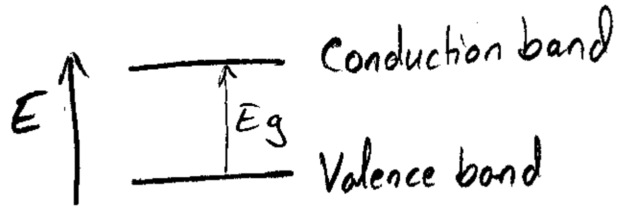
Solid-State physics

Semiconductor materials

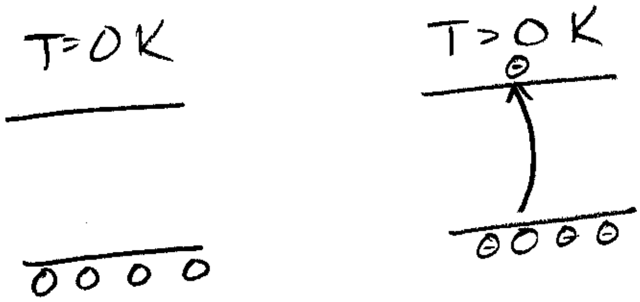
= Materials with conductivity between metals and isolators

= Bandgap $E_g \sim 0{,}3 - 4\ eV$



= At absolute zero temp $T = 0K$ all electrons are located in the valence band → Isolator.

= $T > 0\ K$ a fraction of the electrons are ionized - conductivity > 0



= Elemental (IV)
  Si, Ge

= IV-compound
  SiC, SiGe

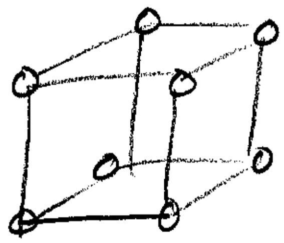
= III-V compounds
  GaAs, GaN, InP

= II-VI compounds
  ZnSe, CdTe

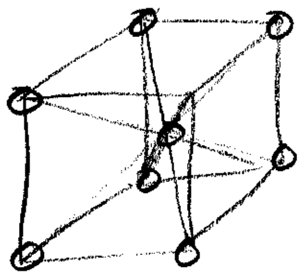
= Mostly Crystalline structure

= Amorphous and polycrystaline materials can sometimes be used as semi-conductors with bad performance.

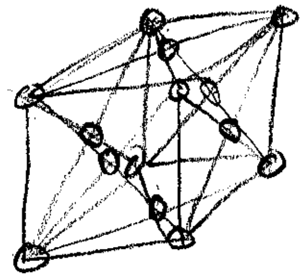
Lattices



Simple cubic
SC



Body centered cubic
BCC



Face centered cubic
FCC

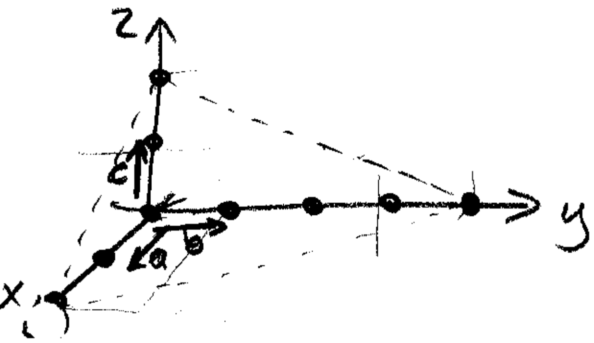
Electronic grade Si (EGS)

Impurities: parts per billion $1 \cdot 10^{-12}$

Single crystal ingots of 140 kg sliced into wafers

Plane and directions

Basis vector a,b,c



Plane interceps the crystal axis at

$2a, 4b, 2c$

reciprocals $\frac{1}{2}, \frac{1}{4}, \frac{1}{2}$

Multiply with least comon multiple

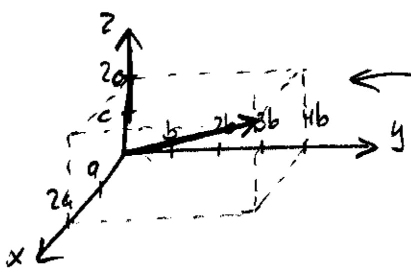
$\times 4 \Rightarrow 212$

Plane is labeled in brackets (212)

With symetrical crystal axis many planes are indistiguisable

$(100), (010), (001) \Rightarrow \{100\}$

Direction is labelled with [ ] and expressed in the basis vectors



$\leftarrow \bar{r} = [242] = [121]$

Indistiguisable directions are labelled wit <>

$[100], [010], [001] \Rightarrow \langle 100 \rangle$

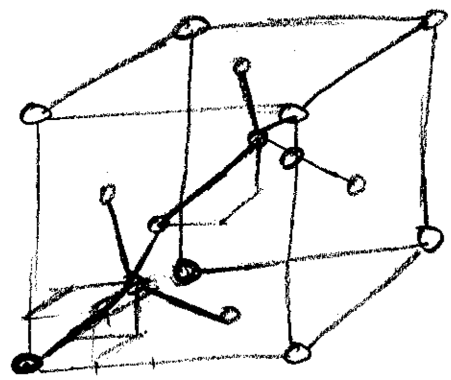
In cubic symetry

Direction [hkl] ⊥ Plane (hkl) !!!

↓↓45 min

Diamond lattice

Lattice structure for many important semiconductors such as Si and Ge.



⇒ FCC

⇒ Additional atom at $\frac{a}{4}+\frac{b}{4}+\frac{c}{4}$

If the additional atoms are different from the orignalal

⇒ zincblende (GaAs)

Silicon

Electronic grade silicon (EGS)

Impurites: parts per billion $1 \cdot 10^{-9}$

Single crystal ingots of 140 kg sliced into wafers.

Commonly sliced in the {100} plane

{111} Silicon is also used

Czoehralsk is the most commonly used growth technique

Floatin zone is also common in detector applications.

Atoms and Electrons (Important, must be understood)

⇒ Electromagnetic waves (Radio, Microwave, IR, visible light, UV, X and γ-Ray) are all quantified into photons!!

$$E = \frac{h \cdot c}{\lambda}$$

⇒ Energy levels in atoms and crystals are discretized with a limited number of states in each level.

- Electrons moving from a higher to a lower free energy level **emits** a photon with

$$\lambda = \frac{h \cdot c}{E_{Diff}}$$

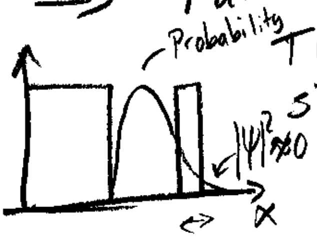
- A photon can be absorbed in the material and **exite** an electron if there exist a free state with the exact energy.

$$E = E_0 + \frac{hc}{\lambda}$$

⇒ Quantum well

In a sufficiently narrow potential barrier ($L < 20\,nm$) the allowed energy states becomes quantified

⇒ Tunneling



There is a small that an electron is on the opposite side of a barrier.
Carrier can "tunnel through" a thin energy barrier

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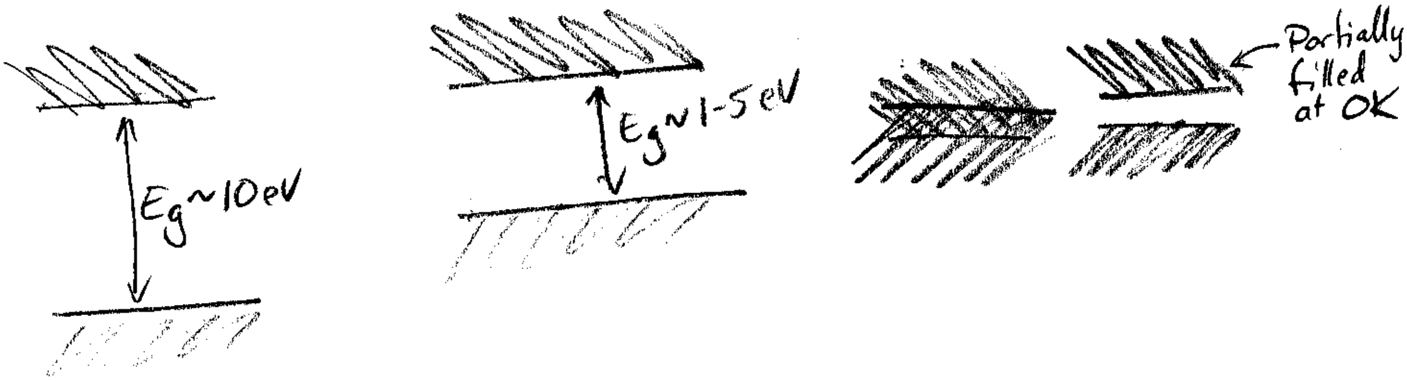
Energy bands and charge carriers

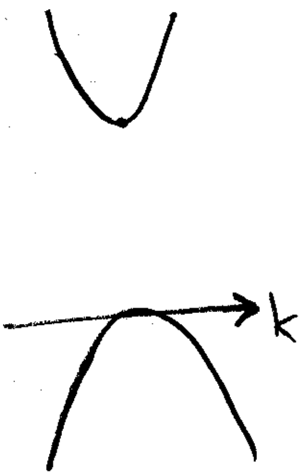
Isolator Semiconductor Metall



K-space (Momentum space)

Matematically the Fourier transformation of the real space.

Energy bands in k-space are parabolic



- An electron in the bottom of the band is in rest
- The electron velocity is proportional to the gradiend

$$v = \frac{1}{\hbar}\frac{\partial E}{\partial k}$$

- When an electrical field is applied the carriers moves with constant velocity in k-space

  → Parabolic bands $E(k) \propto k^2$

  → Accelerating in real space

- Carrier momentum

$$p = m \cdot v = \hbar k$$

Classic energy

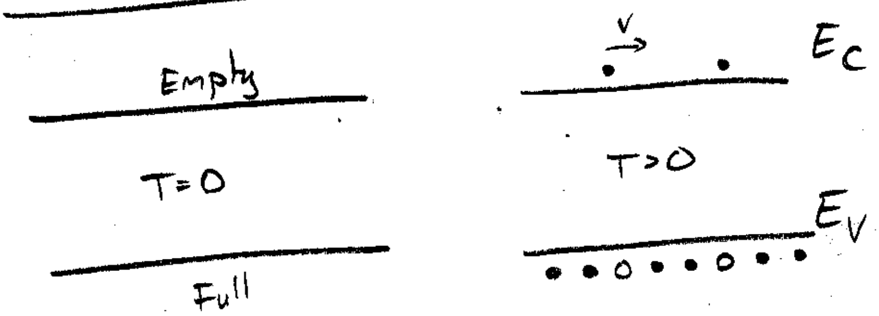
$$E = \frac{1}{2} m v^2 = / m v = p = \hbar k / = \frac{1}{2} \frac{\hbar^2 k^2}{m}$$

$$\frac{\partial E}{\partial k} = \frac{\hbar^2 k}{m} \qquad \frac{\partial^2 E}{\partial k^2} = \frac{\hbar^2}{m}$$

An effective mass, m^* can be extracted from the curvature of Energy bands in k-space

$$m^* = \frac{\hbar^2}{\frac{\partial^2 E}{\partial k^2}}$$

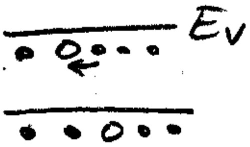
Electrons and holes



At temperatures above T=0 some electrons in the valece band is thermally excited to the conduction band. The vacancies that are created in the valence band is called "holes".

- The electrons in the conduction band are movable as plenty of nerby free states exists.

- Electrons in the valence band can move to the vacancy created by the hole
    - This is equivialent to saying that the hole is moving in the opposite direction.

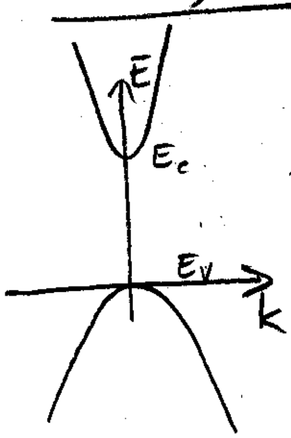


- The effective mass for holes is **often** lower than for electrons as it requires many electrons to participate in the transport.

- Excited electrons and holes are always created in pairs.
    - In room temperature

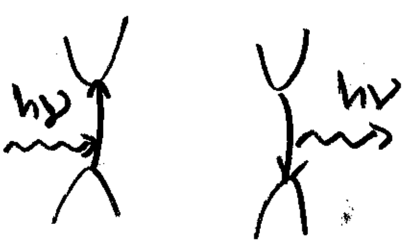
      $N_i \sim 10^{10}\ cm^{-3}$ $\quad N_{Si} \sim 5 \cdot 10^{22}\ cm^{-3}$

Direct, indirect bandgap



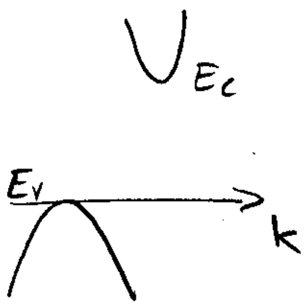
Materials where the conduction band minima and valence band maxima coincides in k-space are called direct bandgap.

Between the two states electrons can excite and deexcite ineracting with just a photon



- Very important in optical devices such as lasers and LEDs (Light emitting diodes)

Materials: GaAs, GaN, InP



In Si E_v-max and E_c-min occurs at different k-vectors.

A transition between the two bands requires a change in momentum for the electron.

As photons are mass-less ($p=0$) the momentum change must come from some other transition
- Phonons (Lattice vibrations)
- Defect levels

Intrinsic materials

- Perfect semiconductor without any impurities

  electron conc = hole conc = intrinsic conc

  $$n = p = n_i$$

  (Temperatur dependent according to Fermi-Dirac statistics)

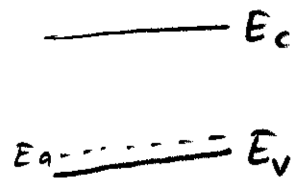
  Generation rate = Recombination rate

  $$g_i = r_i \quad \text{(Temp dependent)}$$

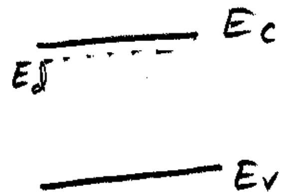
Extrinsic materials

- Carrier can also be created intentionally by doping
  - During growth
  - Implantation
  - Diffusion

- By adding an impurity with just 3 valence electrons doping level is created just above E_v.
  - The doping level easily accepts an valence electron and leaves a hole in the valence band
    - Acceptor level
    - B, Al, Ga, In



- Impurity from group V P, As, Sb
  - Doping energy just below E_c
  - Donates its electron to the conductiond band, donor level.



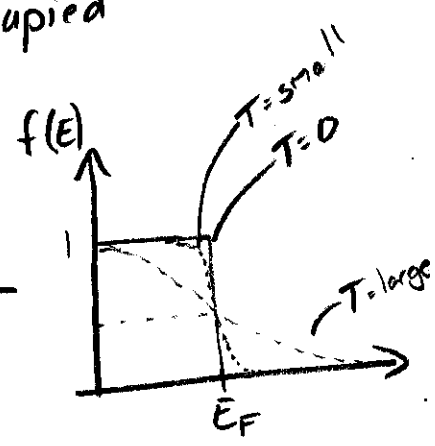
Carrier concentration

Electrons in solids obey Fermi-Dirac statistics

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

k - Boltzmann's const
T - Temp in K
E_F - Fermi energy level
$f(E)$ - Probability that an state is occupied

$$f(E_F) = \frac{1}{1+e^{(E_F-E_F)/kT}} = \frac{1}{1+e^0} = \frac{1}{1+1} = \frac{1}{2}$$



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Photon energy

$$E = \frac{hc}{\lambda}$$

Direct bandgap mtrl emits photons with

$$\lambda = \frac{hc}{E_g}$$

All semiconductor mtrl absorbs photons with

$$\lambda \leqslant \frac{hc}{E_g}$$

The electron velocity is proportional to the gradient of the energy bands in k-space

$$V = \frac{1}{\hbar} \frac{\partial E}{\partial k}$$

Effective mass is proportional to the curvature of the bands

$$m^* = \frac{\hbar^2}{\frac{\partial^2 E}{\partial k^2}}$$

Fermi-Dirac distribution

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

Density of states

Conc in conduction band

$$n_0 = \int_{E_0}^{\infty} f(E) N(E) \, dE \qquad 3\text{-}12$$

Equilibrium

$N(E)$ - Density of states

To simplify calculations an effective density of states (N_c, N_v) is defined located at the band edge

$$n_0 = N_c \cdot f(E_c) \qquad 3\text{-}13$$

For holes

$$p_0 = N_v \cdot (1 - f(E)) \qquad 3\text{-}17$$

For intrinsic material

E_F close to middle of bandgap

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \approx e^{-(E-E_F)/kT}$$

$$n_i^\circ = N_c \, e^{-\frac{E_c - E_i^\circ}{kT}} \qquad p_i^\circ = N_v \, e^{-\frac{E_i^\circ - E_v}{kT}}$$

$$n_i p_i = N_c \cdot N_v \, e^{\frac{-(E_c - E_v)}{kT}} = N_c \cdot N_v \, e^{-E_g/kT}$$

$n_i^\circ = p_i^\circ$ (created in pairs)

$$n_i^\circ = \sqrt{N_c \cdot N_v} \, e^{-\frac{E_g}{2kT}} \qquad 3\text{-}23$$

As long as the fermilevel is a few kT (~26meV) away from valence and conduction band the simplification of f(E) is correct

$$f(E) = e^{-(E-E_F)/kT}$$

$n_o p_o$ can be calculated as above

$$n_o p_o = N_c \cdot N_v e^{-E_g/kT} \qquad 3\text{-}22a$$

$$\underline{n_o p_o = n_i^2} \qquad 3\text{-}24$$

$$n_o = N_c e^{-(E_c-E_F)/kT} = n_i e^{(E_c-E_i)/kT} e^{-(E_c-E_F)/kT}$$

$$n_o = n_i e^{(E_F-E_i)/kT} \qquad 3\text{-}25a$$

$$p_o = n_i e^{(E_i-E_F)/kT} \qquad 3\text{-}25b$$

Carrier concentration is very teperature dependent

In Fig 3-17 $T = 300 \rightarrow 400K$, $n_i = 10^{10} \rightarrow 10^{13}\ cm^{-3}$

- In the presens of both acceptor and donor doping the majority concentration determines the final doping.

- For electrostatic neutral material the free carriers must balance the ionized doping atoms

$$p_0 + N_d^+ = n_0 + N_a^-$$

If $n_0 >> p_0$

$$n_0 \approx N_d - N_a$$

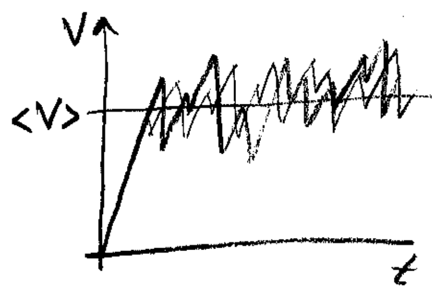
The carriers moves randomly like gas molecules

$\Rightarrow$ Diffusion   Higher conc $\rightarrow$ Lower conc

The carriers are affected by electric and magnetic fields

$\Rightarrow$ Drift

-Accelerated by the field
-Scattered against the lattice and other carriers
- Equlibrium mean velocity $\langle v \rangle$



Mobility μ

$$\mu = -\frac{\langle V \rangle}{E} \qquad \frac{cm^2}{Vs}$$

Current density

$$J = q \cdot n \cdot \mu \cdot E$$

Both electrons and holes

$$J = q\left(n \cdot \mu_n + p \cdot \mu_p\right) E = \sigma E$$

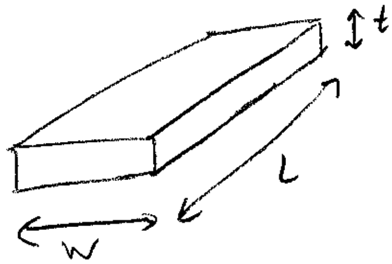
σ - Conductivity

$$\sigma = q\left(n\mu_n \cdot p\mu_p\right) = \frac{1}{\rho}$$

ρ - Resistivity

Resistance

$$R = \frac{\rho L}{wt} = \frac{L}{wt}\frac{1}{\sigma}$$

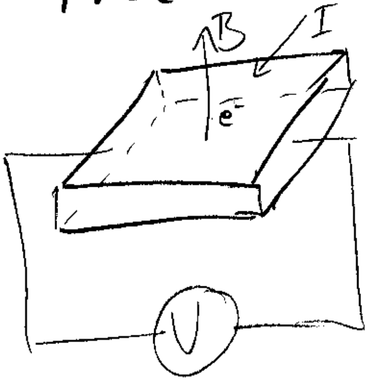


The mobility is not constant

- Decreases with doping concentration Fig 3-23
- Decreases at high temperatures Fig 3-22
- Decreases at high electrical fields
  - → The mean velocity saturates Fig 3-24 Lect3

Hall effect

The carriers are affected by a magnetic field



$$F = q(E + v \times B)$$

$$F_y = qE_y - q_v B_z$$

- Carriers moves in the y-direction
- A Voltage are built up → generates a Electrical counter-field to the magnetic field
- Stationary condition.

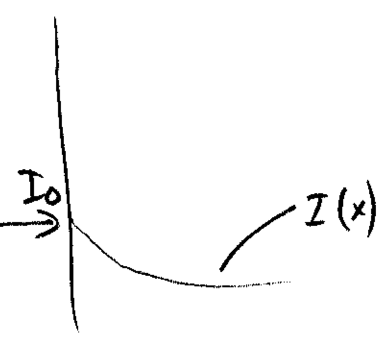
Excess Carriers in Semiconductors

- Carriers in excess of thermal equilibrium values

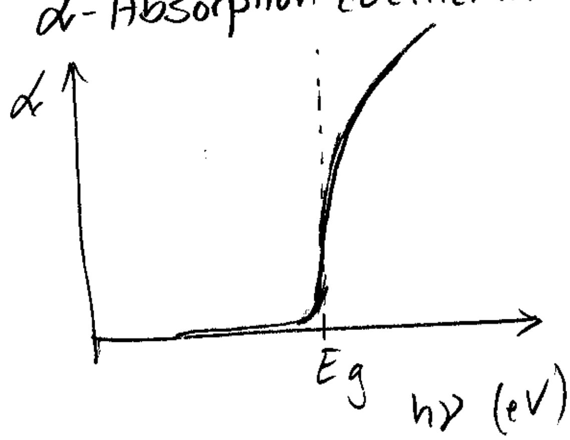
- Optical excitation
- Electron bombardment
- Injection in p-n junction diodes

Optical absorption

$$I(x) = I_0 e^{-\alpha x}$$



α - Absorption coefficient



α - Energy or wavelengd dependent $\quad \lambda = \frac{hc}{E_g}$

Semiconductor material transparent below bandgap.

Photoconductivity

$\Rightarrow$ Optical luminiscence

→ Generates excess electrons <u>and</u> holes

→ Increased conductivity

Direct recombination

Excess electrons falls back to the valence band spontaneously.

Energy is given up as a photon with energy

$$h\nu \approx E_g$$

Decay rate proportional to # electrons and # holes

$$\alpha_r \cdot n(t) p(t)$$

Net change in carrier conc

$$\frac{dn(t)}{dt} = \underbrace{\alpha_r n_i^2}_{\text{Thermal generation rate}} - \underbrace{\alpha_r n(t) p(t)}_{\text{Recombination}}$$

In excess carriers

$$\frac{d\,\delta n(t)}{dt} = \alpha_r n_i^2 - \alpha_r (n_0 + \delta n(t))(p_0 + \delta p(t)) =$$

$$\alpha_r n_i^2 - \alpha_r (\underbrace{n_0 p_0}_{n_i^2} + n_0 \delta p(t) + p_0 \delta n(t) + \delta n(t)\,\delta p(t)) =$$

$$/\delta n(t) = \delta p(t)/ = -\alpha_r (n_0 + p_0)\delta n(t) + \delta n^2(t)$$

neglect as $\delta n(t)$ is small

In p-type material

$$\frac{d\delta n(t)}{dt} = -\alpha_r p_o \delta n(t)$$

$$\delta n(t) = \Delta n\, e^{-\alpha_r p_o t} = \Delta n\, e^{-t/\tau_n}$$

Orignal excess conc

τ_n - Recombination lifetime

$$\tau_n = \frac{1}{\alpha_r p_o}$$

In n-type material

$$\tau_p = \frac{1}{\alpha_r n_o}$$

Indirect recombination

In semiconductors with indirect bandgap such as Silicon or Germanium falls back via recombination levels in the band-gap

$\Rightarrow$ Energy is given to the lattice as heat.

Impurities and lattice defect can act as trapping center if an electron or hole can be captured and sub sequently be annihilated by an opposite carrier type.

If the opposite carrier isn't trapped soon enough the trapped carrier may be thermally reexcited again.

Indirect recombination is much more difficult analyzing than direct.

Steady-state carrier generation

In many problems a steady state carrier generation is maintained

$$\frac{\partial}{\partial t} = 0$$

$$\frac{d^2 \delta n}{dx^2} = \frac{\delta n}{D_n \tau_n} \equiv \frac{\delta n}{L_n^2}$$ — Diffusion length

Represents the length where the conc is reduced to $1/e$

$$\frac{d^2 \delta p}{dx^2} = \frac{\delta p}{D_p \tau_p} \equiv \frac{\delta p}{L_p^2}$$

- A simple fermi level is not meaningful in presens of excess carriers.

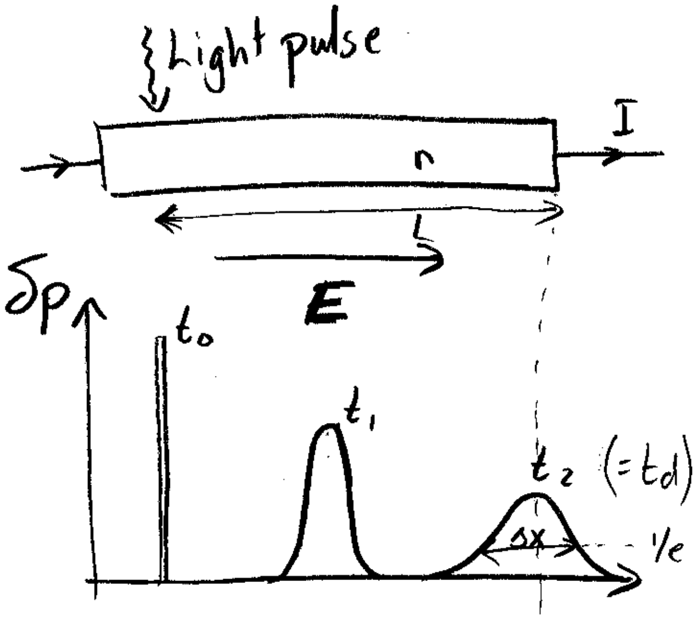
By defining separate quasi-fermi levels carrier conc eq like 3-25 can be achived

$$n = n_i e^{(F_n - E_i)/kT}$$

$$p = n_i e^{(E_i - F_p)/kT}$$

F_n and F_p electron and hole quasi fermi levels

Haynes-Shockley experiment



By measuring the current at the right contact the mobility and Diffusio coefficient can be measured simultaniously

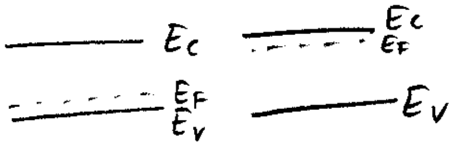
$$\mu_p = \frac{v_d}{E} = \frac{L}{t_d \cdot E}$$

$$D_p = \frac{(\Delta x)^2}{16 t_d} \quad \text{(Details in book)}$$

μ_p and D_p should fulfill einsteins relation

$$\frac{D_p}{\mu_p} = \frac{kT}{q}$$

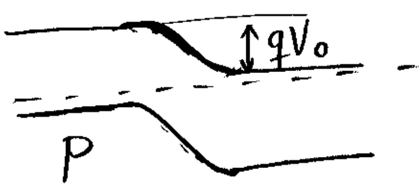
Junctions



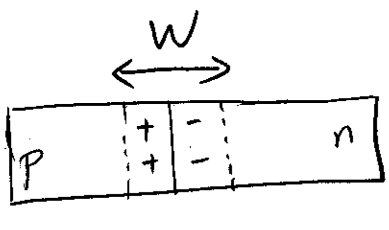
Bring together
- Diffusion of holes $p \rightarrow n$
  electrons $n \rightarrow p$
  - → elec. and holes meet and recombine
  - Fixed doping charges buid up an electrical field
  - → Drift current which exactly canceles the diffusion current

⇒ No net current at equilibrium

Fermi level is constant at equilibrium



Energy bands



W - Space charge region

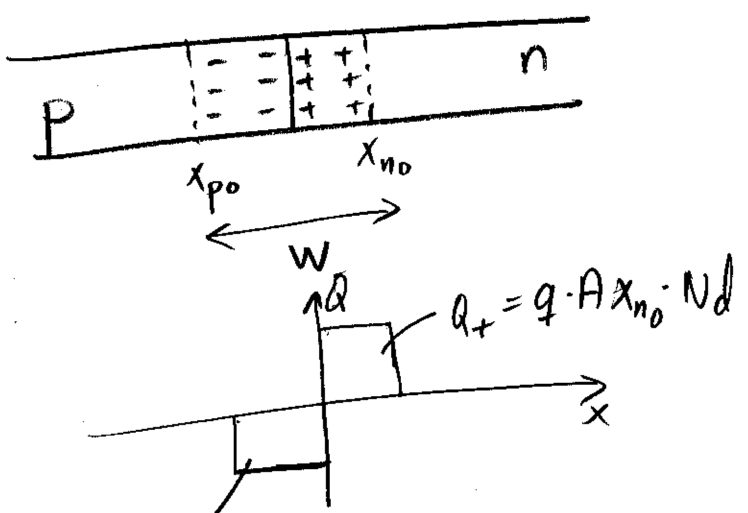
Electrostatic potential

Built in potential, V_0 (Contact potential)

- Cannot be measured by just adding probes
  - Additional contact potentials are created at the probes cancelling V_0

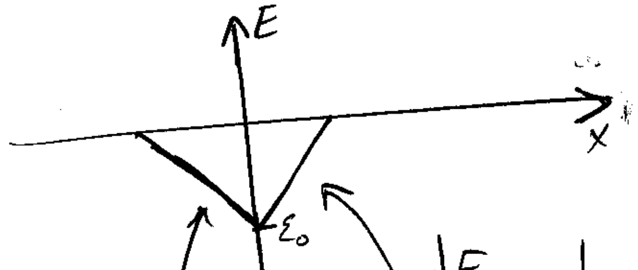
Lect 4

Space charge



$Q_- = q A x_{po} N_a$

$$Q_+ = Q_- \Rightarrow x_{no} N_d = x_{po} \cdot N_a$$



$$\frac{dE}{dx} = \frac{1}{\varepsilon}(qN_d)$$

$$\frac{dE}{dx} = \frac{1}{\varepsilon}(-qN_a)$$

From Poissons eq

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon}$$

The maximal electrical field

$$E_0 = -\frac{q}{\varepsilon} N_d x_{n0} = -\frac{q}{\varepsilon} N_a x_{p0}$$

Contact potential

$$-V_0 = \int_{-x_{p0}}^{x_{n0}} E(x)\,dx = /\text{triangle}/ = \frac{1}{2} E_0 W$$

$$V_0 = -\frac{1}{2}\frac{q}{\varepsilon} N_d x_{n0} \cdot W$$

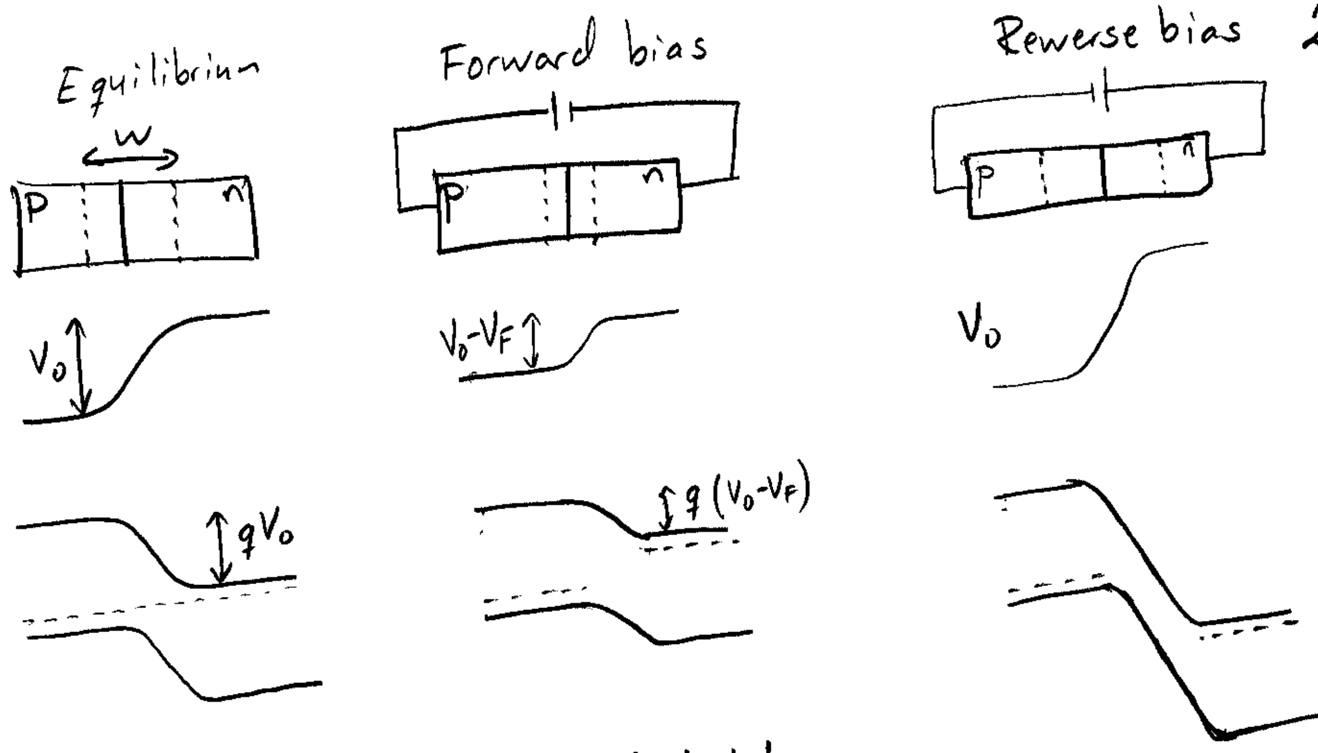
$$x_{n0} = x_{p0}\frac{N_a}{N_d} = (W - x_{n0})\frac{N_a}{N_d} = W\frac{N_a}{N_d} - x_{n0}\frac{N_a}{N_d}$$

$$\Rightarrow x_{n0}\left(1+\frac{N_a}{N_d}\right) = W\frac{N_a}{N_d} \qquad x_{n0} = \frac{W N_a}{N_d\left(1+\frac{N_a}{N_d}\right)} = \frac{W N_a}{N_d + N_a}$$

$$V_0 = -\frac{1}{2}\frac{q}{\varepsilon}\frac{N_a N_d}{N_a + N_d} W^2$$

$$W = \sqrt{\frac{2\varepsilon V_0}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)}$$

Junction capacitance $C = \frac{\varepsilon \cdot A}{W}$ Parallel plate formula



Reduced depletion

$\Rightarrow$ Diffusion current increases

$\Rightarrow$ Diffusion current reduced

EHP:s are constantly thermally and optically generated.

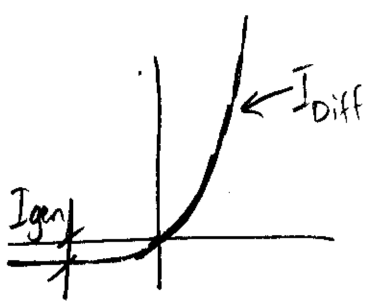
EHP:s in the depletion region drifts in the field.

$\Rightarrow$ Generation current (Drift current)

$\Rightarrow$ Rather insensitive to the potential barrier (E field). It's the number of carriers that determines the current not the velocity.

At equilibrium the diffusion and generation current cancels.

$$I = I_{diff} - |I_{gen}| = 0 \text{ for } V = 0$$



The probability for a carrier to diffuse across the junction increases exponentioally

$$\Rightarrow I = I_0 \left(e^{\frac{qV}{kT}} - 1\right)$$

The drift and diffusion current must cancel at equilibrium

$$J_p(x) = q\left(\mu_p p(x) E(x) - D_p \frac{dp(x)}{dx}\right) = 0$$

$$\underbrace{\frac{\mu_p}{D_p}}_{\frac{q}{kT}} \underbrace{E(x)}_{\frac{dV}{dx}} = \frac{1}{p(x)} \frac{dp(x)}{dx}$$

$\frac{q}{kT}$ = Einsteins relation

$$-\frac{q}{kT}\frac{dV}{dx} = \frac{1}{p(x)}\frac{dp(x)}{dx}$$

$$-\frac{q}{kT}\int_{V_p}^{V_n} dV = \int_{p_p}^{p_n} \frac{1}{p}\, dp$$

$$-\frac{q}{kT}\underbrace{(V_n - V_p)}_{V_0} = \ln p_n - \ln p_p$$

$$V_0 = \frac{kT}{q} \ln \frac{p_p}{p_n}$$

$$\frac{p_p}{p_n} = e^{\frac{qV_0}{kT}} \quad \left(= \frac{n_n}{n_p}\right) \qquad 5.10$$

Equilibrium values

Applying bias

$$\frac{p(-x_{p_0})}{p(x_{n_0})} = e^{\frac{q(V_0 - V)}{kT}} \simeq \frac{p_p}{p(x_{n_0})}$$

Majority carrier is approx constant under low injection

Non equlibrium

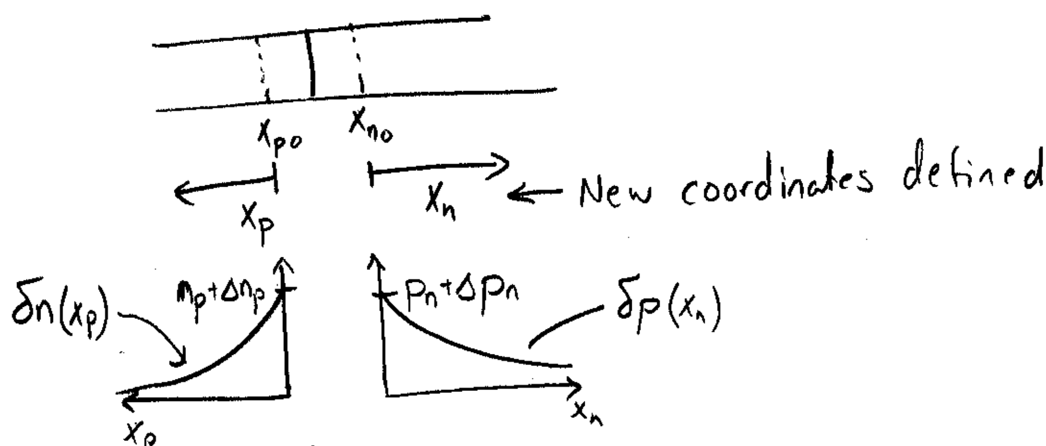
$$\frac{P_p/p_n}{P_p/p(x_{no})} = \frac{e^{\frac{qV_0}{kT}}}{e^{\frac{q(V_0-V)}{kT}}}$$

$$\frac{p(x_{no})}{p_n} = e^{\frac{qV}{kT}}$$

Excess conc at the edge of the transition region

$$\Delta P_n = p(x_{no}) - p_n = p_n\left(e^{\frac{qV}{kT}} - 1\right)$$

$$\Delta n_p = n_p\left(e^{\frac{qV}{kT}} - 1\right)$$



Assuming long n and p region:

$$\delta n(x_p) = \Delta n_p\, e^{-x_p/L_n} = n_p\left(e^{qV/kT} - 1\right) e^{-x_p/L_n}$$

$$\delta p(x_n) = \Delta p_n\, e^{-x_n/L_p} = p_n\left(e^{qV/kT} - 1\right) e^{-x_n/L_p}$$

Hole diffusion current

$$I_p(x_n) = -qAD_p \frac{d\,\delta p(x_n)}{dx_n} = qA\frac{D_p}{L_p}\Delta p_n\, e^{-x_n/L_p}$$

$$= qA\frac{D_p}{L_p}\delta p(x_n)$$

Hole current injection

$$I_p(x_n=0) = \frac{qAD_p}{L_p} p_n \left(e^{qV/kT} - 1\right)$$

elec inj.

$$I_n(x_p=0) = -\frac{qAD_n}{L_n} n_p \left(e^{qV/kT} - 1\right)$$

$-x_p$ direction

Total current

$$\underline{\underline{I = qA\left(\frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p\right)\left(e^{qV/kT} - 1\right)}}$$

Diode equation.

Both in forward and reverse direction

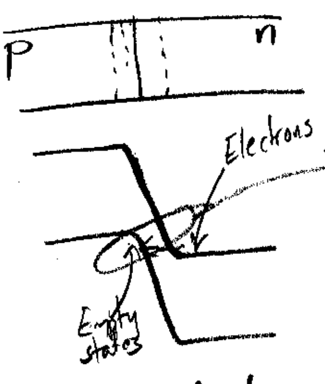
Lect 5 ⇊

High reverse bias

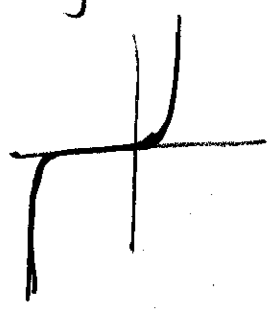
Two different cases

High doping

→ Thin depletion region

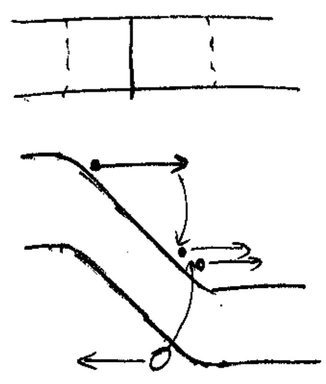


The probability of tunneling becomes significant



⇒ Zener effect

Low doping



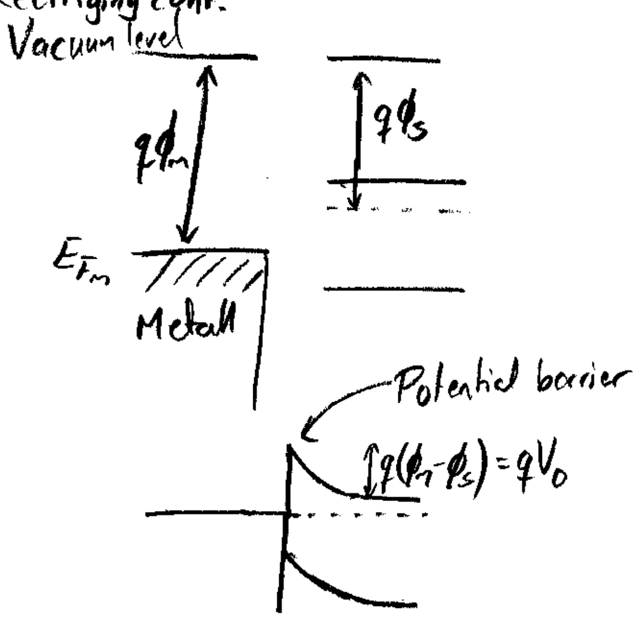
- Electron gains high energy as they drift in the field
- When the energy > E_g it is sufficient to create a new electron-hole pair
- The new carriers can create even more EHP:s

$\Rightarrow$ Avalanche effect

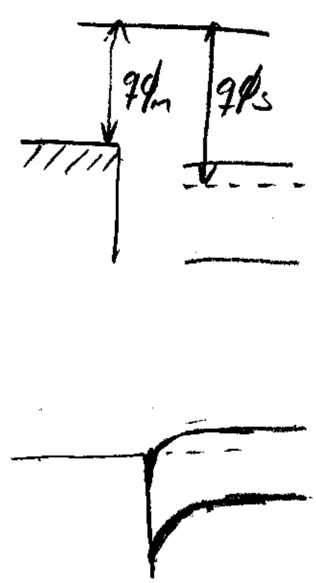
Metal - Semiconductor junctions

A metal-semiconductor junction can either be rectifying or ohmic depending on the "work-function" of the metall.

Rectifying cont.



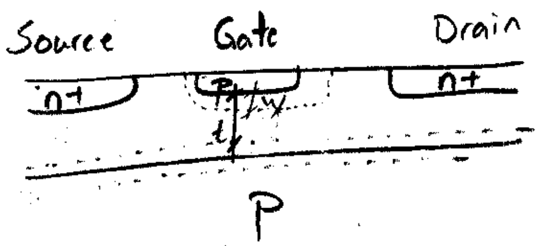
Ohmic



Transistors

- Transistors
  - Bipolar
    - NPN
    - PNP
  - Field effect, FET
    - Metal Oxide FET MOSFET
    - Junction FET JFET
    - Metal-Semiconductor MESFET

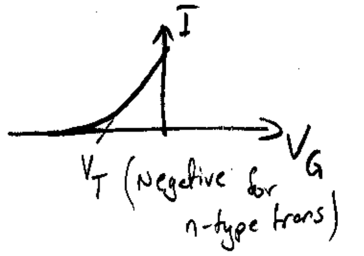
Junction FET, JFET

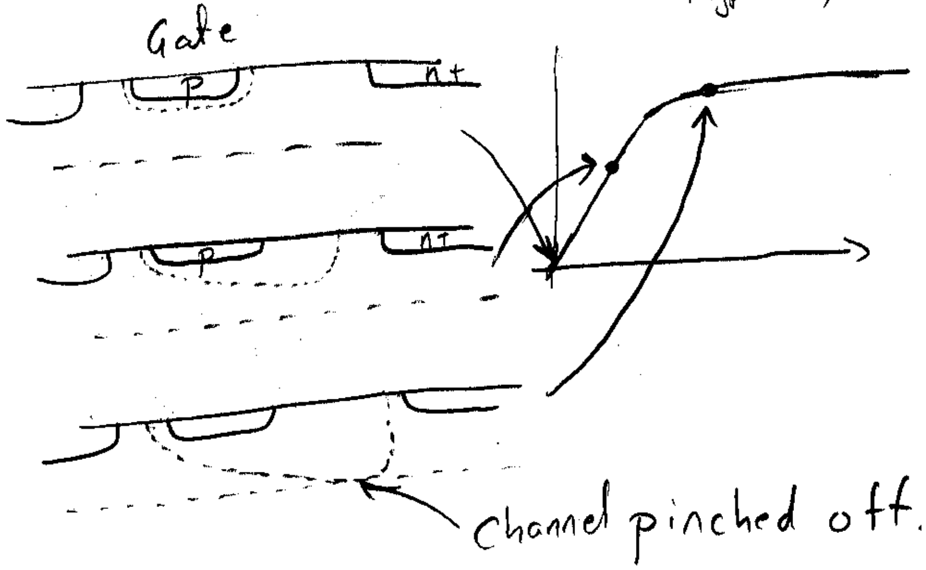


$w = f(V_G)$

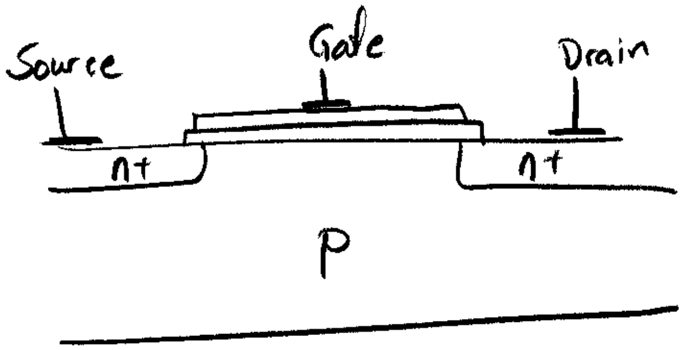
$I = qn\mu E \cdot A$ wher $A \propto t - w$

At $V_G = 0 \Rightarrow w = \text{min} \Rightarrow I = \text{max}$





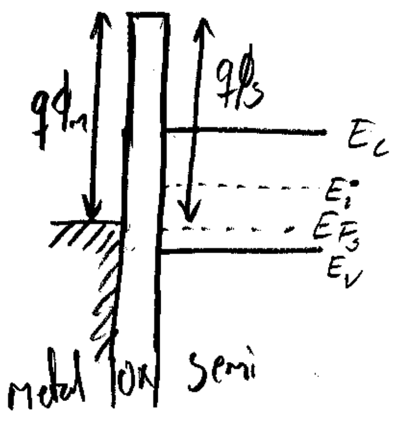
Metal Insulator Semiconductor FET MISFET
Oxide MOSFES

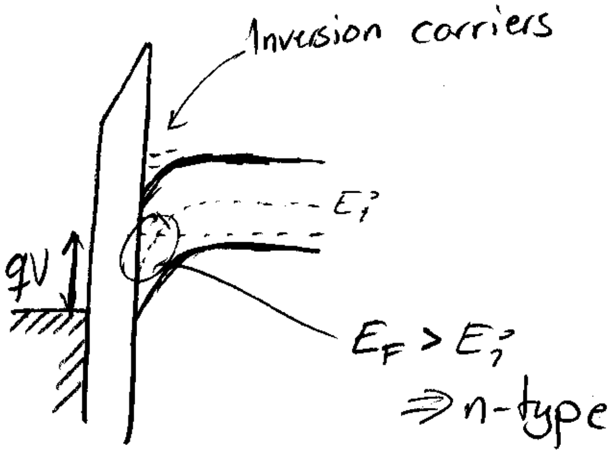


- Positive gate bias
  - Depletes the holes from the surface
  - Attracts electrons to the surface

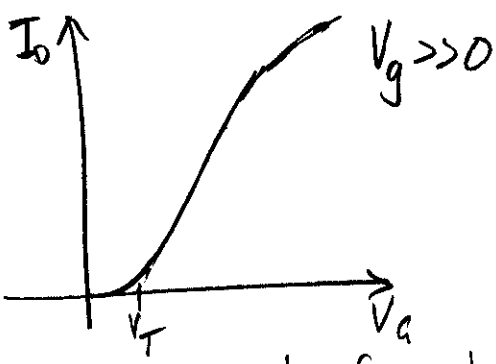
⇒ Inversion layer (p-mtrl becomes n-conductive

Band diagram



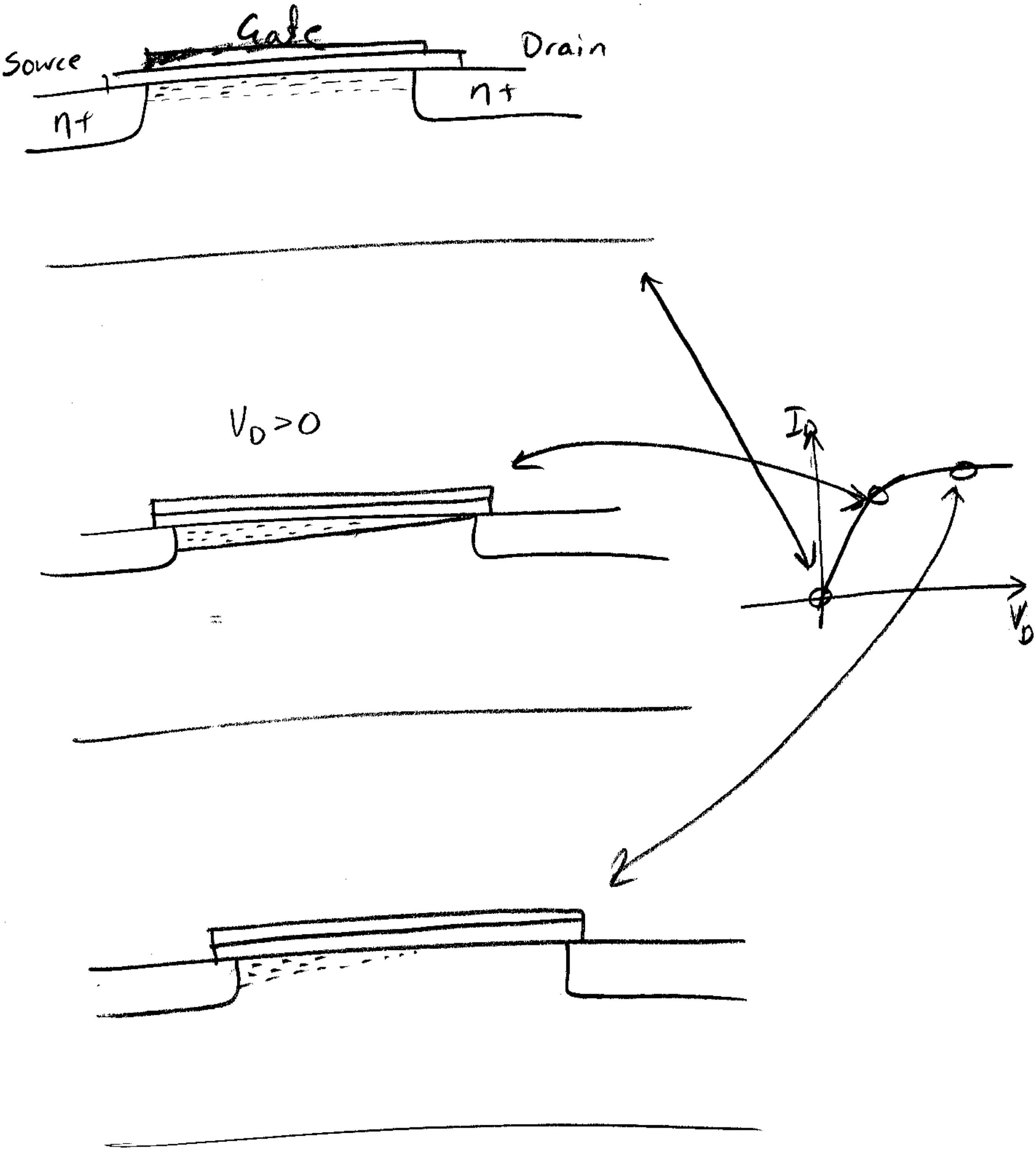


$V_g = 0$



V_T positive for n-channel

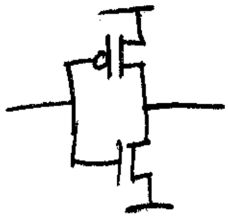
$V_G > V_T$, $V_D = 0$



MOSFETs totally dominates in digital applications

Contributing factors

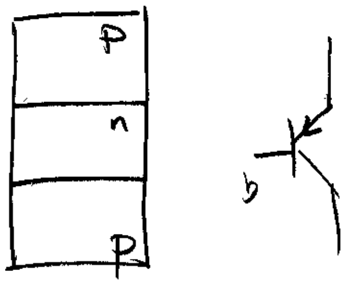
- Complementary logic CMOS
  nmos & pmos integrated
  - Allows simple logic buildig blocks

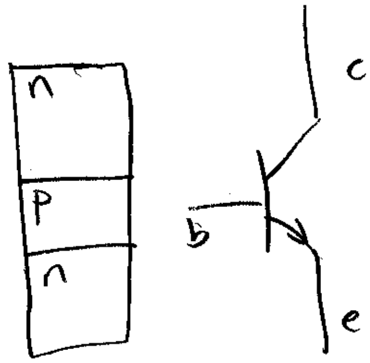


CMOS - inverters

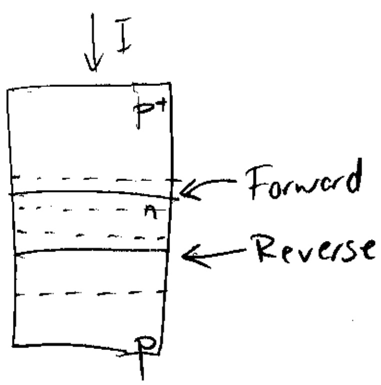
  - Scaling has allowed shrinked size and improved performance
    - However, scaling cannot be continued much longer due to fundamental limitations
      - Gate leakage
      - Source-Drain leakage current

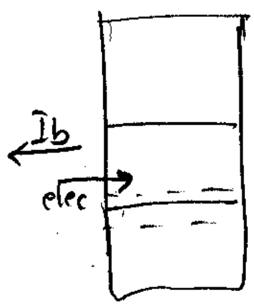
Bipolar transistor





- Two diodes coupled facing each other
- the reverse current is determined by the number of minority carriers





Electrons injected into the the

- Minority carriers are injected in the base throught the forward biased junction
  - → Increased reverse bias
  - → Small amount of injected carriers recombine in the base region
    - Must be resupplied from the base current.

Photo detector

- Reverse biased pn-junction diode

Light emitting diode

- Forward biased diode in direct bandgap material.

Semiconductor lasers

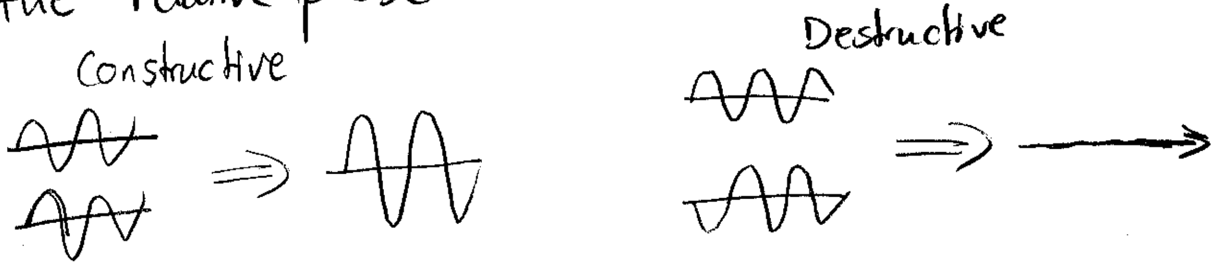
- pn-junction diode surrounded by cavity mirrors.

Optics

- Interference
- Diffraction
- Reflection in thin films
  (Transfer matrix)

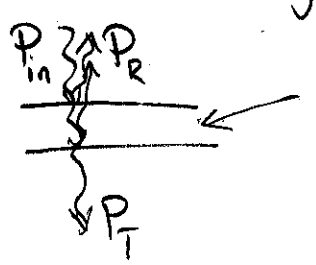
Interference

- Two waves with the same wavelength can eithe summarize or concel depending on the relative phase



Interference in thin films

Two reflecting layers



Maximal transmittanc if the two reflected waves interfere destructively

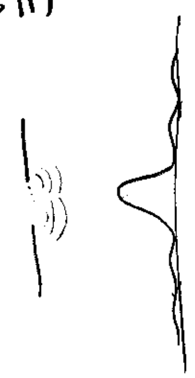
$$P_T = P_{in} - P_R$$

Diffraction

Monocromatic ray of light passing an edge or slit is spread.

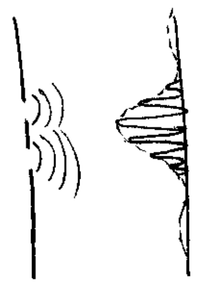
? Heygen's principle
- The wave front can be replaced by multiple point sources.

Single slit



Ons strong interference peak and additial weak peaks

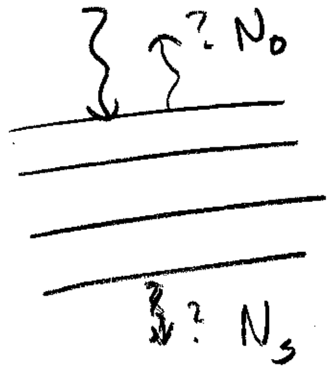
Double slit



Many slits (Bragg grating)

Theory of multilayer films

Transmission matrix (Perpendicular incidence)



For each layer in a dielectric stack a transfer matrix can be calculated such as

$$M_x = \begin{bmatrix} \cos l\delta & -\frac{i}{N}\sin l\delta \\ -iN\sin l\delta & \cos l\delta \end{bmatrix}$$

l is the layer thickness

$\delta \equiv \frac{2\pi N}{\lambda_0}$ where $N(\lambda) = n(\lambda) + ik(\lambda)$
Complex refractive index
Including absorption

The transfer matrix can be calculated by matrix multiplication

$$M = M_1 \cdot M_2 \cdot \ldots \cdot M_3 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

The transmission coefficient for the whole stack

$$t = \frac{2N_0}{A \cdot N_0 + N_0 \cdot N_s \cdot B + C + N_s \cdot D}$$

N_0 - Incidence medium (Air $N_0 = 1 + i0$)
N_s - Substrate

The transmittance

$$T = \frac{Re(N_s)}{Re(N_0)} |t|^2$$