail-to-Rail can deliver all the way to the supply voltage.

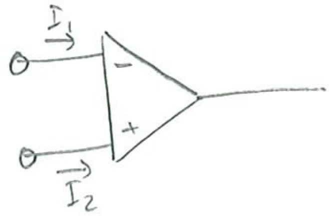
Input offset voltage

Ideally $V_{out} = 0$ when $V_{in} = 0$ but in practice due to mismatch $V_{out} = 0$ when a small voltage difference occurs at the input V_{io} which is typically in the range 2mV or less.

	KA 741 typ	KA 741 max
V_{IO}	2mV	6mV

Input bias current, I_{BIAS}

In reality $I_{in} \neq 0$



I_{BIAS} is the mean value of the two small input currents

$$I_{BIAS} = \frac{I^+ + I^-}{2}$$

As $I_1 \neq I_2$ the difference is called Input offset current

$$I_{OS} = |I_1 - I_2|$$

	KA 741 typ	KA 741 max
I_{BIAS}	80nA	500nA
I_{OS}	20nA	200nA

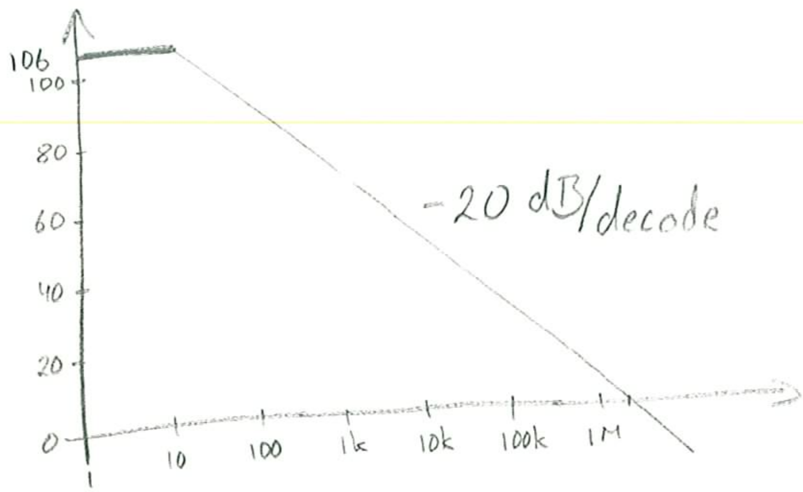
Input and Output resistances

	KA 741	
	min	typ
R_I	$0{,}3M\Omega$	$2M\Omega$

R_{out} is very low but the output current is often limited to protect the device from destruction

Frequency responce

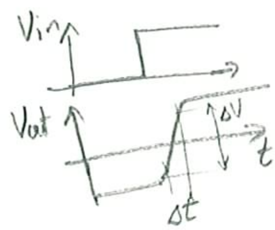
Open-loop gain A_{ol} decreases with frequency



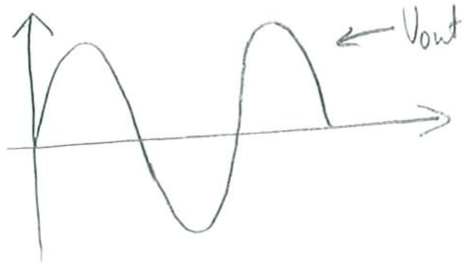
∴ At high frequencies the gain is less.

The output rate of change is limited by the "Slew rate"

$$\text{Slew rate} = \frac{\Delta V_{out}}{\Delta t}$$



Ex/ Calculate the cut-off frequency due to slew rate.



$\leftarrow V_{out} = \hat{U} \cdot \sin(\omega t)$

The derivative V_{out}' is the rate of chang

$$V_{out}' = \frac{dV_{out}}{dt} = \hat{U} \cdot \omega \cos(\omega t)$$

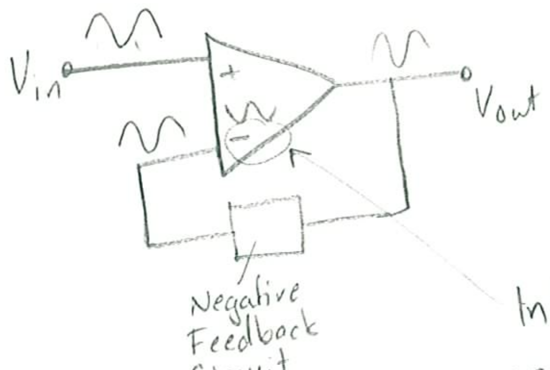
$$\left.\frac{dV_{out}}{dt}\right|_{max} = \text{slew rate} \qquad SR = \hat{U} \cdot \omega = \hat{U} \cdot 2\pi f$$

$$f_{max} = \frac{SR}{2\pi \hat{U}}$$

$\therefore$ At large signals the Bandwidth is less

Negative feed-back

- A portion of the output is returend and subtracted from the input



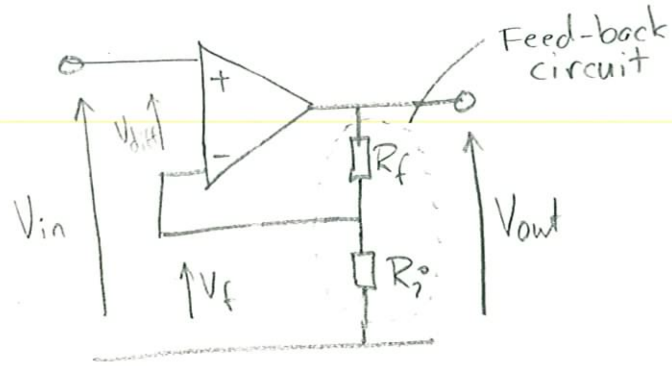
Internal inversion makes V_F 180° out of phase compared to V_{in}

Why use negative feedback

- Gain is reduced to desierd value for stable operation
- The bandwidth is significantly increased
- The linearity is significantly improved
- Input resistance can be increased or reduced to som desiered value
- Output resistance can be reduced to a desierd value.

OP-Amp configurations

- Non inverting amplifier



$$V_{out} = A_{ol}(V_{in} - V_f)$$

The feedback factor

$$B = \frac{R_i}{R_i + R_f}$$ (Portion of signal that are fed back)

$$V_{out} = A_{ol}(V_{in} - V_{out} \cdot B$$

$$V_{out} = A_{ol} \cdot V_{in} - V_{out} A_{ol} \cdot B$$

$$V_{out}(1 + A_{ol} \cdot B) = A_{ol} \cdot V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{A_{ol}}{1 + A_{ol} \cdot B}$$

$A_{ol} \cdot B$ is often $\gg 1$

$$\frac{V_{out}}{V_{in}} \approx \frac{A_{ol}}{A_{ol} \cdot B} = \frac{1}{B}$$

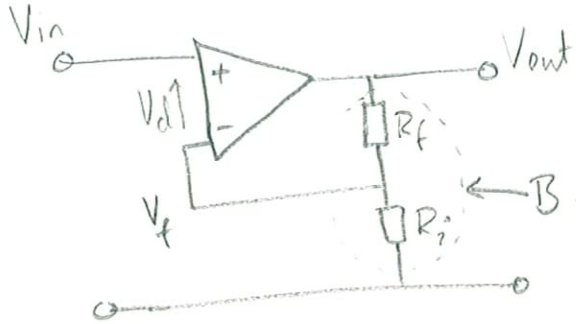
$$\frac{V_{out}}{V_{in}} = A_{cl} \approx \frac{1}{B} = \frac{R_i + R_f}{R_i}$$

Closed loop

$\therefore$ Independent on A_{ol} as long as $A_{ol} \cdot B \gg 1$

Effect on impedances

Input Impedance



$$B = \frac{R_i}{R_i + R_f}$$

$$V_f = B \cdot V_{out} \implies V_{in} = V_d + B \cdot \underbrace{V_{out}}_{= V_d \cdot A_{o1}} = V_d + B V_d \cdot A_{o1} = \underbrace{V_d}_{= I_{in} \cdot Z_{in}} (1 + B A_{o1})$$

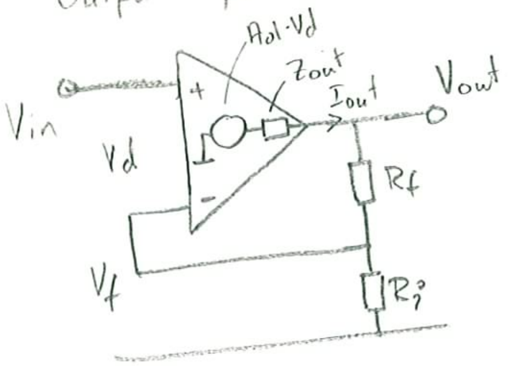
Open loop Zin

$$V_{in} = (1 + B \cdot A_{o1}) I_{in} \cdot Z_{in} \qquad \underbrace{\frac{V_{in}}{I_{in}}}_{Z_{in(NI)}} = (1 + B \cdot A_{o1}) \cdot Z_{in}$$

$$Z_{in(NI)} = (1 + B \cdot A_{o1}) \cdot Z_{in} \qquad \therefore Z_{in(NI)} \gg Z_{in}$$

Lect 3

Output impedance



$$V_{out} = A_{o1} \cdot V_d - \underbrace{I_{out} \cdot Z_{out}}_{\ll A_{o1} \cdot V_d} \simeq A_{o1} (V_{in} - \underbrace{V_f}_{= B \cdot V_{out}}) = A_{o1} (V_{in} - B \cdot V_{out})$$

$$V_{out} \simeq A_{o1} \cdot V_{in} - A_{o1} \cdot B \cdot V_{out}$$

$$\underbrace{V_{out}}_{I_{out} \cdot Z_{out(NI)}} (1 + A_{o1} \cdot B) \simeq A_{o1} \cdot V_{in}$$

$$I_{out} \cdot Z_{out(NI)} (1 + A_{o1} \cdot B) \simeq A_{o1} \cdot V_{in}$$

$$\underbrace{\frac{\overbrace{A_{o1} \cdot V_{in}}^{V_{out(OL)}}}{I_{out}}}_{Z_{out}} = (1 + A_{o1} \cdot B) Z_{out(NI)}$$

$$\implies Z_{out(NI)} = \frac{Z_{out}}{1 + A_{o1} \cdot B}$$

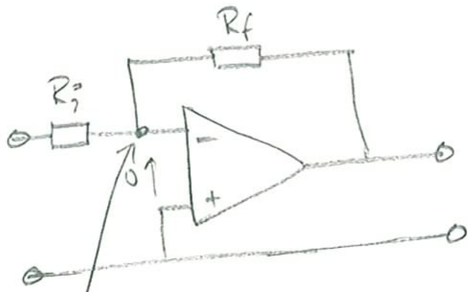
$\therefore Z_{out}$ is reduced with feedback

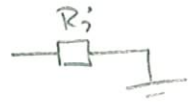
A Voltage follower is a non-inverting amplifier with $B=1$

$$\Rightarrow Z_{in(VF)} = (1+A_{ol})Z_{in}$$

$$Z_{out(VF)} = \frac{Z_{out}}{1+A_{ol}}$$

For an inverting amplifier



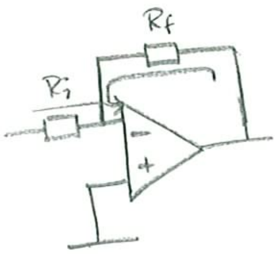


$Z_{in} \simeq R_i$

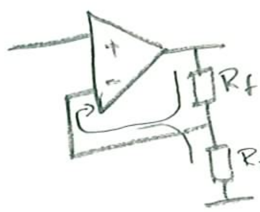
$$Z_{out(I)} = \frac{Z_{out}}{1+A_{ol}B}$$

Bias current compensation

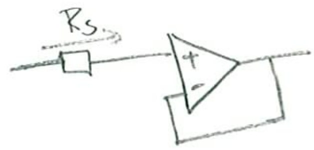
Inverting



Non inverting

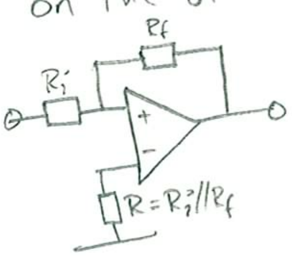


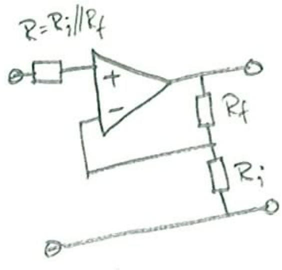
Voltage follower

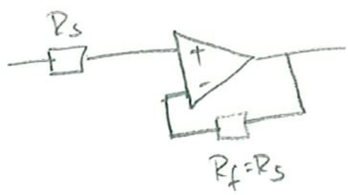


$V_{OO} = I_{IO} \cdot R_f$ $\quad$ $V_{OO} = I_{IO} \cdot R_f$ $\quad$ $V_{OO} = I_{IO} \cdot R_s$

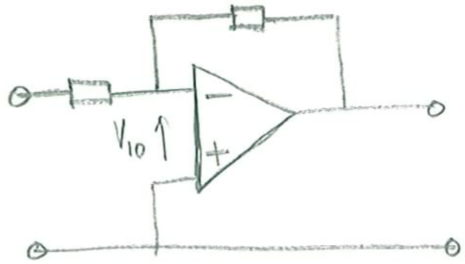
Bias current compensation is achieved by adding a resistor on the other input oposing the first one







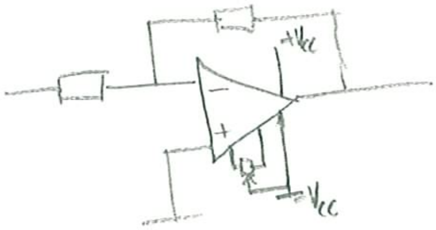
Input offset voltage compensation



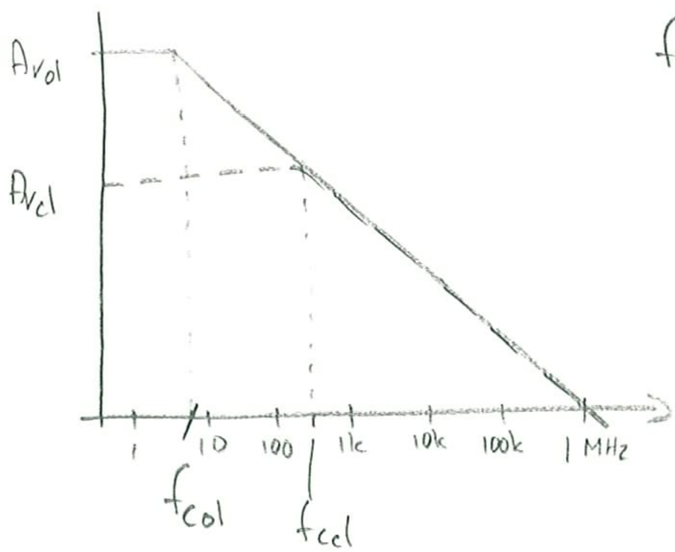
V_{IO} is generally amplified by A_{cl}

$$V_{OO} = V_{IO} \cdot A_{cl}$$

OP-Amps have often designated pins for compensation for instance μA741



Closed loop frequency response



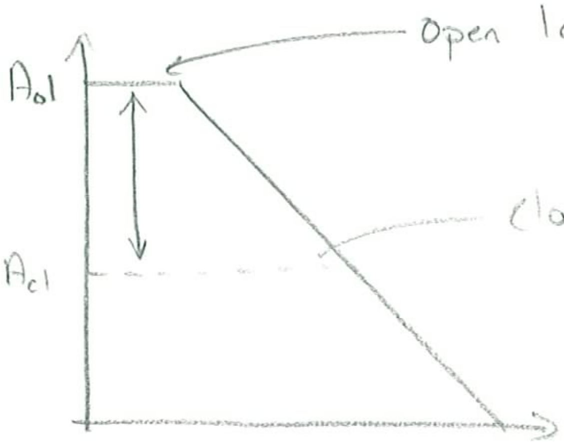
$$f_{ccl} = f_{col}\left(1 + B \cdot A_{ol}\right)$$

∴ High gain → small bandwidth
low gain → large bandwidth

Gain Bandwidth product

$A_{cl} f_{ccl} = A_{ol} \cdot f_{col} = f_T$ (the frequency where $A_{ol} = 0\,dB$)

Stability



$$A_{cl} = \frac{A_{ol}}{1 + B \cdot A_{ol}}$$

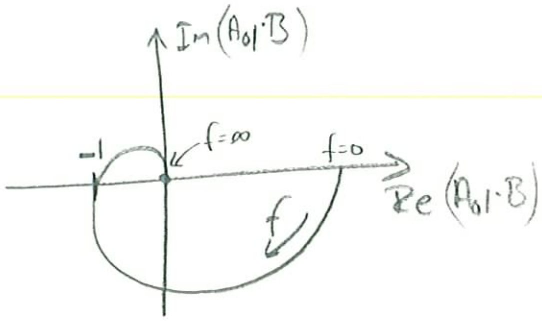
If $B \cdot A_{ol}$ reaches -1

$\Rightarrow A_{cl} \to \infty \to$ Instability

$B \cdot A_{ol}$ is called loop gain and is the key for study of instability

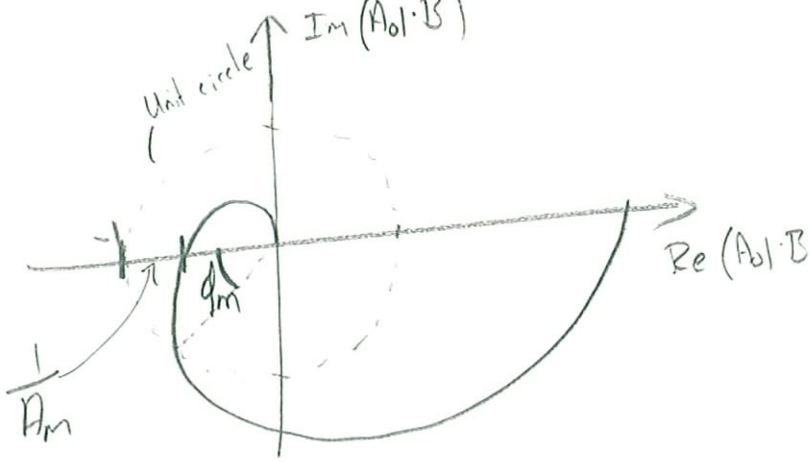
Stability criteria $\quad A_{ol} \cdot B < -1$ or $>|1| \angle -180^\circ$

Nyquist digram



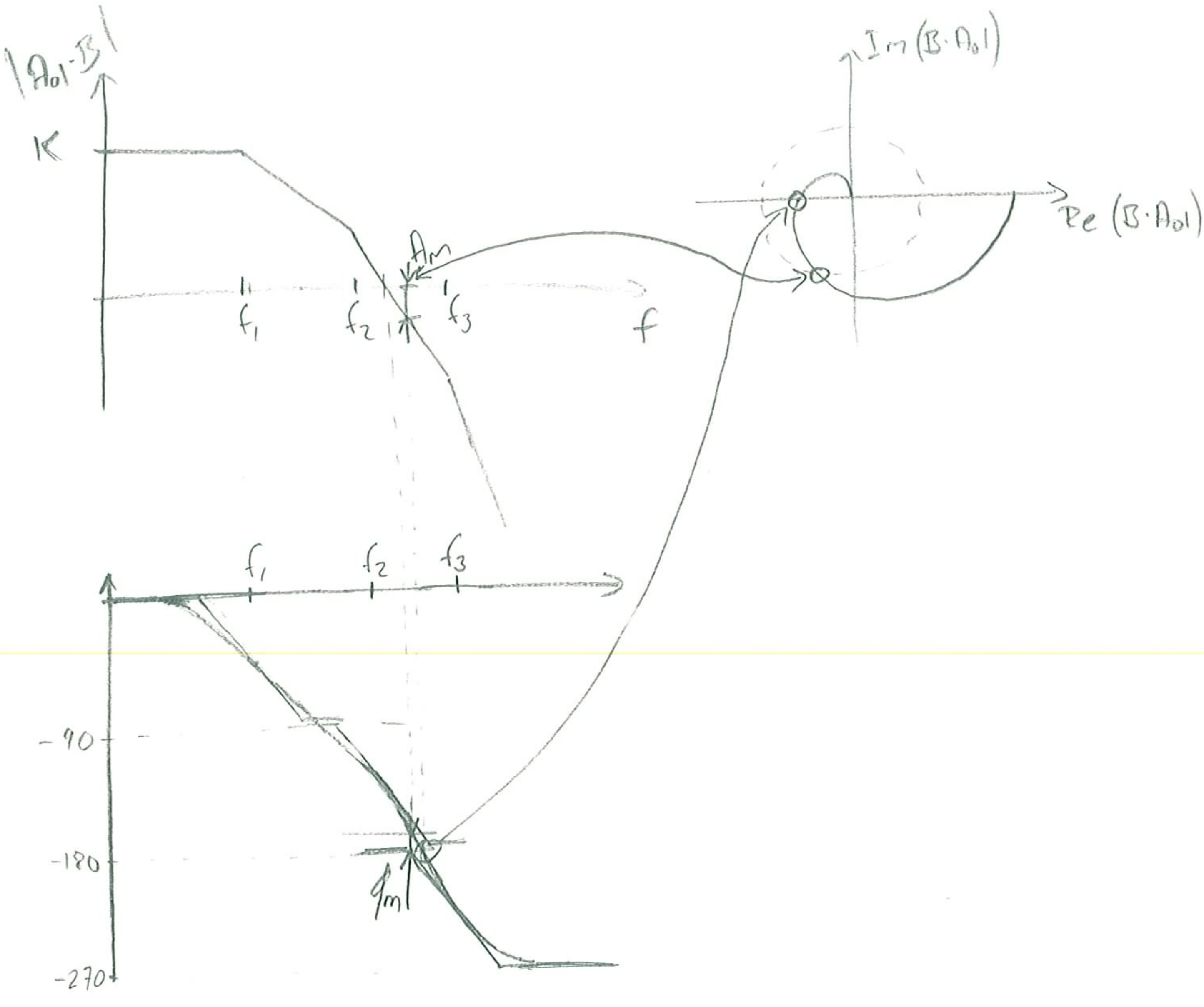
For stable operation some margin is requred

Either Amplitude margin, A_m or Phase margin, ϕ_m



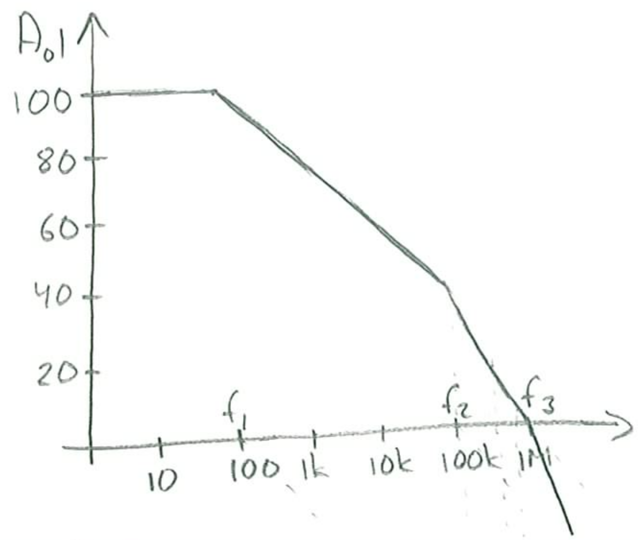
Rule of thumb : $\phi_m > 45^\circ$ and $A_m > 2$ (6 dB) for stable operation

$$A_{ol} \cdot B = \frac{K}{\left(1+j\frac{f}{f_1}\right)\left(1+j\frac{f}{f_2}\right)\left(1+j\frac{f}{f_3}\right)}$$



Ex/ In what ranges of resistive feed-back is the following amplifier stable, nearly stable and unstable

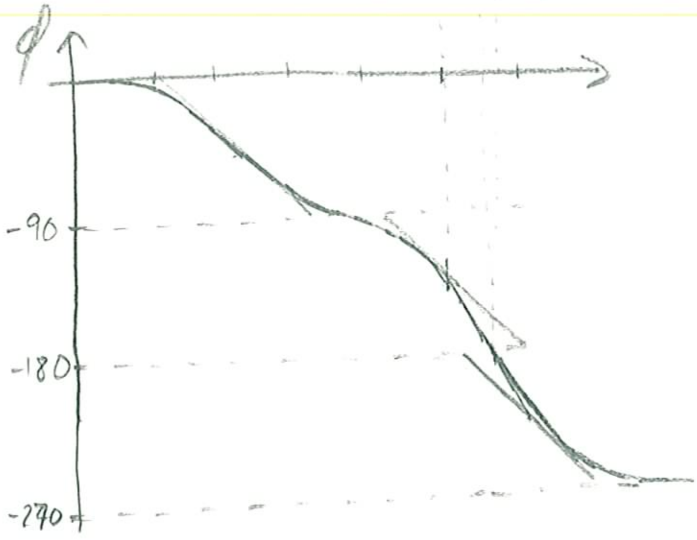
$$A_{ol} = \frac{100\,000}{\left(1+j\frac{f}{100}\right)\left(1+j\frac{f}{100k}\right)\left(1+j\frac{f}{1M}\right)}$$



	Phase margin ϕ_m
Unstable	< 0
Nearly stable	$0 - 45°$
Stable	$> 45°$

$\phi = -180°$ ~ $f = 300\,kHz$, $A_{ol} = 20\,dB$

$\phi = -135°$ ~ $f = 100\,kHz$, $A_{ol} = 40\,dB$



for $\beta = 0$ (No feed-back)

$A_{ol} \cdot \beta < 0\,dB \Rightarrow$ stable

for $\beta = 1$ (Unit gain)

$A_{ol} \cdot \beta > 0$ where $\phi = -180°$

$\Rightarrow$ Unstable

With $\beta = -20\,dB$ or 0,1

$A_{ol} \cdot \beta = 0$ where $\phi = -180°$

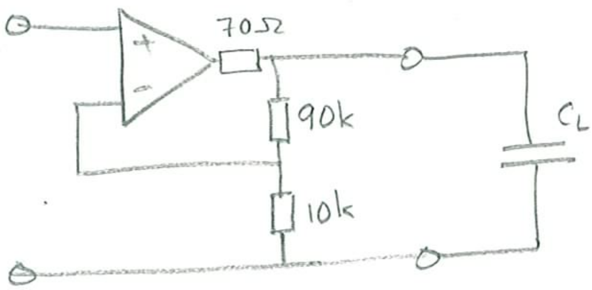
With $\beta = -40\,dB$ or 0,01

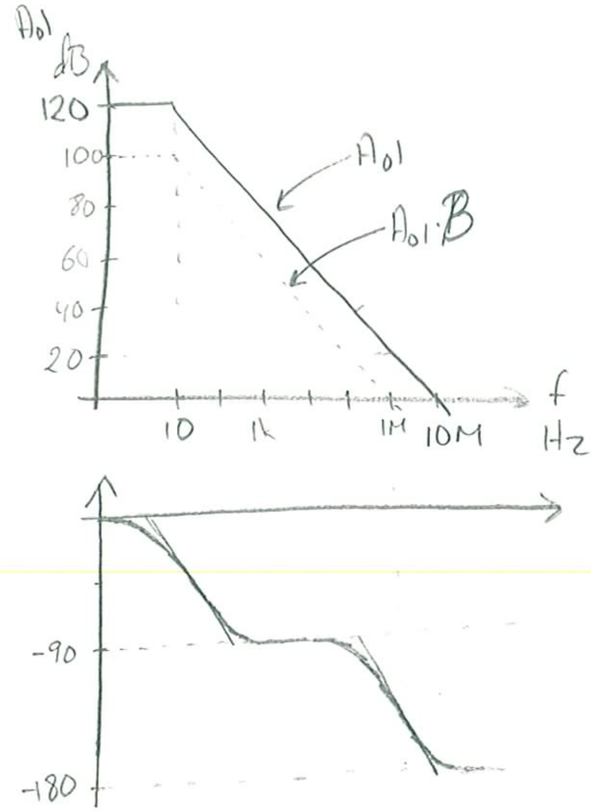
$A_{ol} \cdot \beta = 0$ where $\phi = -135°$

	β	A_v
Stable	$0 \to 0{,}01$	$A_{ol} - 40\,dB$
Nearly stable	$0{,}01 \to 0{,}1$	$40 - 20\,dB$
Unstable	$0{,}1 \to 1$	$20 - 0\,dB$

Driving capacitive loads can give stability problems for othewise stable circuits

Ex1 Calculate how long wire with $C = 100pF/m$ the following amplifier can drive.





$$A_V = \frac{90+10}{10} = 10 \text{ times or } 20 \text{ dB}$$

$$B = \frac{10k}{10k + 90k} = 0{,}1$$

With $B = 10$, $A_{ol} \cdot B = 0\,dB$ @ 1MHz

The pole @ 10 Hz gives $-90°$ phase $> 100Hz$

Placing the 2nd pole at 1MHz gives $\phi = -90 - 45° = -135°$ or

$\phi_M = 45°$ @ 1 MHz

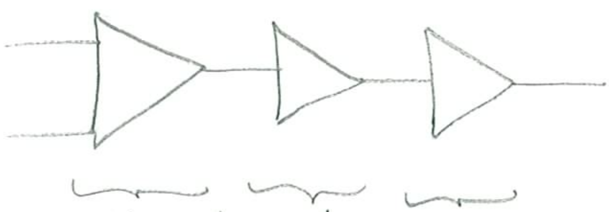
$$f_c = \frac{1}{2\pi RC} \Rightarrow \quad C_{max} = \frac{1}{2\pi R f_c} = \frac{1}{2\pi 70 \cdot 1 \cdot 10^6} = 2{,}3nF$$

wire length $$l = \frac{C_{max}}{C_{wire}} = \frac{2{,}3nF}{100pF/m} = 23\,m$$

OP-amp compensation

Most sold OP-amps are internally compensated to give stable operation in most cases.

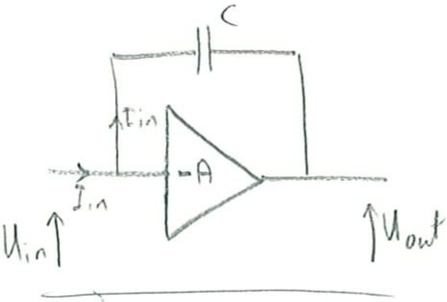
OP-amp block diagram



Differential stage	2nd stage	Driver stage
High gain	High gain	Unit gain

Internal compensation is often achieved by "Dominant pole compensation" using a "miller capacitor" on the 2nd stage

Miller Capacitor

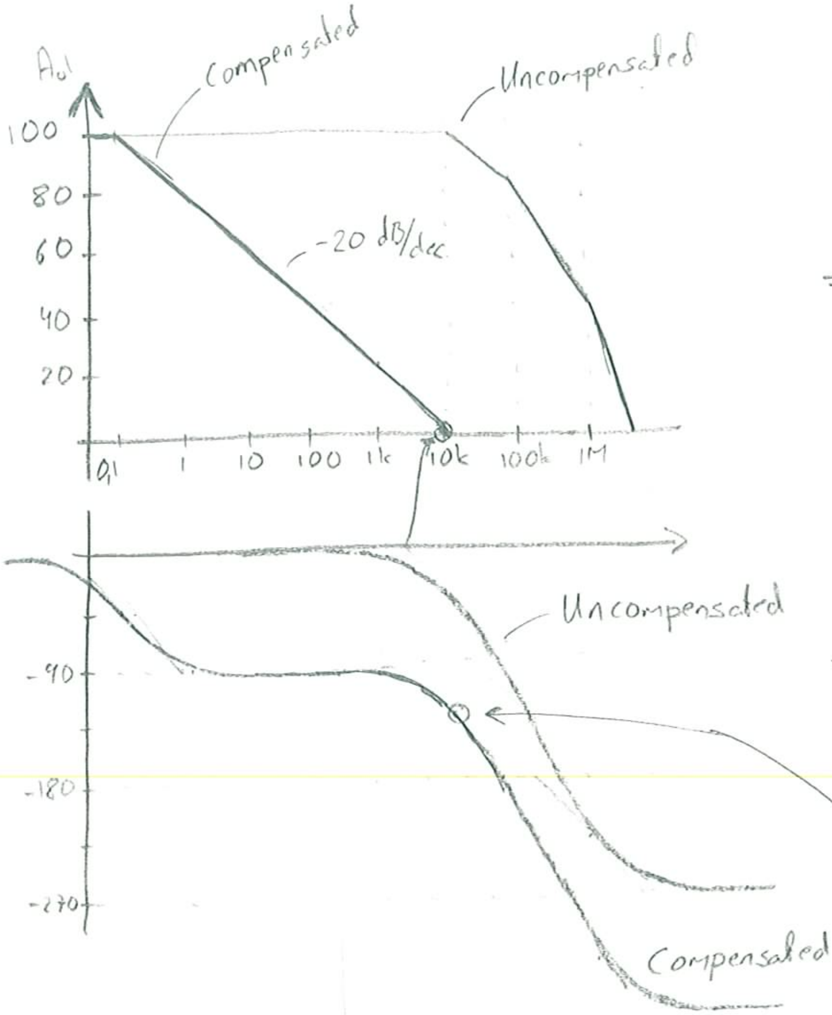


$$\left.\begin{aligned} U_{in} - U_{out} &= I_{in} \cdot \frac{1}{j\omega C} \\ U_{out} &= U_{in} \cdot (-A) \end{aligned}\right\} \quad U_{in}(1+A) = \frac{I_{in}}{j\omega C}$$

$$Z_{in} = \frac{U_{in}}{I_{in}} = \frac{1}{j\omega C(1+A)}$$

Dominant pole compensation

Assume $A_{ol} = \dfrac{100\,000}{\left(1+j\frac{f}{10k}\right)\left(1+j\frac{f}{100k}\right)\left(1+j\frac{f}{1M}\right)}$



For unit gain ($B = 1$)

$B \cdot A_{ol} > 1$ where $\phi_M = 45°$

⇒ Unstable

Compensate the OP-amp with a dominant pole to be stable for unit gain.

⇒ A new pole at low frequencies will give additional $-90°$ in phase shift

⇒ $\phi = -135°$ ($\phi_M = 45°$) the occur at the first pole for the amplifier, 10kHz

⇒ The gain must be reduced to 0dB at 10kHz for stable operation

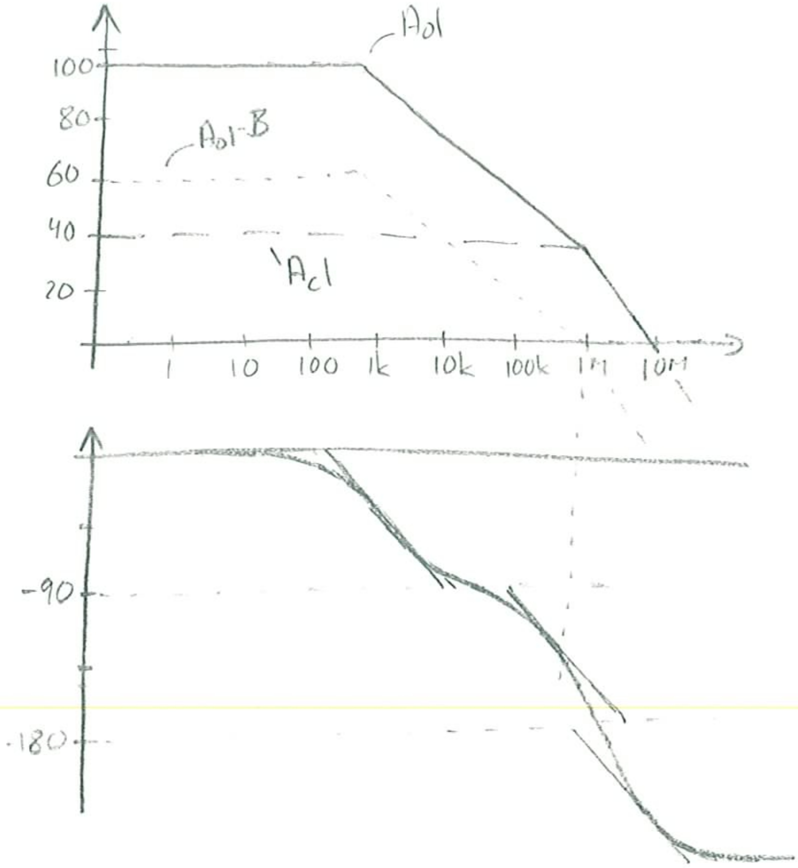
⇒ -20 dB/decade slope gives $f_0 = 0{,}1$ Hz

⇒ stable operation down to unit gain

⇒ Much reduced possible bandwidth.

Gain compensation

Assume $A_{ol} = \dfrac{100\,000}{\left(1+j\frac{f}{1k}\right)\left(1+j\frac{f}{1M}\right)\left(1+j\frac{f}{10M}\right)}$



$\phi = -135^\circ$ or $\phi_m = 45^\circ$ @ 1MHz

where $A_{ol} = 40$ dB

For $B = -40$ dB or 0,01

$B \cdot A_{ol} = 0$ dB where $\phi_m = 45^\circ$

$A_{cl} = \dfrac{1}{B} = 100$ or 40 dB

Bandwidth BW = 1MHz

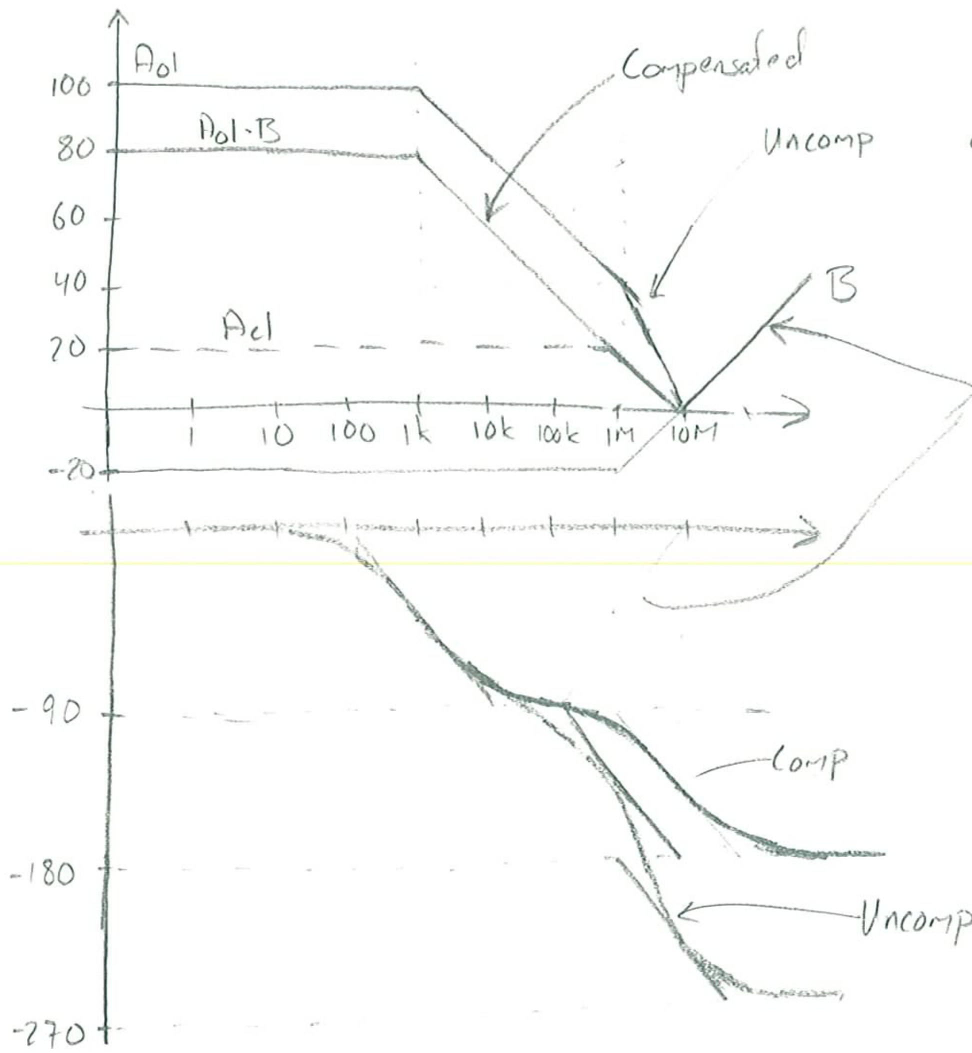
⇒ As long as the application can stand the higher gain gain compensation is the best method used.

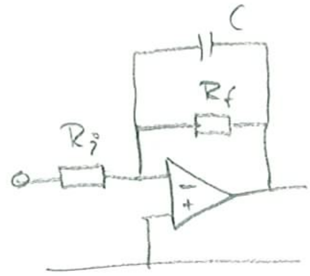
⇒ Uncompensated versions of normally internally compensated ones are avaliable for tailored external compensation.

Lead compensation

⇒ A phase advancing filter is used to improve ϕ_m at high freq.

Assume $$A_{ol} = \frac{100\,000}{\left(1+j\frac{f}{1k}\right)\left(1+j\frac{f}{1M}\right)\left(1+j\frac{f}{10M}\right)}$$





$$A_v = -\frac{R_f \| C}{R_i} = \frac{R_f \frac{1}{j\omega C}}{R_i\left(R_f + \frac{1}{j\omega C}\right)}$$

$$= \frac{R_f}{R_i(1+j\omega R_f C)} = \frac{1}{B}$$

$$B = \frac{R_i}{R_f}(1+j\omega R_f C)$$

Ex1 Assume 10x amplification is desired ($B = 0{,}1$ or -20 dB)

Let $R_i = 10\,k\Omega$ and $R_f = 100\,k\Omega$

∴ 20 dB gain with BW = 1 MHz